

11.6 Exponential Equations

Solve over the set of real numbers. Express each solution to three significant digits.

$$1. \left(\frac{1}{3}\right)^{x-6} = 3^x = 3$$

$$\begin{aligned} (3^{-1})^{x-6} &= 3^x \\ 3^{-x+6} &= 3^x \\ -x+6 &= x \\ 6 &= 2x \\ 3 &= x \end{aligned}$$

$$2. \left(\frac{1}{8}\right)^x = 2^{x-6} = \frac{3}{2}$$

$$\begin{aligned} 2^{-3x} &= 2^{x-6} \\ -3x &= x-6 \\ -4x &= -6 \\ x &= \frac{6}{4} = \frac{3}{2} \end{aligned}$$

$$3. 9^{3x} = 27^{x+2} = 2$$

$$\begin{aligned} 3^{6x} &= 3^{3x+6} \\ 6x &= 3x+6 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

$$4. 9^x = 45 = 1.732$$

$$\begin{aligned} \log 9^x &= \log 45 \\ x \cdot \log 9 &= \log 45 \\ x &= \frac{\log 45}{\log 9} \\ \text{or } \log_9 45 \end{aligned}$$

$$5. 5^x = 52 = 2.455$$

$$\begin{aligned} \log 5^x &= \log 52 \\ x \log 5 &= \log 52 \\ x &= \frac{\log 52}{\log 5} \\ \text{or } \log_5 52 \end{aligned}$$

$$6. 4^{3p} = 10 = .5537$$

$$\begin{aligned} \log 4^{3p} &= \log 10 \\ 3p \log 4 &= \log 10 \\ p &= \frac{\log 10}{3 \log 4} \\ \text{or } \frac{1}{\log 64} \end{aligned}$$

$$7. 3^{n+2} = 14.5 = .4341$$

$$\begin{aligned} (n+2) \log 3 &= \log 14.5 \\ n \log 3 + 2 \log 3 &= \log 14.5 \\ n \log 3 &= \log 14.5 - 2 \log 3 \\ n &= \frac{\log 14.5 - 2 \log 3}{\log 3} \end{aligned}$$

$$8. 9^{z-4} = 6.28 = 4.836$$

$$\begin{aligned} (z-4) \log 9 &= \log 6.28 \\ z \log 9 - 4 \log 9 &= \log 6.28 \\ z \log 9 &= \log 6.28 + 4 \log 9 \\ z &= \frac{\log 6.28 + 4 \log 9}{\log 9} \end{aligned}$$

$$9. 8.2^{n-3} = 42.5 = 4.782$$

$$\begin{aligned} (n-3) \log 8.2 &= \log 42.5 \\ n \log 8.2 - 3 \log 8.2 &= \log 42.5 \\ n \log 8.2 &= \log 42.5 + 3 \log 8.2 \\ n &= \frac{\log 42.5 + 3 \log 8.2}{\log 8.2} \end{aligned}$$

$$10. 2.1^{t-5} = 9.32 = 8.009$$

$$\begin{aligned} (t-5) \log 2.1 &= \log 9.32 \\ t \log 2.1 - 5 \log 2.1 &= \log 9.32 \\ t \log 2.1 &= \log 9.32 + 5 \log 2.1 \\ t &= \frac{\log 9.32 + 5 \log 2.1}{\log 2.1} \end{aligned}$$

$$11. 5^{x^2} = 68 = \pm 1.619$$

$$\begin{aligned} x^2 \log 5 &= \log 68 \\ x^2 &= \frac{\log 68}{\log 5} \\ x &= \pm \sqrt{\frac{\log 68}{\log 5}} \end{aligned}$$

$$12. 5^x = 3^x = 0$$

$$\begin{aligned} x \log 5 &= x \log 3 \\ x \log 5 - x \log 3 &= 0 \\ x &= 0 \end{aligned}$$

$$13. 3^{x+1} = 7^{x-2} = 5.89 \quad 14. 2^{x-1} = 6^{2x+3} = -2.1 \quad 15. x = \log_4 19.5 = 2.143$$

$$(x+1) \log 3 = (x-2) \log 7 \quad (x-1) \log 2 = 2x+3(\log 6) \quad x = \frac{\log 19.5}{\log 4}$$

$$x \log 3 + \log 3 = x \log 7 - 2 \log 7 \quad x \log 2 - \log 2 = 2x \log 6 + 3 \log 6$$

$$x \log 3 - x \log 7 = -2 \log 7 - \log 3 \quad x \log 2 - 2x \log 6 = 3 \log 6 + \log 2$$

$$x(\log 3 - \log 7) = -2 \log 7 - \log 3 \quad x(\log 2 - 2 \log 6) = 3 \log 6 + \log 2$$

$$x = \frac{-2 \log 7 - \log 3}{\log 3 - \log 7}$$

$$x = \frac{3 \log 6 + \log 2}{\log 2 - 2 \log 6}$$

$$16. \log_{27} \frac{1}{3} = x = -\frac{1}{3}$$

$$17. 5a^{\frac{2}{5}} = 15.35 = 16.514$$

$$18. 3x^{\frac{4}{3}} = 21.3 = 4.350$$

$$27^x = \frac{1}{3}$$

$$a^{\frac{2}{5}} = 3.07$$

$$x^{\frac{4}{3}} = 7.1$$

$$3^{3x} = 3^{-1}$$

$$a = 3.07^{\frac{5}{2}}$$

$$x = 7.1^{\frac{3}{4}}$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

Homework: page 639 #16, 17, 19-21, 23, 25-30, 32, 35, 36