Geometric:

Algebraic $\mathbf{u} - \mathbf{v} = \langle u_1 - v_1, u_2 - v_2 \rangle$

Ex: Let $\mathbf{m} = \langle -1, 5 \rangle$, $\mathbf{n} = \langle 4, -2 \rangle$

1. Find $-3\mathbf{m}$
2. Find $\mathbf{n} - \mathbf{m}$
3. Find $-2\mathbf{m} + \mathbf{n}$

Unit Vector:

The unit vector **u** has length 1 and the same direction as vector **v**.

To find: divide **v** by its' length $\frac{v}{\| v \|}$.

Ex: $\mathbf{v} = \langle -7, 8 \rangle$

Unit vector: $\left\langle \frac{-7}{\sqrt{113}}, \frac{8}{\sqrt{113}} \right\rangle$

Standard Unit Vectors $\langle 1, 0 \rangle$ and $\langle 0, 1 \rangle$

We always use $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$

$$\begin{aligned} v &= \langle v_1, v_2 \rangle \\ &= v_1 \langle 1, 0 \rangle + v_2 \langle 0, 1 \rangle \\ &= v_1\mathbf{i} + v_2\mathbf{j} \end{aligned}$$

$v = v_1\mathbf{i} + v_2\mathbf{j}$ is called a <u>linear combination</u>.

To write Linear Combination:

1) Write the vector in component form
2) Use **i** and **j** to write the equation

Ex: **v** = vector from (1, 4) to (-3, 6), write as a linear combination

component form $\langle -4, 2 \rangle$

linear combination $-4i + 2j$

Ex: **u** = -2**i** – 6**j** and v = -4**i** + 2**j**

Find 3**u** + 2**v**: $u = \langle -2, -6 \rangle$ $v = \langle -4, 2 \rangle$

$\langle -14, -14 \rangle$

You could solve by converting back to component form but it is not necessary.

Ex: unit vector **w** = -5**i** – 3**j** $\langle -5, -3 \rangle$ $\sqrt{(-5)^2 + (-3)^2} = \sqrt{34}$

$\left\langle \frac{-5}{\sqrt{34}}, \frac{-3}{\sqrt{34}} \right\rangle$

Show **u** + **v** graphically with a vector from (1,3) to (-3,-4) and a vector from (2,-2) to (4,-5). Move to standard position first.

Algebraically:

Direction Angles:

If u is a unit vector and θ is the angle (counter-clockwise) from the x-axis, then **u** = $\langle \cos\theta, \sin\theta \rangle$ because its' terminal point is on the unit circle.

θ = direction angle

to find θ remember

unit vector = $\langle \cos\theta, \sin\theta \rangle$

$\tan\theta = \frac{y}{x}$

$$\frac{v}{\|v\|} = \langle \cos\theta, \sin\theta \rangle$$

$$\mathbf{v} = \|v\| \langle \cos\theta, \sin\theta \rangle \text{ or } \mathbf{v} = \|v\| [(\cos\theta)\mathbf{i} + (\sin\theta)\mathbf{j}]$$

To find Directional Angles:

1) Put the vector in compound form or as a linear combination
2) Find tan θ
3) Find θ

Ex: Find the direction angle for:

A: $\mathbf{v} = 6\mathbf{i} + 6\mathbf{j}$ or $< 6, 6 >$

This is Quadrant I

$\tan\theta = \frac{6}{6}$ $\quad \|v\| = \sqrt{6^2+6^2} = \sqrt{72}$

$\tan\theta = 1$

$\theta = \pi/4$ $\quad v = \sqrt{72} < \cos \pi/4, \sin \pi/4 >$

B: $\mathbf{v} = 2\mathbf{i} - 5\mathbf{j}$ or $< 2, -5 >$

Quadrant IV

$\tan\theta = \frac{-5}{2}$

$\arctan(-5/2)$ $\quad v = \sqrt{29} < \cos(\arctan^{-5/2}), \sin \arctan^{-5/2})$

$\|v\| = \sqrt{29}$

Dot Product – different from vector addition and scalar multiplication because in those you get a vector answer & in this you get a scalar answer.

Definition: Dot product of $\mathbf{u} = < u_1, u_2 >$ and $\mathbf{v} = < v_1, v_2 >$

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2$$

Properties:

1) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2) $0 \cdot \mathbf{v} = 0$
3) $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
4) $\mathbf{v} \cdot \mathbf{v} = \|v\|^2$
5) $k(\mathbf{u} \cdot \mathbf{v}) = k\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot k\mathbf{v}$

Ex: $< 6,8 > \cdot < 1,2 > =$ $\quad 6 + 16 = 22$
Ex: $< 3,-5 > \cdot < 3,2 > =$ $\quad 9 + -10 = -1$
Ex: $< 0, 4 > \cdot < 2,1 > =$ $\quad 0 + 4 = 4$