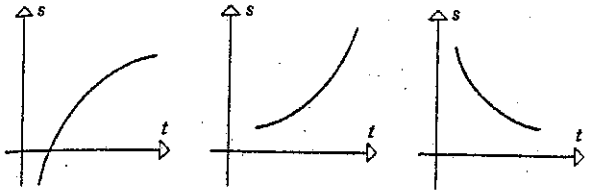
Velocity Unit

Problem Set 7 Q1

1. The graphs of three position functions are shown in Figure 4.11.4. In each case determine the signs of the velocity and acceleration, then determine whether the particle is speeding up or slowing down.

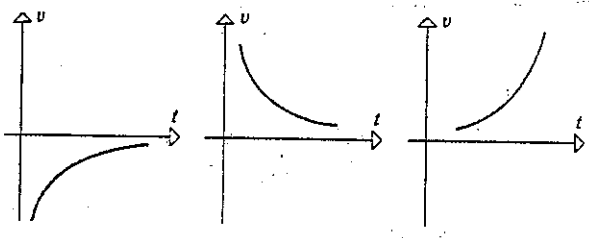


(a) (b) (c)

Figure 4.11.4

	velocity	acceleration	speed
a.			
b.			
c.			

2. The graphs of three velocity functions are shown in Figure 4.11.5. In each case determine the sign of the acceleration, then determine whether the particle is speeding up or slowing down.



(a) (b) (c)

Figure 4.11.5

	acceleration	speed
a.		
b.		
c.		

3. The position function of a particle moving on a coordinate line is shown in Figure 4.11.6.
   (a) Is the particle moving left or right at time t_0?
   (b) Is the acceleration positive or negative at time t_0?
   (c) Is the particle speeding up or slowing down at time t_0?
   (d) Is the particle speeding up or slowing down at time t_1?

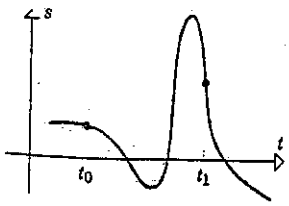
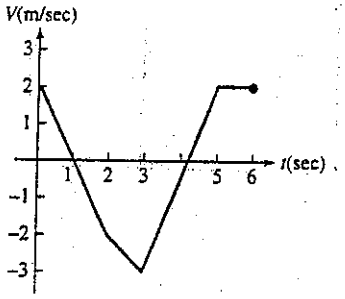


Figure 4.11.6

4. The graph shows the velocity $v = f(t)$ of a particle moving along a horizontal coordinate axis.

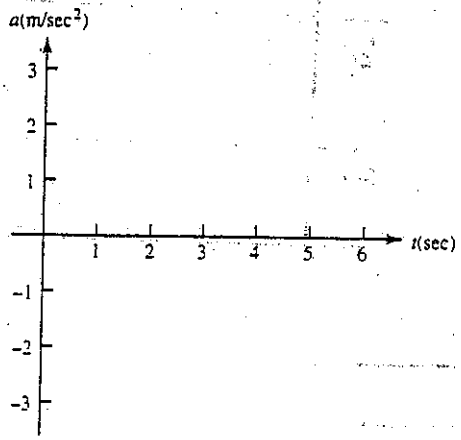


(a) When does the particle reverse direction? _____

(b) When is the particle moving at a constant speed? _____

(c) When is the particle moving at its greatest speed? _____

(d) Graph the acceleration (where defined).



5. A particle moves along a vertical coordinate axis so that its position at any time $t \geq 0$ is given by the function $s(t) = \frac{1}{3}t^3 - 3t^2 + 8t - 4$, where s is measured in centimeters and t is measured in seconds.

(a) Find the displacement during the first 6 seconds.

_____

(b) Find the average velocity during the first 6 seconds.

_____

(c) Find expressions for the velocity and acceleration at time t.

$v(t) =$ _____ $a(t) =$ _____

(d) For what values of t is the particle moving downward?

_____

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6. The position (x-coordinate) of a particle moving on the line $y = 2$ is given by $s(t) = 2t^3 - 13t^2 + 22t - 5$ where t is time in seconds.
   a) ~~Describe the motion of the particle for $t \geq 0$.~~
   b) When does the particle speed up? Slow down?
   c) When does the particle change direction?
   d) When is the particle at rest?
   e) ~~Describe the velocity and speed of the particle.~~
   f) When is the particle at the point (5, 2)?

7. A 45-caliber bullet fired straight up from the surface of the moon would reach a height of $s = 832t - 2.6t^2$ feet after t seconds. On Earth, in the absence of air, its height would be $s = 832t - 16t^2$ feet after t seconds. How long would it take the bullet to get back down in each case?

8. A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec (about 86 km/h) reaches a height of $s = 24t - 0.8t^2$ meters in t seconds.
   a) Find the rock's velocity and acceleration as a function of time. (The acceleration in this case is the acceleration of gravity on the moon.)
   b) How long did it take the rock to reach its highest point?
   c) How high did the rock go?
   d) How long did it take the rock to reach half its maximum height?
   e) How long was the rock aloft?

a) $V =$
$a =$

9. Figure 3.1.17 shows the position versus time curve for a certain particle moving on a straight line.
   (a) Is the particle moving faster at time t_0 or time t_2? Explain.
   (b) At the origin, the tangent is horizontal. What does this tell us about the initial velocity of the particle?
   (c) Is the particle speeding up or slowing down in the interval $[t_0, t_1]$? Explain.
   (d) Is the particle speeding up or slowing down in the interval $[t_1, t_2]$? Explain.

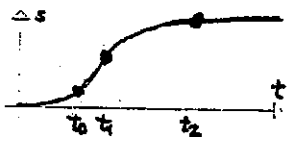


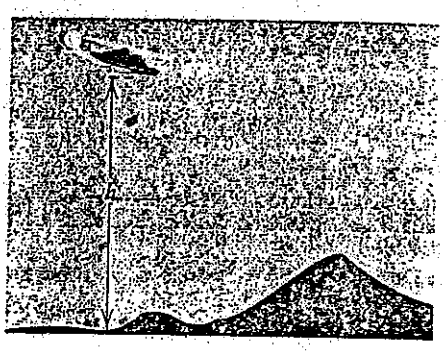
Figure 3.1.17

a)

b)

10. Emergency food supplies are dropped from a helicopter and hit the ground 10 sec later.
   (a) What is the height h of the helicopter?

Exercise 10



   (b) The box in which the supplies are packed is strong enough to withstand a speed of 180 mi/hr on impact. Will the supplies be intact?
   (c) What is the maximum height at which the helicopter can be positioned to guarantee that the box will not break up when it hits the ground?

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Exer. 11 – 12: A projectile is fired directly upward with an initial velocity of v_0 ft/sec, and its height (in feet) above the ground after t seconds is given by $s(t)$. Find (a) the velocity and acceleration after t seconds, (b) the maximum height, and (c) the duration of the flight.

11 $v_0 = 144$: $s(t) = 144t - 16t^2$

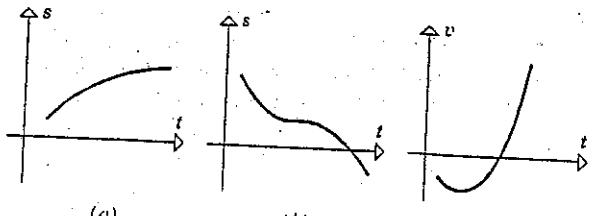
12 $v_0 = 192$: $s(t) = 100 + 192t - 16t^2$

13 A cork bobs up and down in a lake. The distance from the bottom of the lake to the center of the cork at time $t \geq 0$ is given by $s(t) = \cos \pi t + 12$, where $s(t)$ is in inches and t is in seconds (see figure).

(a) Find the velocity of the cork at $t = 0, \frac{1}{2}, 1, \frac{3}{2}$, and 2.

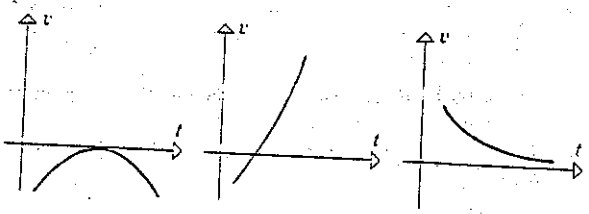
(b) During what time intervals is the cork rising?

14. Match the graphs of the position functions shown in Figure 4.11.7 with their velocity functions shown in Figure 4.11.8.



(a) (b) (c)

Figure 4.11.7



(a) (b) (c)

Figure 4.11.8

15. Figure 4.11.9 shows the velocity versus time graph for a test run of the Grand Prix GTP. Using this graph, estimate

(a) the acceleration at 60 mi/hr (in units of ft/sec^2)

(b) the time at which the maximum acceleration occurs.

[Data from *Car and Driver*, October 1990.]

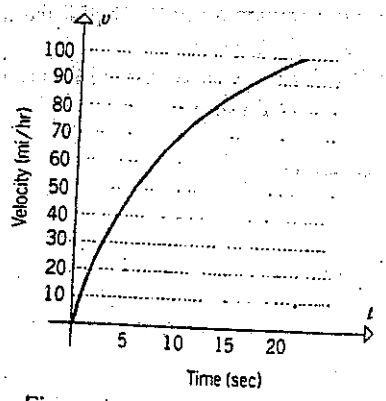


Figure 4.11.9

16. Let $s = t^3 - 6t^2 + 1$.
    (a) Find s and v when $a = 0$.
    (b) Find s and a when $v = 0$.

a) s _____ v _____ b) s _____ v _____

17. Let $s = 4t^{3/2} - 3t^2$ for $t > 0$.
    (a) Find s and v when $a = 0$.
    (b) Find s and a when $v = 0$.

a) s _____ v _____ b) s _____ v _____

18. During the first 40 sec of a rocket flight, the rocket is propelled straight up so that in t sec it reaches a height of $s = 5t^3$ ft.
    (a) How high does the rocket travel in 40 sec?
    (b) What is the average velocity of the rocket during the first 40 sec?
    (c) What is the average velocity of the rocket during the first 135 ft of its flight?
    (d) What is the instantaneous velocity of the rocket at the end of 40 sec?

a)

b)

c)

d)

19. A car is traveling on a straight road that is 120 mi long. For the first 100 mi the car travels at an average velocity of 50 mi/hr. Show that no matter how fast the car travels for the final 20 mi it cannot bring the average velocity up to 60 mi/hr for the entire trip.

20. If $s = t/(t^2 + 5)$ is the position function of a moving particle for $t \geq 0$, at what instant of time will the particle start to reverse its direction of motion, and where is it at that instant?

21. A particle moves on a line away from its initial position so that after t hr it is $s = 3t^2 + t$ mi from its initial position.
    (a) Find the average velocity of the particle over the interval $[1, 3]$.
    (b) Find the instantaneous velocity at $t = 1$.

a)

b)

22. A particle moves in one direction along a straight line so that after t min its distance is $s = 6t^4$ ft from the origin.
    (a) Find the average velocity of the particle over the interval $[2, 4]$.
    (b) Find the instantaneous velocity at $t = 2$.

a)

b)

23. (a) Let $s(t) = t^3 - 6t^2$. Make a table showing the position, velocity, speed, and acceleration at times $t = 1$, $t = 2$, $t = 3$, $t = 4$, and $t = 5$.
(b) At each of these times specify the direction of motion, if any, and whether the particle is speeding up, slowing down, or neither.

24. Let $s = \frac{100}{t^2 + 12}$ for $t \geq 0$. Find the maximum speed of the particle and the direction of motion of the particle when it has this speed.

Let $s = 5t^2 - 22t$.
(a) Find the maximum speed of the particle during the time interval $1 \leq t \leq 3$.
(b) When, during the time interval $1 \leq t \leq 3$, is the particle farthest from the origin? What is its position at that instant?

a)

25. If a particle moves at constant velocity, what can you say about its position versus time curve?

23)

t	s(t)	v(t)	a(t)	speed	direction	speeding up / slowing down / neither
1						
2						
3						
4						
5						