Question 1

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

(a) $v(5.5) = -0.45337$, $a(5.5) = -1.35851$ The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.	2 : conclusion with reason
(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$	2 : { 1 : integral 1 : answer
(c) Distance $= \int_0^6 \lvert v(t)\rvert\,dt = 12.573$	2 : { 1 : integral 1 : answer
(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$. $v(t)$ changes sign from positive to negative at time $t = b$. $x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135	3 : { 1 : considers $v(t) = 0$ 1 : integral 1 : answer

Question 3

The wind chill is the temperature, in degrees Fahrenheit (°F), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is 32°F, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant 32°F. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286

When $v = 20$ mph, the wind chill is decreasing at 0.286 °F/mph.

$2: \begin{cases} 1: \text{value} \\ 1: \text{explanation} \end{cases}$

(b) The average rate of change of W over the interval $5 \le v \le 60$ is $\dfrac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254.

$W'(v) = \dfrac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.

$3: \begin{cases} 1: \text{average rate of change} \\ 1: W'(v) = \text{average rate of change} \\ 1: \text{value of } v \end{cases}$

(c) $\left.\dfrac{dW}{dt}\right|_{t=3} = \left.\left(\dfrac{dW}{dv} \cdot \dfrac{dv}{dt}\right)\right|_{t=3} = W'(35) \cdot 5 = -0.892$ °F/hr

OR

$W = 55.6 - 22.1(20 + 5t)^{0.16}$

$\left.\dfrac{dW}{dt}\right|_{t=3} = -0.892$ °F/hr

$3: \begin{cases} 1: \dfrac{dv}{dt} = 5 \\ 1: \text{uses } v(3) = 35, \text{ or uses } v(t) = 20 + 5t \\ 1: \text{answer} \end{cases}$

Units of °F/mph in (a) and °F/hr in (c)

1 : units in (a) and (c)

Question 3

Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

(a) Find Mighty's profit on the sale of a 25-meter cable.

(b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\,dx$ in the context of this problem.

(c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

(d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

(a) Profit $= 120 \cdot 25 - \int_0^{25} 6\sqrt{x}\,dx = 2500$ dollars	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) $\int_{25}^{30} 6\sqrt{x}\,dx$ is the difference in cost, in dollars, of producing a cable of length 30 meters and a cable of length 25 meters.	1 : answer with units
(c) Profit $= 120k - \int_0^{k} 6\sqrt{x}\,dx$ dollars	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{expression} \end{cases}$
(d) Let $P(k)$ be the profit for a cable of length k. $P'(k) = 120 - 6\sqrt{k} = 0$ when $k = 400$. This is the only critical point for P, and P' changes from positive to negative at $k = 400$. Therefore, the maximum profit is $P(400) = 16{,}000$ dollars.	$4 : \begin{cases} 1 : P'(k) = 0 \\ 1 : k = 400 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$