12.1 Arithmetic Sequences & Series

1. A **sequence** is an ordered list of numbers. Each number in the list is called a **term** of the sequence. The first term of a sequence is denoted as a_1. The second term is denoted as a_2. The term in the n^{th} position is called the n^{th} term and is denoted as a_n. The term before a_n is a_{n-1}.

   A sequence is a function whose <u>range</u> is the <u>terms</u> of the sequence and the <u>domain</u> is the <u>position</u> of each term.

2. **Finding the Next Term:** To find the next term in an arithmetic sequence, first find the common difference, then add the common difference to the next term.

3. Find the next three terms.

   a. $-12,\ -1,\ 10,\ .\ .\ .$ b. 7, 10, 13, … c. r – 4, r – 1, r + 2

4. **Arithmetic sequences:**
   The sequence $a_1,\ a_2,\ a_3,\ a_4,\ \ldots.a_n$ is arithmetic if there is a number d such that:

   $a_2 - a_1 = d$
   $a_3 - a_2 = d$ Where d is the common difference.

   Ex: 6, 9, 12, 15, ….3n + 3 The common difference is 3 because 9 – 6 = 3. $d = 3$

   Ex: 2, -3, -8, -13, …, -5n + 7 The common difference is –5 because -3–2 = -5. $d = -5$

5. **<u>Explicit Formula:</u>** a formula that defines the n^{th} term. $\boxed{\mathbf{a_n = d n + c}}$

   We will look at c as being the a_0 **(a sub not) term**. Think y-intercept!

   The book will use $a_n = a_1 + (n-1)d$ or $a_{n+1} = a_n + d$ (when recursive)

6. **Recursive Sequence:** is a sequence in which each term is defined using the previous terms.
   Each Arithmetic Sequence can be written recursively using $a_{n+1} = a_n + d$

Ex: Find a formula of an arithmetic sequence whose common difference is 4 and whose first term is 3.

$a_n = d$n + c We know $d = 4$. $a_1 = 3$. So $a_0 = 3 - 4$. $a_0 = -1$

$a_n = 4$n – 1. The terms of this sequence are: 3, 7, 11, 15, …, 4n – 1.

Ex: Find the formula of the arithmetic sequence whose first term is 3 and whose second term is –1.

$a_n = d$n + c We know $a_1 = 3$ and $a_2 = -1$. So $d = -4$. a_0 must be 3–(-4) = 7

$a_n = -4$n + 7 The terms of this sequence are: 3, -1, -5, -9, …, -4n + 7.

7. Find the 35^{th} term in the sequence 11, 4, -3, …

8. Find the 20^{th} term in the sequence for which $a_1 = -27$ and $d = 3$.

9. Find the first term in the sequence for which $a_4 = 229$ and $d = 8$.

10. The fifth term of an arithmetic sequence is 25 and the 12^{th} term is 60.
Write the first several terms of this sequence.

$a_5 = 25$ $\qquad\qquad a_{12} = 60$

$a_{12} = a_5 + 7d$ (where 7 is the difference in the term numbers).

$60 = 25 + 7d \qquad 35 = 7d \qquad d = 5$

Since $a_5 = 25$ we can subtract 5 to get each term in the sequence down to the first.

5, 10, 15, 20, 25

11. **Arithmetic means: the terms between any two nonconsecutive terms of an arithmetic sequence.**

The terms between 2 given terms of an arithmetic sequence are called *arithmetic means*.

$10, \underbrace{13, 16, 19}_{\text{3 arithmetic means}}, 22$

$10, \underbrace{14, 18}_{\text{2 arithmetic means}}, 22$

12. Form an arithmetic sequence that has five arithmetic means between -11 and 19.

13. Form an arithmetic sequence that has six arithmetic means between -12 and 23.

14. **Summation Notation:** the sum of a sequence is also known as an **Arithmetic Series**.

$$\sum_{k=1}^{m} c_k = c_1 + c_2 + c_3 + \ldots + c_m$$

15. **Sigma Notation**: the sum of the first n terms of a sequence (called a series)

Ex: $\sum_{i=2}^{5} 2i$ Ex: $\sum_{k=3}^{8} (2k+3)$ Ex: $\sum_{j=5}^{10} (3-j)$

16. **The Sum of an Arithmetic Series: S_n =** $\frac{n}{2}(a_1 + a_n)$ This means that we add the first and last terms, then multiply by the number of terms divided by 2.

Ex: Find the sum of the integers from 1 to 500.

$S_n = \frac{n}{2}(a_1 + a_n)$ n = 500, a_1 = 1 and a_n = 500 $S_n = \frac{500}{2}(1+500)$ = 250(501) = 125, 250

17. Find the sum of the first 27 terms in the series -14, -8, -2, ……+ 142.

18. Find the sum of the first 32 terms in the series -12, -6, 0, ….

19. Find n for a series for which a_1 = 5, d = 3, and $S_n = 440$.

20. Nimisha starts a college savings account for her daughter on her sixth birthday. She plans to deposit $25 the first month and then increase the deposit by $5 each month. How much will she have deposited in twelve years?

21. The number of seats in the first row is 20, the second row is 23, the third row is 26, and so on. How many seats are in Row 16? How many seats is there altogether in those 16 rows?

DAY 1 HW

12.1 pg. 660-661 #5-37 odd, 41, 43

Find the next five terms in each arithmetic sequence.

5. 5, 9, 13, … 7. 5, -1, -7, … 9. 1.5, 3, 4.5, …

11. –n, 0, n, … 13. b, -b, -3b,…

15. Find the 79th term in the sequence -7, -4, -1, ….

17. Form an arithmetic sequence that has one arithmetic mean between 12 and 21.

Solve. Assume that each sequence is an arithmetic sequence.

19. Find the 19th term in the sequence for which a_1 = 11 and d = -2.

21. Find n for the sequence for which a_n = 37, a_1 = -13, and d = 5.

23. Find the first term in the sequence for which d = -2 and a_7 = 3.

25. Find d for the sequence for which a_1 = 4 and a_{11} = 64.

27. Find the sixth term in the sequence $-2+\sqrt{3}, -1-\sqrt{3},...$

29. Find the 43rd term in the sequence -19, -15, -11, …

31. Form a sequence that has one arithmetic mean between 36 and 48.

33. Form a sequence that has two arithmetic means between $\sqrt{2}$ and 10.

35. Find the sum of the first 11 terms in the series – 1 + 1 + 3 + …

37. Find n for a series for which a_1 = -7, d = 1.5, and S_n = -14.

41. Terri works after school at the Find Foods Supermarket. One day, Terri had to stack cans of soup in a grocery display in the form of a triangle. On the top row, there was only one can. Each row below it contained one more can that the one above it. On the bottom row, there were 21 cans. If all the cans were the same size, how many cans were in the display?

43. Michael is a chocoholic. On New Year's Day, he ate one piece of chocolate. On the next day, he ate 2 pieces. On each subsequent day, he ate one additional piece of candy.
    a. How many pieces of candy did he eat on the last day of January?

    b. How many pieces of did he eat during the month of January?

Day 2 Sequences and Series Notes

1. **Recursive Formula:**
   A formula for a sequence that gives the value of a term t_n in terms of the preceding term t_{n-1}. The first term is represented by t_1, the second term in represented by t_2, the third term in represented by t_3, and so forth.

   Explicit Formula: $\boxed{a_n = a_{n-1} + d}$ **or** $\boxed{a_{n+1} = a_n + d}$ **if Arithmetic Sequence!**

2. Find the next three terms in each sequence.

   a. 80, 77, 74, 71, 68, … b. 4, 8, 16, 32, 64, … *c. 0, 3, 7, 12, 18, … * d. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32} ...$

3. Now write the recursive formula for the sequences above.

4. If $a_1 = 22$ and $a_n = a_{n-1} - 3$, find the next three terms.

5. If $t_1 = 64$ and $t_n = \frac{1}{2} t_{n-1}$, find the next four terms.

6. If $a_1 = 3$, $a_2 = 5$ and $a_n = a_{n-2} + 4a_{n-1}$, find the third, fourth and fifth terms..

7. Find the first four terms.

   a. $t_n = 16 - 3n$ b. $a_k = 3 + 4(k-1)$ c. $a_n = \frac{1}{n} + \frac{1}{n+2}$

8. If the domain values are $\{1, 0, 3, 5\}$, find the corresponding range values for $t_n = 2n - 5$.

10. Sigma Notation

Simplifies the process of writing out the sum of a series

$$\sum_{n=2}^{10} (2n)$$

last value of n

is read as the sum of $2n$ **as** n **increases from 2 to 10**

first value of n

11. $\displaystyle\sum_{k=1}^{5} (2k)$

12. $\displaystyle\sum_{k=-1}^{2} (k+2)$

13. $\displaystyle\sum_{k=0}^{3} k+2$

(no parentheses, careful)

14. $\displaystyle\sum_{k=2}^{5} 2^k$

15. $\displaystyle\sum_{n=1}^{3} n^2 - \sum_{n=2}^{5} n$

16. $\displaystyle\sum_{k=5}^{9} [3-4(n-1)]$

(typo: must be k's)

Homework: Worksheet #1 and Worksheet #2

DAY 2 HW

Worksheet 1

I. Give the first four terms of each sequence:

1. $t_1 = 5, t_{n+1} = t_n + 3$

2. $t_1 = 10, t_n = t_{n-1} + n$

3. $t_1 = 3, t_n = 2t_{n-1}$

4. $t_1 = 4, t_{n+1} = 2t_n - 1$

II. Give the third, fourth, and fifth terms of each sequence:

5. $t_1 = 6, t_{n+1} = t_n + 4$

6. $t_1 = 9, t_n = \frac{1}{3} t_{n-1}$

7. $t_1 = 1, t_{n+1} = 3t_n - 1$

8. $t_1 = 4, t_n = (t_{n-1})^2 - 10$

9. $t_1 = 1, t_{n+1} = t_n + 2n - 1$

10. $t_1 = \frac{1}{2}, t_n = \frac{n}{n+1}(t_{n-1} + 1)$

11. $t_1 = 2 \; and \; t_2 = 4, \; t_n = t_{n-1} + t_{n-2}$

12. $t_1 = 2 \; and \; t_2 = 4, \; t_n = t_{n-1} \bullet t_{n-2}$

13. $t_1 = 5 \; and \; t_2 = 8, \; t_n = (t_{n-1} - t_{n-2})^2$

14. $t_1 = 7 \; and \; t_2 = 3, \; t_n = t_{n-1} - 2t_{n-2}$

Worksheet 2

I. The domain of the sequence in each exercise consists of the integers 1,2,3,4,5. Write the corresponding range values:

1. $a_n = 2n - 1$

2. $a_k = (-1)^k$

3. $a_n = \left(\frac{1}{2}\right)^{n-3}$

II. Write the first four terms of the sequence given by the formula in each exercise.

4. $a_n = (-1)^n \bullet n^2$

5. $a_n = 3(0.1)^n$

6. $a_n = 3(0.1)^{2n}$

7. $a_n = \frac{1}{n} - \frac{1}{n+1}$

8. $a_k = (2k - 10)^2$

9. $a_n = -2 + (n-1) \bullet 3$

III. Find the sum of the first five terms of the sequence given by the formula in each exercise:

10. $a_n = 3n$

11. $a_n = -6 + 2(n-1)$

12. Let n = 7, find the sum: $2 + 4 + + \ldots + 2^n$

IV. Evaluate each of the following:

13. $5\left(\sum_{k=1}^{6} k\right)$

14. $\sum_{n=1}^{4} n^2 + \sum_{n=1}^{4} n$

15. $\sum_{k=1}^{8} (-1)^k$

16. $\sum_{j=1}^{6} [-3 + (j-1)5]$

17. $\sum_{k=-3}^{3} \frac{1}{10^k}$

12.2 Geometric Sequences and Series Notes

Geometric Sequence: the <u>ratio</u> of any term to the previous term is <u>constant</u>.

r = common ratio

Find the next three terms in the geometric sequence:

Ex: 27, 135, 675, … Ex: 2, 4, 8, 16… Ex: 18, 54, 162 … Ex: $a_n = \left(-\frac{1}{4}\right)^n$

<u>Explicit Formula:</u> a formula that defines the n^{th} term. $\boxed{a_n = a_1 r^{n-1}}$

Ex: Find the first 5 terms of the geometric sequence whose first term is $a_1 = 4$ and whose ratio is r = 3.

Ex: Write a rule for the n^{th} term. -8, -12, -18, -27, …

Ex: $a_4 = 3$, r = 3. Write a rule for the n^{th} term.

Ex: Find the 18^{th} term of the geometric sequence whose first term is 20 and whose common ratio is 1.5.

Ex: Find the 12^{th} term in the geometric sequence $4, 6, 9, \frac{27}{2}...$

Geometric Means: the terms between any two nonconsecutive terms of a geometric sequence.
Ex: Insert 2 geometric means between 8 and 512.

8, ____, ____, 512

Ex: Form a sequence that has two geometric means between 136 and 459.

Ex: Form a sequence that has two geometric means between 128 and 54..

Geometric series $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ r = common ratio n = number of terms

works for a finite sequence

Ex: Find the sum of the first 10 terms of the geometric series 1 + 5 + 25 + 125 + 625 …

Ex: Find the sum of the 1^{st} 8 terms of the geometric series where $a_1 = 8$ and $a_4 = 512$.

Ex: During the first week of training of a marathon, you run a total of 10 miles? You increase the distance you run each week by twenty percent. How many miles do you run during your twelfth week of training?

Ex: A certain chemical decays at the rate of one-half its weight every 6 months. If 100 grams is the initial weight of the chemical, how much remains after 4 years?

Ex: When a ball bounces, the heights of consecutive bounces form a geometric sequence. Suppose a ball is dropped from 10 meters and rebounds 95% of the height of the previous bounce. What is the height on the fifth bounce? **What is the total distance traveled in the first five bounces?

DAY 3 HW

12.2 Book problems pg. 668-669 #7-31 odd

7. Find the next four terms for the following geometric sequence: 7, 3.5, …

9. Determine whether $\sqrt{2}, 2, \sqrt{8}, ...$ form a geometric sequence. Write yes or no.

11. Determine whether $t^{-2}, t^{-1}, 1, ...$ form a geometric sequence. Write yes or no.

13. If r = 2 and a_3 = 24, find the first term of the geometric sequence.

15. Find the sum of the first five terms of the series $\frac{5}{3} + 5 + 15 + ...$

17. The first term of a geometric sequence is 8, and the common ration is $\frac{3}{2}$, find the next three terms.

19. Find the sixth term of the geometric sequence 10, 0.1, 0.001, …

21. Find the first three terms of the geometric sequence for which a_4 = 2.5 and r = 2.

23. Form a sequence that has two geometric means between 1 and 27.

25. Find the sum of the first six terms of the series 2 + 3 + 4.5 + …

27. Find the sum of the first ten terms of the series $1 + \sqrt{2} + 2 + 2\sqrt{2} + ...$

29. The Landbury Museum has been investing in paintings for many years. Twenty years ago, the museum purchased a painting by one of the French impressionist painters for $180,000. The value of the paintings has appreciated at a rate of 14% per year. Find the value of the painting after 10, 20, 30, 40, and 50 years, assuming that the rate of appreciation remains constant.

31. The population of a certain bacteria doubles every 30 minutes. The initial population is 150. Determine the number of organisms that would exist after 12 hours and after 24 hours.

Worksheet Day 4: Geometric/Arithmetic Sequences and Series:

Find the nth term of the Arithmetic Sequence having the given values of a, d, and n.

1. $a_1 = 6,\ d = \frac{2}{3},\ n = 10$

2. $a_1 = 25x,\ d = -3x,\ n = 33$

Find the specified term of each arithmetic sequence:

3. The twentieth term of $x - 11y, x - 9y, x - 7y, \ldots$

4. Which term of $115, 108, 101, \ldots is\ 17$?

Insert the specified number of arithmetic means in each case:

5. Seven, between 26 and – 30

6. two, between 2x + 5y and 5x – 4y

Find the missing terms:

7. $t_4 = 13,\ t_6 = 7, t_1 =$

8. $t_2 = -11,\ t_8 = 19,\ t_7 =$

Solve.

9. A rocket fired vertically traveled 6 meters during the first second and (as long as its engines provided thrust) traveled 34 meters farther during each following second than in the one before. How far did it travel during the 12th second?

10. If an employee hired at $6850 a year is guaranteed annual salary increases of $450, in which year of his employment will he first earn at least $10,000 a year?

11. A pile of bricks has 53 bricks in the first row, 51 in the second row, 49 in the third row, and so on, and 1 brick in the top row. How many bricks are in the 14th row?

Use the given data to find the sum of the arithmetic series.

12. $a_1 = -53,\ n = 45, d = 3$

13. $a_1 = 7,\ a_n = 127, d = 2$

Solve:

14. A lecture hall has 20 seats in the front row and two seats more in each following row than in the preceding one. If there are 15 rows, what is the seating capacity of the hall?

15. Roger paid off a debt to his father in 8 months by paying $60 the 1st month, $55 the second, $50 the third, and so on. How much was the original debt?

Write the next three terms of the given geometric sequence, then write an expression for the nth term.

16. 3, 6, 12, …….

17. $\frac{9}{2}, -\frac{3}{2}, \frac{1}{2}$

Find the specified term of the geometric sequence described.

18. tenth term if $a_1 = \frac{3}{32}$ *and* $r = -2$

19. Eighth term of 24, 12, 6,…

Insert the given number of positive geometric means and write the resulting geometric sequence.

20. Two, between 2 and 54

21. Four, between 48 and 3/2

Solve:

22. Assuming no duplication of ancestors, how many great, great, great grandparents did you have?

23. A piece of real estate bought 5 years ago for $25,600 increased in value 25% each year since then. What is it worth now?

24. A car traveled 32 meters in the first second after the brakes were applied and in each second after that traveled half as far as it had in the second before. How far did the car travel in the ten seconds after the brakes were applied?

Find the sum of the geometric series described.

25. $a_1 = 3, r = 2, n = 6$

26. $a_1 = 8, r = 1.5, a_n = 40.5$

Of the five quantities a, a_n, r, n, and S_n, three are given. Find the other two. Some exercises have two sets of answers.

27. $n = 8, r = 2, S_n = 765$

28. $a_1 = 2, n = 3, S_n = 42$

12.3 and 12.4 Infinite Sequences & Series

1. **Infinite Sequence:** A sequence that never ends.

2. The sequence $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ . \ . \ ., \frac{1}{n}$ has infinitely many terms; as n increases the value of the terms decrease and get closer and closer to 0. Think about the graph of this function.

   0 is called the Limit of the terms in this sequence and can be expressed as follows:

   $\lim\limits_{n\to\infty} \frac{1}{n} = 0$ *"the limit of* $\frac{1}{n}$*, as n approaches infinity equals zero".*

3. Rules for Finding Limits (**Same as horizontal asymptote rules**):

   a. If the largest exponents are the same in the numerator and denominator, the limit is the ratio of the coefficients of the terms containing the largest exponent.

   b. If largest exponent is in the numerator, there is no limit.

   c. If the largest exponent is in the denominator, the limit is 0.

4. Find each limit:

   a. $\lim\limits_{n\to\infty} \frac{1-2n}{5n}$ b. $\lim\limits_{n\to\infty} \frac{4n^2-6}{3n-1}$ c. $\lim\limits_{n\to\infty} \frac{2n^2-3n+4}{3n^2+1}$

   d. $\lim\limits_{n\to\infty} \frac{n^2+4}{n^3}$ *e. $\lim\limits_{n\to\infty} \frac{1}{5^n}$ *f. $\lim\limits_{n\to\infty} (2)^n$

5. **Infinite Series:** An infinite series is the indicated sum of the terms of an infinite sequence

6. **Sum of an Infinite Series:** The sum of an infinite geometric series for which $|r|<1$ is

   given by $\boxed{S_n = \frac{a_1}{1-r}}$

7. Find the sum of each infinite series, or state that the sum does not exist.

a. $\frac{1}{20}+\frac{1}{40}+\frac{1}{80}+\ . \ . \ .$

b. $2\sqrt{2}+8+16\sqrt{2}\ . \ . \ .$

c. $2+\sqrt{2}+1+\ . \ . \ .$

8. **Convergent Series:** Has a sum or limit $(|r|<1)$

9. **Divergent Series:** Does not have a sum or limit, $(|r|\geq 1)$ and it diverges

10. Determine whether each series is arithmetic or geometric, then determine if it is convergent or divergent.

a. $\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\ . \ . \ .$

b. $-4-2-0+2+\ . \ . \ .$

c. $1+3+9+27+\ . \ . \ .$

d. $\sum_{k=1}^{\infty} 2^k$

f. $\sum_{n=1}^{\infty} 2(.4)^k$

11. Repeating Decimals as a Fraction:
You can use what you know about infinite series to write repeating decimals as fractions - - you must write the repeating decimal as an infinite geometric series

12. Write each repeating decimal as a fraction.

a. 0.454545...

b. 0.888...

c. 7.259259...

Homework: page 676 # 17-24; Worksheet Day 5

DAY 5 HW

12.4 Infinite Geometric Series

Determine whether each series is convergent (has a sum) or divergent. If the series is convergent, find the sum.

1. 5 + 10 + 15 + 20 + …

2. 16 + 8 + 4 + 2 + …

3. 1 + .1 + .01 + .001 + .0001 + …

4. 4 + 2 + 0 – 2 - …

5. 2 – 4 + 8 – 16 + …

6. $1-\frac{1}{5}+\frac{1}{25}-\frac{1}{125}+....$

7. $-\frac{5}{3}+\frac{10}{9}-\frac{20}{27}+\frac{40}{81}-....$

Decide whether or not the infinite series has a sum. If it does, find it.

8. $\sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^{i-1}$

9. $\sum_{i=1}^{\infty}(2)(0.1)^{i-1}$

Using the new information, convert the following to fraction form:

10. 0.7777….

11. 0.131313…..

12. 0.013131313…..

Solve each word problem.

13. Geologists estimate that the continents of Europe and North America are drifting apart at a rate of an average of 12 miles every 1 million years, or about 0.75 inch per year. If the continents continue to drift apart at that rate, how many inches will they drift in 50 years?

Contagious diseases can spread very quickly. Suppose five people are ill during the first week of an epidemic and that each person who is ill spreads the disease to four people by the end of the next week. By the end of the tenth week of the epidemic, how many people have been affected by the illness **12.3 Book pg 676 #17-24**

DAY 5 HW

Evaluate each limit, or state that the limit does not exist.

17. $\lim_{n \to \infty} \frac{1}{3^n}$

18. $\lim_{n \to \infty} \frac{2n^2 - 6n}{5n^2}$

19. $\lim_{n \to \infty} \frac{n^2 - 4}{2n}$

20. $\lim_{n \to \infty} \frac{(n+2)(2n-1)}{n^2}$

21. $\lim_{n \to \infty} \frac{n^2 + n - 3}{n^2}$

22. $\lim_{n \to \infty} \frac{3n+1}{n-3}$

23. $\lim_{n \to \infty} \frac{4n^2 + 5}{3n^2 + 2n}$

24. $\lim_{n \to \infty} \frac{2n^8}{n^2 + 4n}$

12.5 Sigma Notation & The n^{th} Term

I. Write each of the following in expanded form and find the sum.

1. $\sum_{n=4}^{7}(3^n+1)$

2. $\sum_{n=1}^{\infty}2\left(\frac{1}{3}\right)^n$

3. Express $16+19+22+25+28$ using sigma notation. $\sum_{n=}$

4. Express $25-6.25+1.5625-.390625$ using sigma notation. $\sum_{n=}$

5. Not all sequences are arithmetic or geometric;
Some important sequences are generated by products of consecutive integers

Factorial: The product $n(n-1)(n-2)...1$ **is called *n factorial* and is symbolized by n!**

6. By Definition $0!=1$

7. Evaluate each expression:

a. $5!$ b. $20!$ c. $\frac{10!}{3!\bullet 6!}$ d. $\frac{3!}{6!-3!}$

e. $\frac{n!}{(n+1)!}$ f. $\frac{9!}{0!}$ g. $\frac{3!+6!}{6!-3!}$ h. $\frac{(x+1)!(x-2)!}{(x-4)!(x-1)!}$

12.6 Pascal's Triangle

1. Use Pascal's Triangle to expand each binomial.

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1
\end{array}
$$

a. $(x+y)^3$

b. $(x-3)^5$

c. $(x+2y)^6$

d. $(a-5)^3$

e. $(3x-2y)^5$

Homework page 688 # 23-33 odd, 35-38, 41, 42; Worksheet Day 6 on Factorials and Pascal's

DAY 6 HW

12.5 Book pg 688 #23-33 odd

Write each expression in expanded form and find the sum.

23. $\sum_{r=1}^{3}(r-3)$

25. $\sum_{b=4}^{8}(4-2b)$

27. $\sum_{b=2}^{5}(b^2+b)$

29. $\sum_{n=3}^{6}(3^n+1)$

31. $\sum_{p=1}^{4}(3^{p-1}+\frac{1}{2})$

33. $\sum_{k=1}^{\infty}4(\frac{1}{2})^k$

Worksheet --Factorials: Day 6

Write each product in factorial notation:

1. $1\bullet2\bullet3\bullet4\bullet5\bullet6$
2. $8\bullet7\bullet6\bullet 5$
3. $12\bullet11\bullet10\bullet9\bullet8$
4. $100\bullet99\bullet98$

Evaluate each expression using factorial knowledge:

5. $5!$
6. $\frac{6!}{4!\bullet 3!}$
7. $3\,(6!)$
8. $3!4!$
9. $\frac{10!}{8!}$
10. $\frac{(8-2)!}{(4+1)!}$
11. $\frac{12!}{11!}$
12. $\frac{6!}{7!-6!}$
13. $\frac{4!\bullet 6!}{7!}$
14. $\frac{3!}{0!}$
15. $\frac{5!}{4!-3!}$
16. $\frac{3!+5!}{5!-3!}$
17. $\frac{n!}{(n-1)!}$
18. $\frac{(n+2)!}{n!}$
19. $\frac{(n+1)!}{(n-1)!}$
20. $\frac{(x-y)!}{(x-y-1)!}$
21. $\frac{(x-3)!\,x!}{(x-2)!(x-1)!}$
22. $\frac{(x+3)!(x-1)!}{(x-2)!(x+1)!}$

Expand using Pascal's Triangle:

1. $(2x+y)^6$
2. $(3a-5b)^4$
3. $(x^2-3y)^4$
4. $(a^3-2b^2)^6$
5. $\left(1+\frac{1}{x}\right)^6$

12.6 Binomial Theorem

1. **Binomial Theorem:** If n is a nonnegative integer then

$$(x+y)^n = x^n + nx^{n-1} \bullet y + \dots {}_nC_r x^{n-r} \bullet y^r$$

where ${}_nC_r = \dfrac{n!}{(n-r)!r!}$ if you are looking for the fifth term then r = 4

$$(a+b)^n = 1a^n b^0 + \frac{n}{1}a^{n-1}b^1 + \frac{n(n-1)}{1\bullet 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1\bullet 2\bullet 3}a^{n-3}b^3 + \dots\dots + 1a^0 b^n$$

$$= \frac{n!}{n!\bullet 0!}a^n b^0 + \frac{n!}{(n-1)!\bullet 1!}a^{n-1}b^2 + \frac{n!}{(n-2)!\bullet 2!}a^{n-3}b^3 + \dots + \frac{n!}{0!\bullet n!}$$

$$= \sum_{k=0}^{n} \frac{n!}{(n-k)!\bullet k!}a^{n-k}b^k$$

2. Use the binomial theorem to expand each expression.
   a. $(x-5)^4$

3. Find the indicated term of each expression.
   a. Fourth term of $(x+2)^7$
   b. Sixth term of $(x-y)^9$
   c. Fifth term of $(2x+3y)^9$
   d. Third term of $\left(a-2\sqrt{3}\right)^6$
   e. Find the middle term of $\left(4x^2-9y\right)^{12}$

4. Find the term containing y^8 in the expansion of $\left(2x+3y^2\right)^9$

Homework Worksheet on Specific Term Day 7

DAY 7 HW

12.6 Specific Term

Use the binomial theorem to find the indicated term of the following:

1. $(2a - b)^7$; 5^{th} term
2. $(x + y)^{17}$; 4^{th} term
3. $(v + w)^{20}$; 18^{th} term
4. $(x + 2y)^{20}$; 3^{rd} term
5. $(x^2 + y)^{15}$; 11^{th} term
6. $(x^2 - 2y)^{10}$; 4^{th} term
7. $(3a - 2b^2)^5$; 4^{th} term
8. $\left(x + \frac{1}{x}\right)^{12}$; 8^{th} term
9. $\left(x - \frac{3}{y}\right)^{12}$; 7^{th} term
10. $\left(\frac{x^2}{y} - \frac{2}{x}\right)^{10}$; 4^{th} term
11. Find the middle term of $\left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)^{12}$
12. Find the term which does not contain x in the expansion of $\left(6x - \frac{1}{2x}\right)^{10}$.
13. Find the term containing y^6 in the expansion of $(x - 2y^3)^4$.
14. Find the term containing c^3 in the expansion of $\left(c^{\frac{1}{2}} + d^{\frac{1}{2}}\right)^{10}$.
15. Find the term containing x^5 in the expansion of $(2x + 3y)^8$.
16. Find the term containing y^{10} in the expansion of $x - 2y^2)^8$.
17. Find the middle term in the expansion of $\left(3a - \frac{b}{2}\right)^{10}$.
18. Find the last three terms in the expansion of $(a^2 - 2b^3)^7$.

Answers: 1. $280\,a^3b^4$ 2. $680\,x^{14}y^3$ 3. $1140v^3w^{17}$ 4. $760x^{18}y^2$ 5. $3003\,x^{10}y^{10}$ 6. $-960\,x^{14}y^3$

7. $-720\,a^2b^6$ 8. $792/x^2$ 9. $673596x^6/y^6$ 10. $-960\,x^{11}/y^7$ 11. $924\,x^2y^2$ 12. -61236

13. $24x^2y^6$ 14. $210c^3d^2$ 15. $48384x^5y^3$ 16. $-1792x^3y^{10}$ 17. $-15309\,a^5b^5/8$ 18. $-672a^4b^{15}+448a^2b^{18}$

Sequences and Series Word Problems Notes

1. The winner of a contest received \$200 at the end of the first year, with a 25% increase over the preceding year's payment for each subsequent year. How much did the contest winner receive during the first 10 years of payments?

2. Mr. Turtle is moving along a straight line and has traveled 8 meters in one minute. In the next minute he travels 4 m. In each succeeding minute he travels ½ as far as he did in the previous minute. If Mr. Turtle goes on traveling this way forever, how far will he go?

3. The owners of a certain store reduces the price of their items each week the item does not sell. If the original price of a blouse is \$250 and its price at the end of each week is 4/5 of the previous week, what will be the price of the blouse at the end of the 10^{th} week?

4. A pile of logs has 1 log in the top layer, 3 logs in the second layer, 5 logs in the third layer, and so on. How many logs are in the pile if it contains 25 layers?

5. A farmer gathers 35 bushels of potatoes on the first day of the harvest. The farmer estimates that on each successive day of the harvest, the amount gathered will be 4 bushels more than the preceding day. If the harvest lasts 14 days, what is the total number of bushels the farmer can expect to collect?

6. Friction and air resistance cause each swing (after the first) of the pendulum bob to be 75% as long as the previous swing. If the length of the first swing is 16 cm, find the total distance traveled by the bob before coming to rest.

Write each in sigma notation.

7. $3 + 7 + 11 + 15 + 19$

8. $-2+6-18+54....$

DAY 8 HW

Sequences and Series Word Problems

1. Each swing of a pendulum is 3 in. shorter than the preceding swing. The first swing is 6 ft. Write a rule for the length of each swing in inches. How long is the 12^{th} swing?

2. One year before a trip you begin making deposits to an account. The first month you deposit $40. For the next 11 months you deposit $3 more than the previous month. Write a rule for each monthly deposit. How much money do you have after 12 months?

3. You buy a new car for $25,000. The value of the car decreases by 16% each year. Write a rule for the average yearly value of the car (in dollars) in terms of the year. Let n = current year. About how much will the car be worth after 6 years?

4. A tennis ball dropped from a height of 12 feet bounces 70% of the height from which it fell on each bounce. What is the vertical distance it travels before coming to rest?

5. Emily and Kayla begin work and receive $2000 each the first month. Emily will receive a raise of $120 each month thereafter. Kayla will receive a 5% raise each month thereafter.
   a. At the end of 12 months, how much will Emily and Kayla each be making per month? Who has the higher monthly income?

   b. How much did Emily and Kayla make during the 12 month period? Who earned the higher yearly income?

6. Kevin drops a Super Bouncer ball from his balcony, 20 feet above the ground. If the ball rebounds 90% of the height from which it fell on each bounce, find the vertical distance that the ball travels before coming to rest.

7. After one minute, a hot air balloon rose 90 feet. After that time, each succeeding minute the balloon rose 70% as far as it did the previous minute. How far above the Earth was the balloon after 8 minutes? What was the maximum height of the balloon?

8. To prove that objects of different weights fall at the same rate, Galileo dropped two objects with different weights from the Leaning Tower of Pisa in Italy. The objects hit the ground at the same time. When an object is dropped from a tall building, it falls about 16 feet the first second, 48 feet in the second second, and 80 feet in the third second, regardless of its weight. How many feet would an object fall in the tenth second?

9. A construction company will be fined for each day it is late completing its current project. The daily fine will be $4000 for the first day and will increase $1000 each day. Based on its budget, the company can only afford $60,000 in total fines. What is the maximum number of days it can be late?

10. The United States Department of Defense plans to cut the budget on one of its projects by 12% each year. If the current budget is $150 million, what will the budget be in 6 year?

11. A one-ton ice sculpture is melting so that it loses one-fifth of its weight per hour. How much of the sculpture will be left after five hours? Write the answer in pounds. (1 ton=2000 lbs)

12. Rob is helping his dad install a fence. He is using a sledgehammer to drive the pointed posts into the ground. On his first swing, he drives a post 5 inches into the ground. Since the soil is denser the deeper he drives, on each swing after the first, he can only drive the post 30% as far into the ground as he did on the previous swing. How far has he driven the post into the ground after 5 swings?

13. A chessboard has 64 squares. If one penny is placed on the first square, then doubled to two pennies on the second, then doubled to four pennies on the third, how much money will be on the board when the 32^{nd} square is reached?

14. A ball is dropped from a height of 15 m and bounces to 60% of the previous height. How far has the ball traveled when it hits the ground for the fourth time?

Write each in sigma notation.

15. $5+10+15+20+25$

16. $22+20+18+16$

17. $35+29+23+17+...+(-13)$

TEST REVIEW

1. Which term of the arithmetic sequence 18, 11, 4, … is -73?
2. Insert three arithmetic means between 55 and 115.
3. Find the eleventh term of the arithmetic sequence $8, \frac{11}{2}, 3, \ldots$
4. Given $a_4 = 21$ and $a_8 = 33$, find a_1.
5. Find the sum of the first 17 terms in the arithmetic series $3 + 8 + 13 + \ldots$
6. Write $7 + 10 + 13 + \ldots + 31$ using sigma notation.
7. Find $\sum_{k=1}^{22} \left(\frac{k+2}{3}\right)$
8. Find the sum of the arithmetic series with $a_1 = 70, a_n = 7, d = -3$
9. Find the fifth term of the geometric sequence 54, -36, 24, …
10. Insert four geometric means between 48 and $\frac{3}{2}$.
11. Find the eighth term in the geometric sequence 24, 12, 6, …
12. Which term of the geometric sequence 10, -50, 250, … is 6250?
13. Evaluate $\sum_{k=1}^{5} (-2)^{k-1}$
14. Find the sum of the geometric series $a_1 = 48, r = \frac{1}{2},$ and $n = 5$.
15. In the geometric series, given $n = 8, r = 2, S_n = 765,$ find a_1 and a_8.
16. Find the sum of the infinite geometric series $6 - 3 + \frac{3}{2}, \ldots$
17. Expand $(2x - y^2)^4$ using Pascal.
18. Expand $(a + b)^5$
19. Find the fourth term in the expansion $(p^2 - q)^{10}$
20. Find the term in the expansion of $(x - y)^{12}$ containing x^9.
21. Evaluate each.

    a. $7!$ **b.** $\frac{100!}{99!}$ **c.** $\frac{9!}{6!3!}$ **d.** $\frac{(n+1)!}{n!}$
22. Expand $(3a + b)^5$ using Pascal.
23. Find the sum of the infinite series $\frac{1}{2} + \frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \ldots$
24. Write the first four terms in the sequence formed by the pattern $t_n = n + \frac{1}{n}$.
25. Find the limit $\lim_{n \to \infty} \frac{2n+1}{3n+1}$
26. Find the limit $\lim_{n \to \infty} \frac{8n^2 - 3n}{5n^2 + 7}$
27. Find the limit $\lim_{n \to \infty} \frac{2n+1}{3}$
28. Change to a fraction in lowest terms: 0.636363…
29. Given the sequence $t_1 = 3, t_n = 2t_{n-1} + 1$, list the first five terms in the sequence.
30. Give a recursive definition 9, 13, 17, 21, …

Solutions

1. 14th
2. 70, 85, 100
3. -17
4. 12
5. 731
6. $\sum_{k=1}^{9} (3k + 4)$
7. 99
8. 847
9. $\frac{32}{3}$
10. 24, 12, 6, 3
11. $\frac{3}{16}$
12. 5th
13. 11
14. 93
15. 3; 384
16. 4
17. $16x^4 - 32x^3y^2 + 24x^2y^4 - 8xy^6 + y^8$
18. $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$
19. $-120p^{14}q^3$
20. 4th term $-220x^9y^3$
21. a. 5040 b. 100 c. 84 d. $n+1$
22. $243a^5 + 405a^4b + 270a^3b^2 + 90a^2b^3 + 15ab^4 + b^5$
23. $\frac{3}{2}$
24. $2, \frac{5}{2}, \frac{10}{3}, \frac{17}{4}$
25. $\frac{2}{3}$
26. $\frac{8}{5}$
27. Does Not Exist
28. $\frac{7}{11}$
29. 3, 7, 15, 31, 63
30. $t_1 = 9, t_n = t_{n-1} + 4$