Unit 7 Notes
Parabolas:
Ex: reflectors, microphones, (football game), (Davinci) satellites.

Light placed where rays will reflect parallel. This point is the focus.

Parabola – set of all points in a plane that are equidistant from the focus and a line called the directrix.

use point (5, 3) as focus. $\sqrt{(x_1-5)^2+(y_1-3)^2} = \sqrt{(x_1-x_1)^2+(y_1+1)^2}$

Directrix y = -1 and pt(x, y)

Or Notes-

The line segment through the focus and perpendicular to the axis of symmetry whose endpoints are on the parabola is called the **latus rectum**.

x = h Axis of symmetry

length = $|\frac{1}{a}|$ units.

Graph $y = \frac{1}{8}(x-1)^2 + 4$

v = (1, 4)

x = 1

Focus: $(h, k + \frac{1}{4 \cdot \frac{1}{8}}) = (1, 6)$

Directrix y = 2

L.R. = 8 units

Ex: $y = \frac{1}{10}(x+2)^2 - 3$

We can have parabolas that open sideways too (inverses)

$x = a(y-k)^2 + h$ *notice y = k is the A.S.

Focus $(h + \frac{1}{4a}, k)$ Directrix $x = h - \frac{1}{4a}$

Standard Form:	$y = a(x-h)^2 + k$	$x = a(y-k)^2 + h$
Book Uses:	$\mathbf{(x-h)^2 = 4p(y-k)}$	$\mathbf{(y-k)^2 = 4p(x-h)}$
	$\mathbf{p} = \frac{1}{4a}$	
Vertex	(h, k)	(h, k)
Axis of symmetry	x = h	y = k
Focus	$(h, k + \frac{1}{4a})$	$(h + \frac{1}{4a}, k)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
	a > 0 up	a > 0 right
	a < 0 down	a < 0 left
L.R.	$\left\|\frac{1}{a}\right\|$	$\left\|\frac{1}{a}\right\|$

Graph: $6x = y^2 + 6y + 33$

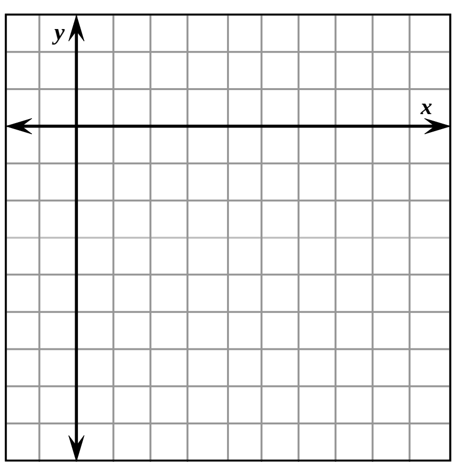
$6x = (y + 3)^2 + 24$

$x = \frac{1}{6}(y + 3)^2 + 4$

* Vertex (,   )   A.S.

Focus (,   )   L.R. =

Directrix:

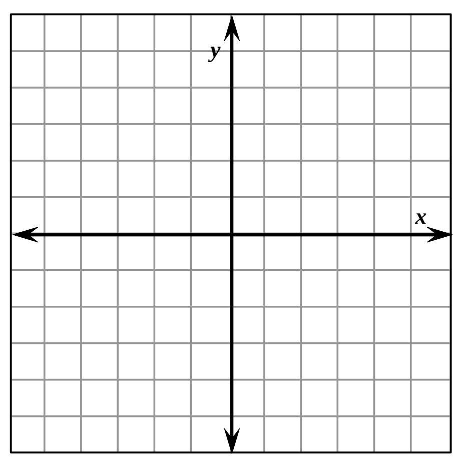


Graph $\frac{1}{2}(y + 1) = (x - 4)^2 - 1$

Vertex (,   )   A.S.

Focus (,   )   L.R =

Directrix:



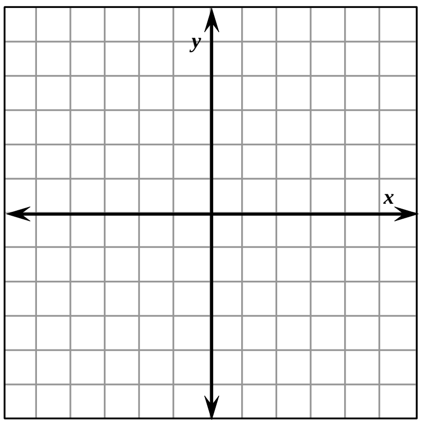
Given the focus and directrix write the equation with a Focus (4, -4) and a directrix of y = 6.

Given the focus and vertex write the equation with a Focus (3, -4) and a vertex (3, 6).

Find each of the following and graph the parabola.

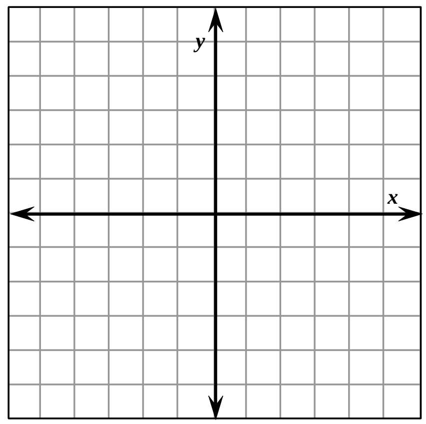
1) $y = -2x^2 + 8x - 3$

Vertex:
Focus:
Axis of Symmetry:
Directrix:
Direction of Opening:



2) $x = -\frac{1}{2}y^2 + 2y + 4$

Vertex:
Focus:
Axis of Symmetry:
Directrix:
Direction of Opening:



Reflective Property of a Parabola: The tangent line to a parabola at point P makes equal angles with the following two lines:

1. The line passing through P and the focus
2. The axis of the parabola

Eccentricty of a Parabola: The eccentricity (how much it deviates from being circular) of a parabola is 1.

Circles- The set of all points in a plane that are equidistant from a given point called the center.

$(x - h)^2 + (y - k)^2 = r^2$

center (h, k) radius = r

Ex: $x^2 + y^2 + 2x - 12y = 35$

Ex: $3x^2 + 3y^2 + 6y + 6x = 2$

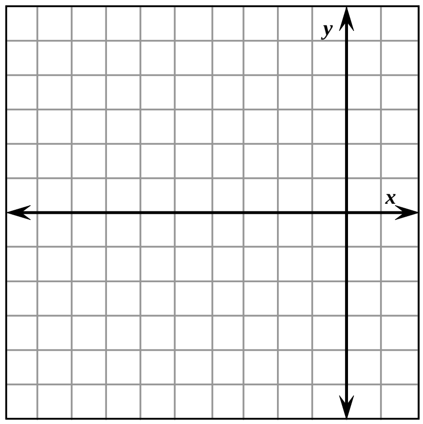
Ex: Write the equation of the circle whose diameter has end pts (3, 5) and (6, 1).

Diameter = Radius =

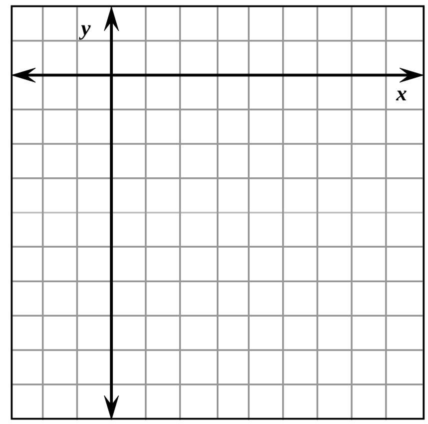
Center (,)

Graph each of the following circles.

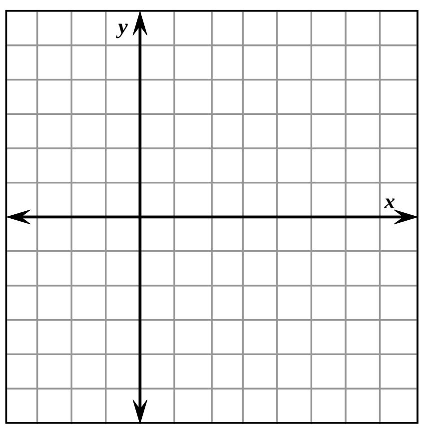
1. $x^2 + 12x + y^2 + 2y = -28$



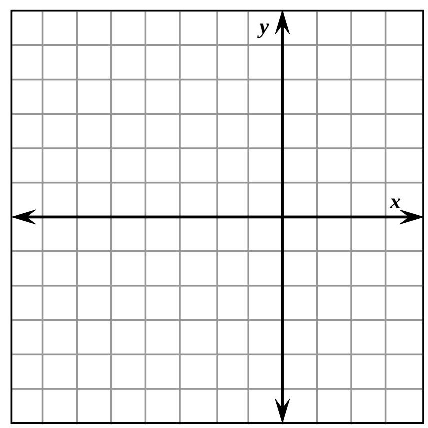
2. $x^2 + y^2 + 8y - 10x + 16 = 0$



3. $x^2 + y^2 - 6y - 16 = 0$



4. $(x + 3)^2 + y^2 = 16$



Ellipse: An ellipse is the set of all points in a plane such that the sum of the distances from two given points (foci) is constant.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Horizontal Vertical

Key $\mathbf{a^2 > b^2}$

If $a^2 = b^2$ then it is a circle

Center (h, k)

Major axis = 2a longer one

Minor axis = 2b

Foci $\mathbf{a^2 - b^2 = c^2}$

c units from center on major axis.

Write the equation in standard form..

Ex: $9x^2 + 25y^2 = 225$

Ex: $x^2 + 9y^2 - 4x + 54y + 49 = 0$

Ex: $x^2 + 25y^2 - 8x + 100y + 91 = 0$

Write the equation:

Ex: The endpoints of major axis (2, 12) & (2, -4)
Endpoints of minor axis are ((4, 4) & (0, 4)

Ex: Foci are at (12, 0) & (-12, 0).
The endpoints of the minor axis are (0, 5) & (0, -5).

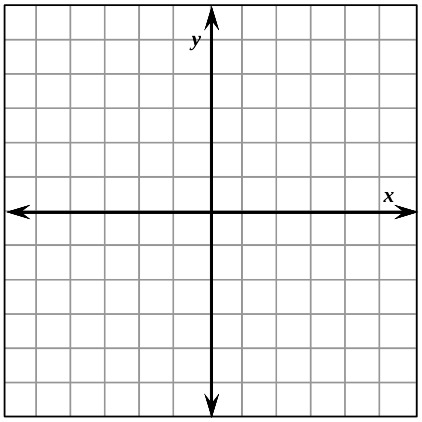
Ex: $64x^2 + 9y^2 = 576$

Ex: $16y^2 + 9x^2 - 96y - 90x + 225 = 0$

Graph:

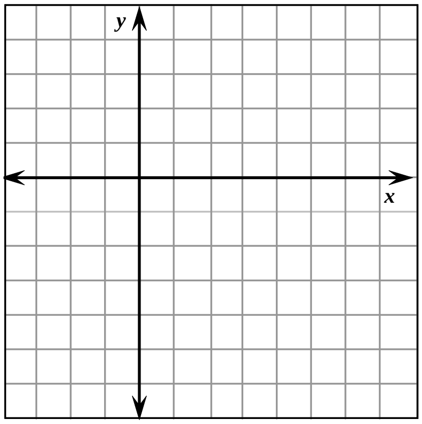
Ex 1: $9x^2 + 25y^2 = 225$

Center:
Vertices:
Foci:



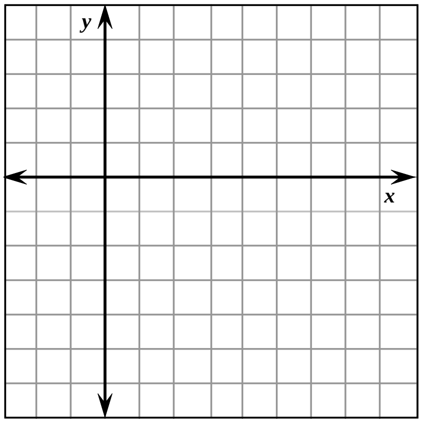
Ex 2: $x^2 + 9y^2 - 4x + 54y + 49 = 0$

Center:
Vertices:
Foci:



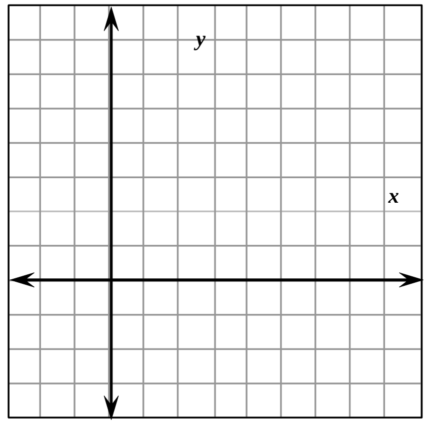
Ex 3: $x^2 + 25y^2 - 8x + 100y + 91 = 0$

Center:
Vertices:
Foci:



Ex 4: $16y^2 + 9x^2 - 96y - 90x + 225 = 0$

Center:
Vertices:
Foci:



Hyperbola- Set of all points in a plane such that the absolute value of the difference of the distance from any point on the Hyperbola to two given points (foci) is constant.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Center (h, k) **key** $\mathbf{a^2}$ **comes first.**

Vertices- a units from center

Asymptotes- as a hyperbola recedes from the center the branches approach lines called asymptotes.

Transverse axis = 2a
Conjugate axis= 2b

Foci $\mathbf{a^2 + b^2 = c^2}$

Ex: $\frac{x^2}{25} - \frac{y^2}{49} = 1$

Ex: $25x^2 - 4y^2 + 100x + 24y - 36 = 0$

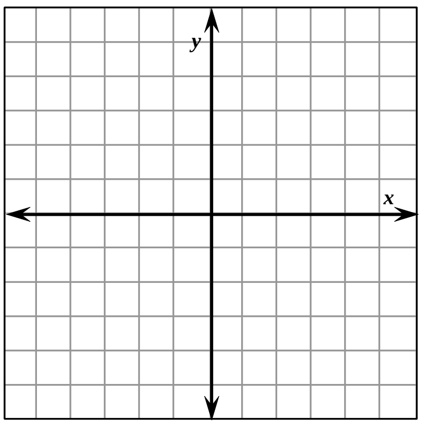
Ex: $y^2 - 4x^2 + 6y + 8x = 59$

Ex: $16x^2 - y^2 + 96x + 8y + 112 = 0$

Ex: $144y^2 - 25x^2 - 576y - 150x = 3{,}249$

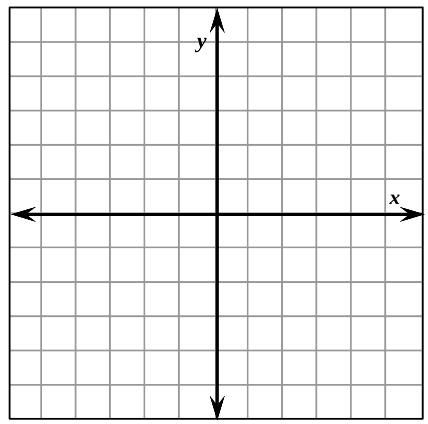
Ex1: $\frac{x^2}{4} - \frac{y^2}{9} = 1$

Vertical or Horizontal:
Center: Vertices:
Foci:
Asymptotes:



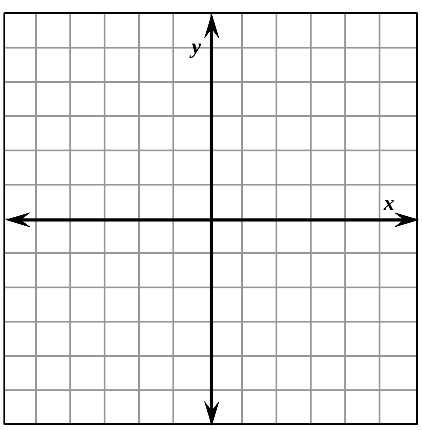
Ex 2: $\frac{(y-3)^2}{4} - \frac{x^2}{9} = 1$

Vertical or Horizontal:
Center: Vertices:
Foci:
Asymptotes:



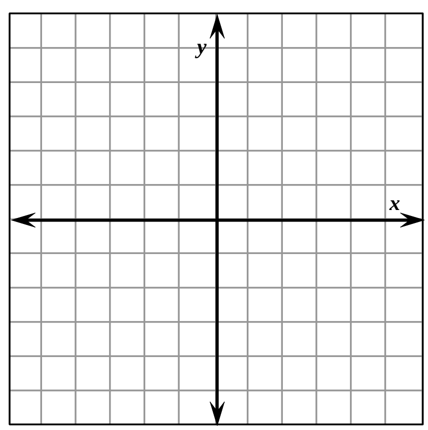
Ex3: $25x^2 - 4y^2 + 100x + 24y - 36 = 0$

Vertical or Horizontal:
Center: Vertices:
Foci:
Asymptotes:



Ex 4: $y^2 - 4x^2 + 6y + 8x = 59$

Vertical or Horizontal:
Center: Vertices:
Foci:
Asymptotes:



Write the equation of a hyperbola with the following characteristics:

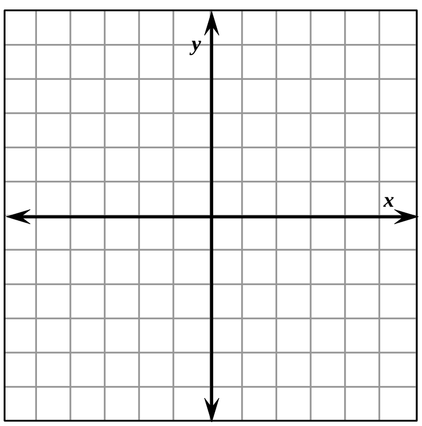
5: The asymptotes: $y = \pm\frac{5}{12}x$; focus (13, 0)

6: Center (2, -3); Vertex (5, -3); Focus (-10, -3)

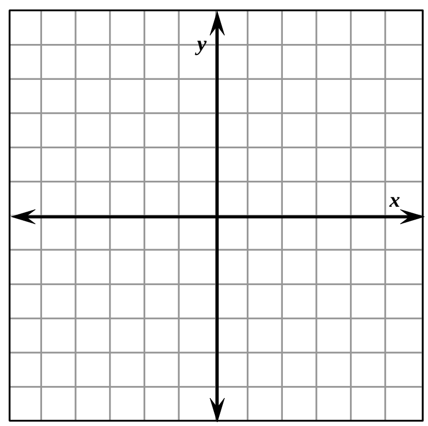
7: Center (-6, -1); a = 4; b = 1; major axis is horizontal

Graphing Quadratic Functions:

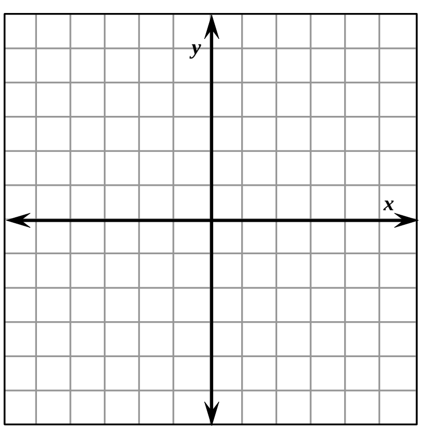
Ex: $y = (x - 2)^2 + 1$
$y = -4x + 5$



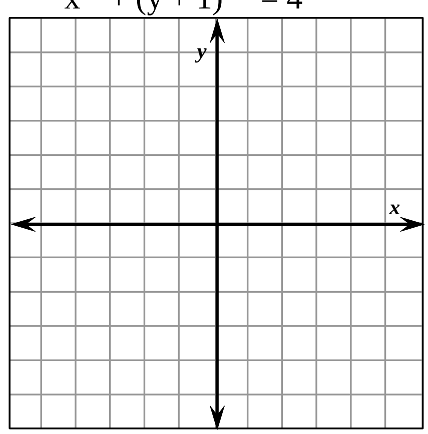
Ex: $4x^2 - y^2 = 36$
$(x - 5)^2 + y^2 = 64$



Ex: $5x^2 + y^2 = 30$
$6x^2 - 2y^2 = 4$



Ex: $x^2 + y^2 = 1$
$y = 3x + 1$
$x^2 + (y + 1)^2 = 4$



Parametric Equation:
We have been doing graphs in x and y and now we will introduce a third variable to represent a curve in the plane. This third variable is called a parameter and often represents "time" though it could mean other things.

Ex: We could graph the curve representing a baseball that is hit at a 45^o angle at a velocity of 50ft per sec.

To introduce t, we will write both x and y as a function of t and get parametric equations

Definitions
Plane curve- the set of ordered pairs (f(t), g(t)) if f and g are continuous functions of t

Parametric equations- x = f(t) and y = g(t) Parameter is t!

Sketching a plane curve
Choose increasing values of t and make an x, y, t table by substituting t into the equation.

Ex: $x = t^2 - 9$ $-3 \le t \le 3$

$y = \frac{t^2}{3}$

Two different sets of parametric equations can have the same graph.

Ex: $x = t^2 - 4$
$y = \frac{t}{2}$ $-2 \le t \le 3$

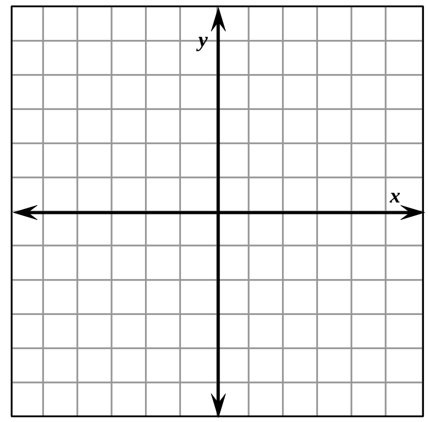
Graph each

Eliminating the parameter:

Converting from parametric equations to rectangular equations (x,y)
1. Solve for t in one equation
2. Substitute what you get into the other equation

Ex: $x = t^2 - 9$
$y = \frac{t^2}{3}$

Ex: sometimes the parameter represents an angle rather than time.
Sketch the curve represented by eliminating the parameter



$x = 2\cos\theta$ $\qquad 0 \leq \theta \leq 2\pi$
$y = 6\sin\theta$

Rectangular equation- good for sketching curve

Parametric equations- good for seeing position, direction, and speed

Examples:

Ex: $x = t$
$y = \frac{1}{2}t$

Ex: $x = t - 1$
$y = \frac{t}{t-1}$

Ex: $x = \cos\theta$
$y = 2\sin 2\theta$

Finding parametric equations for a given graph
You can let t be anything

Ex: $y = x^2 - 4$

Ex: $\frac{x^2}{4} - \frac{y^2}{16} = 1$

Ex: $x = a\cos t \quad y = a\sin t \qquad a > 0$ is a constant

Ex: $x = \frac{3t^2}{4}$ $\qquad -4 \leq t \leq 4$
$y = t$

Ex: $x = 3t^2 + 12t + 12$ $\qquad -4 \leq t \leq 0$
$y = 2t + 4$

Projectile motion- when an object is propelled upward at an inclination θ to the horizontal with initial speed v_0

x = $(v_0 \cos\theta)t$ y = $-1/2gt^2 + (v_0 \sin\theta)t + h$
where t is time and g is constant due to gravity.

Ex: Suppose Jim hit a golf ball with initial velocity of 150 feet per second at an angle of 30^o.

a) find parametric equations to describe position of the ball as a function of time.

b) how long is the golf ball in the air ?

c) when is the ball at max height ?

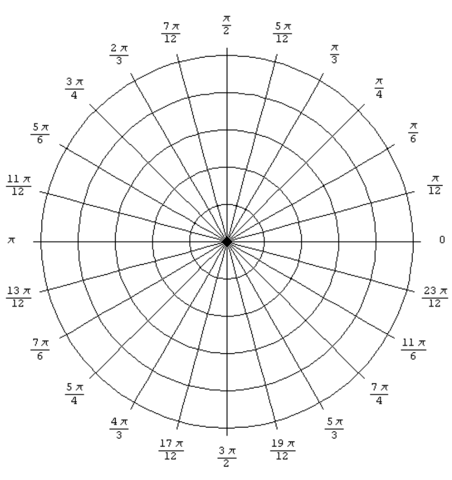
d) determine the distance the ball traveled.

e) graph on calculator

Polar coordinate system:

(r, θ) = polar coordinates
r =
θ =

Plotting points on the Coordinate System:



Plot: A $(3, \frac{\pi}{6})$ B $(2, -\frac{\pi}{3})$ C $(1, \frac{2\pi}{3})$

The same point can represent many polar coordinates:

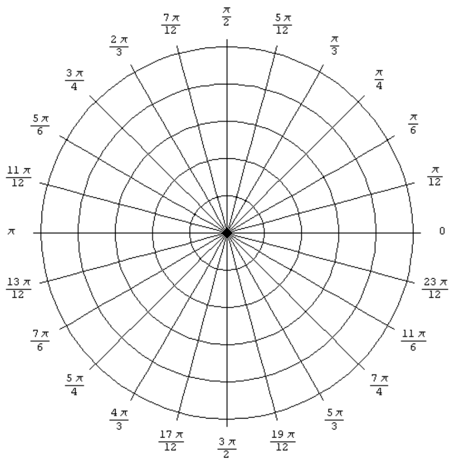
Ex: (r, θ) and (r, 2π + θ) are the same point

(r, θ) and (-r, π + θ) are the same point

Formulas: (r, θ) = (r, θ $\pm$ 2kπ)

(r, θ) = (-r, θ $\pm$ (2kπ+ π)

Example: Plot the point $(2, -\frac{\pi}{6})$ and find 3 additional polar coordinates for this point in the interval -2π < θ < 2π.



Coordinate Conversion:

(r, θ) is related to rectangular coordinates (x, y) by the following equations:

x = r cosθ and tanθ = y/x

y = r sinθ $r^2 = x^2 + y^2$ or $r = \sqrt{(x^2 + y^2)}$

Polar to Rectangular Conversion

Ex: Convert $(3, \frac{\pi}{3})$ and $(-4, \frac{\pi}{4})$ to rectangular coordinates.

Rectangular to Polar Conversion

Find the angle and distance

Remember what quadrant you are in!

Ex: Convert $(-4, 4\sqrt{3})$ and (0, 1) to polar coordinates

Equation Conversion:

Rectangular to Polar:

Ex: Convert $x^2 + y^2 = 4$ to a polar equation.

Ex: $2x – y + 6 = 0$

Ex: $y = x^2$

Polar to Rectangular:

1. r =
2. θ =
3. equations with r and θ

Ex: r = 4

Ex: $\theta = \frac{\pi}{4}$

Ex: r = $\csc\theta + 2$

Ex: $r = \frac{3}{4 + 5\cos\theta}$

Graphs of Polar Equations:

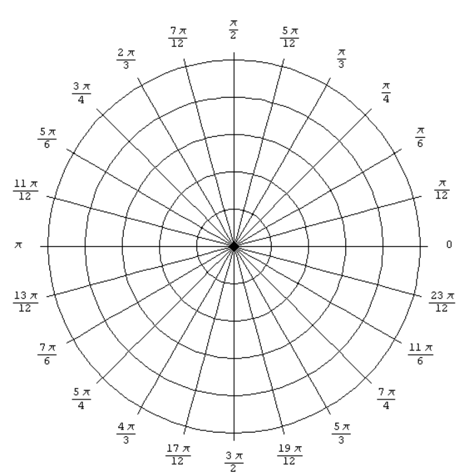
Graphing polar equations by plotting points.

Ex: Sketch the graph $r = 5\cos\theta$

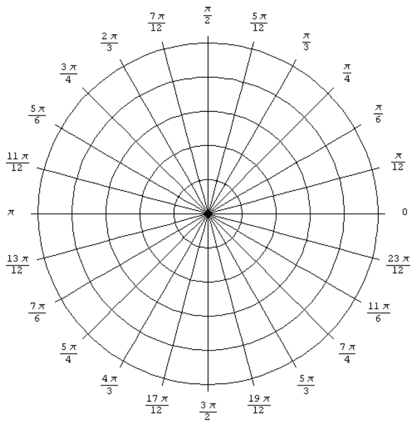
1. choose values of θ in $0 \le \theta \le 2\pi$

Make a Table:

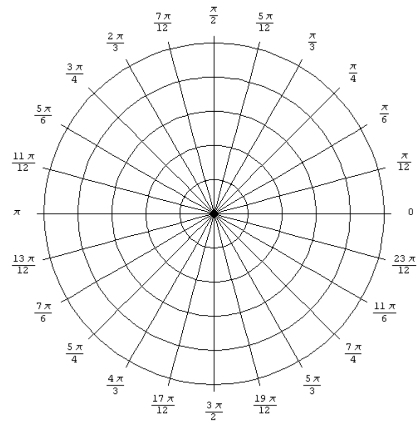
θ												
r												



Ex: Sketch r = $3+2\cos\theta$



Ex: Sketch r = $5\sin 2\theta$



Look at graphs on a calculator:

To graph solve for r

Hint: Use ZOOM SQUARE to make it accurate

Ex: $r=\dfrac{2}{4\cos\theta+5\sin\theta}$

Ex: r = 5

Ex: $r^2=\sin 2\theta$

Ex: $\pm\sqrt{\dfrac{4}{\cos\theta\sin\theta}}$

Different types of Polar Graphs:

- the graphs of these are easier to distinguish in polar form than in rectangular form

I. **Limaçon- 4 Types**

$r=a\pm b\cos\theta$ **a > 0 b >0**

$r=a\pm b\sin\theta$

1. $\dfrac{a}{b}<1$ Limaçon- with an inner loop

   Ex: $r=1+2\cos\theta$

2. $\dfrac{a}{b}=1$ Cardioid

   Ex: $r=3+3\cos\theta$

3. $1<\dfrac{a}{b}<2$ Dimpled Limaçon

   Ex: $r=3+2\sin\theta$

4. $\dfrac{a}{b}\geq 2$ Convex Limaçon

   Ex: $r=8+3\sin\theta$

II. Rose Curves –

$$r = a\cos n\theta \qquad \mathbf{n \geq 2}$$
$$r = a\sin n\theta$$

n petals if n is odd
2n petals if n is even

Ex: $r = -2\cos 3\theta$ Ex: $r = 5\cos 8\theta$

III. Circles –

$$r = a\cos\theta$$
$$r = a\sin\theta$$

IV. Lemniscates –

$$r^2 = a^2\cos 2\theta$$
$$r^2 = a^2\sin 2\theta$$

Symmetry:

Polar	**Rectangular**
1. Line $\theta = \frac{\pi}{2}$	with respect to the y- axis
Replace (r,θ) by $(r,\pi-\theta)$ or $(-r,-\theta)$	
2. Polar Axis	with respect to the x –axis
Replace (r,θ) by $(r,-\theta)$ or $(-r,\pi-\theta)$	
3. Pole	with respect to the origin
Replace (r,θ) by $(r,\pi+\theta)$ or $(-r,\theta)$	

Roses: if n$\geq$2 then they have symmetry about the polar axis, the line $\theta = \frac{\pi}{2}$, or both.

Lemniscates: They have symmetry about the pole.

Graphs of r = *f*(sin θ) have symmetry about the line $\theta = \frac{\pi}{2}$.

Graphs of r = *f*(cos θ) have symmetry about the polar axis.

Graph r = $\theta + 2\pi$ - Spiral of Archimedes Graph r = $e^{\cos\theta} - 2\cos 4\theta + +\sin^5\frac{\theta}{12}$ - Butterfly