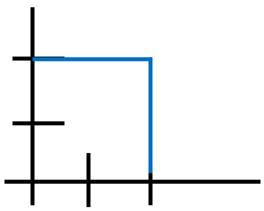
If we have a constant velocity then we can determine the distance travelled.

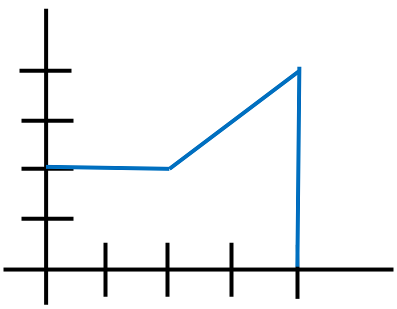
Ex: Travel 30 mph (one direction) for 2 hours = travel 60 miles

On the graph it looks like:



In this case, distance travelled = area under the velocity curve (because v > 0)

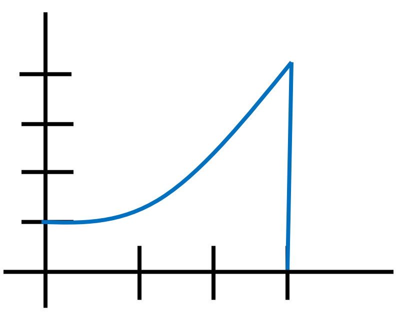
Can we find the total distance travelled?

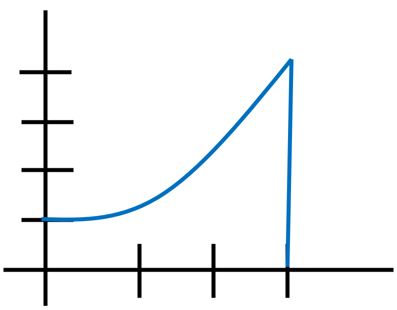


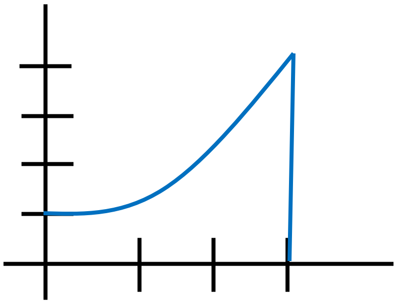
But suppose the velocity curve doesn't have a constant rate of change? (not linear)

Now we don't know formulas for area? What to do?

Estimate! What are our options?







Ex 2: Coffee is poured into a big cup at a rate of *R(t)* measured in in^3/sec.

t	2	4	6	8	10	12	14
R(t)	1	.5	2	3	2.5	1	.5

How much coffee pours into cup over 14 seconds of time?

A. Using LRAM n = 6

B. Using RRAM n = 6

C. Using MRAM n = 3

Ex 3: Ship is travelling on the ocean with velocity each 15 minutes at a rate of *v(t)*.

t	0	15	30	45	60	75	90
v(t)	10	12	11	20	16	17	14

How much coffee pours into cup over 14 seconds of time?

A. Using LRAM n = 6

B. Using RRAM n = 6

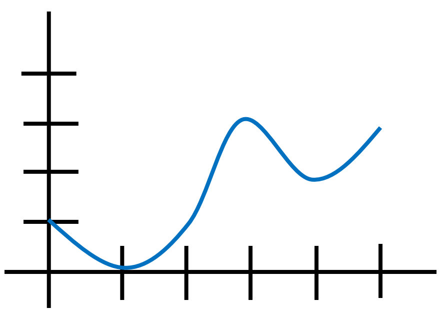
C. Using MRAM n = 3

These are called Riemann Sums:

Ex 4: $y = x^2 + 3$; $[0, 5]$, n = 5, RRAM, LRAM, MRAM Use Calc. table feature and sketch.

Upper Sums – Circumscribed Rectangles

Lower Suns – Inscribed Rectangles



Upper Sums:

Lower Sums:

Summations:

Ex: $\sum_{i=1}^{n} c = cn$

Ex: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Ex: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Ex: $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

HW: pg. 261 #1,5,7,11,13,15,19,23,25 – 31 odd, 32 – 34 all, 35 – 43 odd.

Friday Homework:

1. The odometer on your car is broken. Estimate the distance driven over a 30 second time interval, given the table below: (convert to ft./sec)

t (sec)	0	5	10	15	20	25	30
V(t) (mph)	17	21	24	29	32	31	28

A. Using LRAM n = 6

B. Using RRAM n = 6

C. Using MRAM n = 3

2. Oil is leaking from a tank at R(t) liters/ hr. Estimate the total amunt of oil that leaked out in those 10 hours using the chart below:

t (hrs)	0	2	4	5	8	10	12
R(t)	8.7	7.6	6.8	6.2	5.7	5.3	5

A. Using LRAM n = 6

B. Using RRAM n = 6

C. Using MRAM n = 3

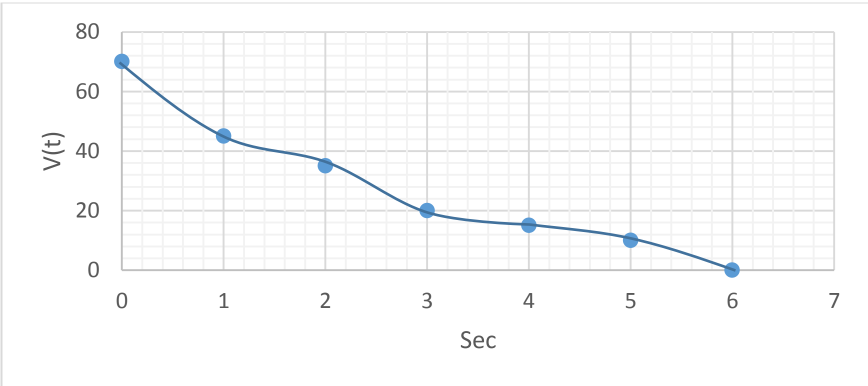
3. The table below, provided by NASA, gives velocity data on the space shuttle Endeavour's Flight. Estimate height in miles above the earth at rocket booster separation.

Event	Launch	Begin Roll	End Roll	Throttle to 89%	Throttle to 67 %	Throttle to 104%	Max Dynamic Pressure	Rocket Booster Separate
t (sec)	0	10	15	20	32	59	62	125
V(t) (mph)	0	185	319	447	742	1325	1445	4151

A. Using LRAM n = 7

B. Using RRAM n = 7

4. A car hits the brakes at t = 0. Estimate the distance it takes the car to stop.



A. Using LRAM n = 6

B. Using RRAM n = 6

C. Using MRAM n = 3

5. An ostrich runs at 20 kph for 2 minutes, 12 kph for 3 minutes, and 40 kph for a minute. How far did the ostrich run? (Assuming it ran in the same direction the entire time)

Approximate the Upper and Lower Sums. (Using circumscribed and inscribed rectangles)

6. $y = 2x^2 + 3$; $[-2, 3]$, n = 5

7. $y = \sqrt{x}$; $[0, 4]$, n = 8

8. $y = \sin(x)$; $[0, \pi]$, n = 4

Approximate the Area Under the Curve using LRAM, RRAM, MRAM

9. $y = x^3$; $[0, 2]$, n = 4

10. $y = \sin(x)$; $[0, \pi]$, n = 4

11. $y = e^{-x}$; $[0, 2]$, n = 4 Use a Calculator for this one.

Warm-up

1. Find $\sum_{i=1}^{n}(2i-3)^2$

2. A particle starts at x = 0 and moves along the x-axis with velocity of $v(t) = t^2$ for time $t \geq 0$. Where is the particle at t = 3?

LRAM:

RRAM:

MRAM:

Upper Sum:

Lower Sum:

Riemann Sums = let # of rectangles be n = 10, 100, 1,000 – exact area on a closed interval.

Ex: Find the area under $y = x^2$; $[0, 2]$, n = 10, n = 100, n = 1,000, exact area

1. Find the width:

2. x's

3. A ~ $\sum_{k=1}^{n} \frac{Area\ of}{k+h\ rect} = \sum_{k=1}^{n} width\ \frac{height\ of}{k+h\ rect}$

Ex: Find the area under $y = 2x^2 + 1$; $[0, 4]$, n = 10, n = 100, n = 1,000, exact area

1. Find the width:

Ex: Find the area under $y = 3x^2 + x$; $[1, 3]$, n = 10, n = 100, n = 1,000, exact area

1. Find the width:

Ex: Find the area under $y = 2x^3 + 5x$; $[0, 3]$, n = 10, n = 100, n = 1,000, exact area

1. Find the width:

Homework pg. 263: # 47, 49, 52, 53

Also $y = 2x + 1$ $[0, 4]$, $y = 3x^2$ $[2, 4]$, $y = x^2 + x$ $[0, 2]$, $y = x^2 - 3x$ $[1, 3]$

Riemann Sums:

Ex: Find the area under $y = x^2 + 1$; $[a, b]$

1. Find the width:

2. Set up the Area Formula:

Ex: Find the area under $y = 2x + 1$; $[a, b]$

1. Find the width:

2. Set up the Area Formula:

Ex: Find the area under $y = x^3$; $[a, b]$

1. Find the width:

2. Set up the Area Formula:

Ex: Find the area under $y = x^2 + x$; $[a, b]$

1. Find the width:

2. Set up the Area Formula:

Homework: $y = 3x + 2$, $y = 3x^2$, $y = x$, $y = x^2 + 3$, $y = 2x^2 - 5$, $y = x^3 + 4$

Go over examples from homework.

The Integrand: (Label parts)

Upper Limit   Integrand

$$\int_a^b f(x)\ dx$$

Lower Limit

Height   Width

Definite Integrands: have the interval $[a, b]$ defined and will result in a numerical value related to the Area under the curve.

Indefinite Integrands: gives us a family of functions whose derivative is *f(x)*.

Ex: $\int x\ dx$

Ex: $\int x^2\ dx$

Ex: $\int (x^2 + 3)\, dx$

Ex: $\int x^3\ dx$

Find:

Ex: $\int \sqrt{x}\ dx$

Ex: $\int x^{-3}\ dx$

Ex: $\int \cos(x)\, dx$

Ex: $\int \sin(x)\, dx$

Are there any integrals we want to know?

Examples:

Ex: $\int_{1}^{3} x\,dx$

Ex: $\int_{0}^{4} x^2\ dx$

Ex: $\int_{1}^{3} 3x^2 + x\ dx$

Ex: $\int_{o}^{\pi} \sin(x)\,dx$

What do we do in these cases?

Ex: $\int_{-1}^{3} |2x-1|\ dx$

Ex: $\int_{-2}^{2} \left(4-x^2\right) dx$

Ex: $\int_{0}^{3} \sqrt{9-x^2}\ dx$

Ex: $\int_{-2}^{2} 2-|x|\ dx$

Ex: $\int_{-3}^{7} 4\ dx$

Ex: $f(x)=\begin{cases} x^2-7 & x \le 1 \\ \sqrt{x} & x > 1 \end{cases}$ Find: $\int_{-1}^{3} f(x)\,dx$

Integral and Area Relationship:

Ex: $\int_{0}^{2} x^3 \, dx$

Ex: $\int_{-2}^{0} x^3 \, dx$

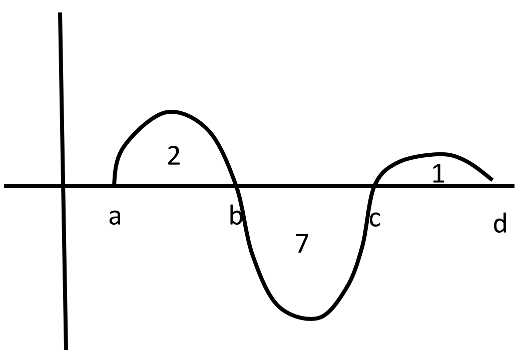
Ex: $\int_{-2}^{2} x^3 \, dx$

Ex: Given that $\int_{2}^{7} f(x)\, dx = 3$ and $\int_{4}^{2} f(x)\, dx = 1$

A. Find $\int_{4}^{7} f(x)\, dx$

B. Find $\int_{2}^{4} f(x)\, dx$

C. Find $\int_{2}^{7} (3f(x) - 4)\, dx$



$\int_{a}^{b} f(x)\, dx =$

$\int_{a}^{d} f(x)\, dx =$

$\int_{a}^{c} f(x)\, dx =$

$\int_{b}^{d} f(x)\, dx =$

$\int_{b}^{c} f(x)\, dx =$

Even Functions: $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ Odd Functions: $\int_{-a}^{a} f(x)\,dx = 0$

Also $\int_{a}^{b} f(x)\,dx = -\int_{b}^{a} f(x)\,dx$

The Fundamental Theorem of Calculus:

States that differentiation and (definite) integration are inverse operations.

If a function *f* is continuous on the closed interval $[a,b]$ and F is an antiderivative of *f* on the interval $[a,b]$, then $\int_{a}^{b} f(x)\,dx = F(b) - F(a)$

Ex: $\int_{1}^{3} x^3\,dx = \frac{x^4}{4}$

Ex: $\int_{1}^{3} (x^2 - 3)\,dx = \frac{x^3}{3} - 3x$

Ex: $\int_{-\pi/4}^{-\pi/6} 2\cos(x)dx =$

Ex: $\int_{-3}^{0} 4x^{\frac{1}{3}}\,dx =$

Ex: $\int_{0}^{2} |2x-1|\,dx = \int_{0}^{1/2} -(2x-1)dx + \int_{1/2}^{2} (2x-1)dx$

Warm Up

Evaluate each definite integral.

1. $\int_{-5}^{1} -\left|x^2 + 4x\right| \, dx$

2. Ex: $f(x) = \begin{cases} \dfrac{x}{2} - 1, & x \le 2 \\ x^2 - 6x + 8, & x > 2 \end{cases}$ Find: $\int_{0}^{3} f(x) \, dx$

3. A car travelling at 75 miles per hour is brought to a stop, at a constant deceleration, 150 feet from where the brakes are applied.

   a. How far has the car moved when its speed has been reduced to 40 mph?

Differential Equations

1. Find *f* given that $f'(x) = 1 - 6x$ and $f(2) = 8$

2. Find *f* given that $f'(x) = \sqrt{x}(6 + 5x)$ and $f(1) = 10$

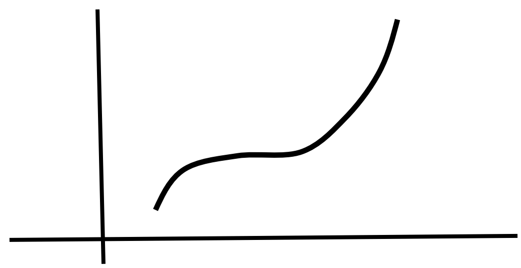
3. Find *f* given that $f''(x) = 2 + \cos(x)$ and $f(0) = -1$ and $f\left(\frac{\pi}{2}\right) = 0$

Homework: pg. 250 # 49 – 52, 55 – 62, 71 – 74, 77, 79 – 84, 87, 89 – 95

Let's think of this from a test score stand point 90 + 88 + 94 + 80

Mean Value Theorem for Integrals:

If a function *f* is continuous on the closed interval $[a,b]$ then there exists a number c in the closed interval $[a,b]$ such that $\int_a^b f(x)\,dx = f(c)(b-a)$



Definition of the Average Value of a Function on an Interval

If *f* is integrable on the closed interval $[a,b]$,then the **average value** of *f* on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Ex: Let's think about $f(x) = x^2$ on the interval $[0,4]$.

Ex: Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[1,4]$.

Ex: Find the average value of $f(x) = \sin(x)$ on the interval $[0,\pi]$.

Ex: Find the average value of $f(x)=\sqrt{x}$ on the interval $[0,9]$.

The Second Fundamental Theorem of Calculus:

If f is continuous on an open interval I containing a, then, for every x in the interval.

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right]=f(x)$$

Ex: $F(x)=\int_0^x \sin(t)\,dt \;\; at \;\; x=0,\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3}, \; and \; \frac{\pi}{2}$

$F(x)=\int_0^x t^3\,dt$

Ex: $F(x)=\int_0^x \sqrt{t}\,dt$

Ex: $F(x)=\int_{\pi/2}^{x^3} \cos(t)\,dt$

Ex: $F(x)=\int_{\sqrt{x}}^{\tan(x)} \ln(t)\,dt$

Hmwk: pg. 284 – 285 # 1 -31 odd, 35, 37, 39, 45 – 51 odd, 54 – 61, 73 – 93 odd

Warm Up

Find the values of c that satisfy the Mean Value Theorem for Integrals.

1. $f(x) = -\frac{x^2}{2} + x + \frac{3}{2}$ $[-3, 1]$

Find the Average Value of the function over the given interval. Then, find the values of c that satisfy the Mean Value Theorem for Integrals.

2. $f(x) = -x + 2$ $[-2, 2]$

Integration by Substitution:

1. $\int \sqrt{3x+2}\,dx =$

Let $u = 3x + 2$

2. $\int x\left(x^2+1\right)^2 dx =$

Let $u = x^2 + 1$

3. $\int 16x^3 \sec^2\left(4x^4 - 2\right)dx =$

4. $\int \left(5x^4+5\right)^{\frac{2}{3}} \bullet 20x^3\,dx =$

5. $\int x\sqrt{3x^2+4}\,dx =$

6. $\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx =$

7. $\int \frac{-8x^3}{\left(-2x^4+5\right)^5}\,dx =$

8. $\int 36x^3\left(3x^4+3\right)^5 dx =$

9. $\int 20x\sin\left(5x^2-3\right)dx =$

10. $\int \frac{4x^3}{\csc\left(x^4-1\right)}dx =$

11. $\int \frac{x+1}{\left(x^2+2x+6\right)^2}dx =$

12. $\int x\cos\left(x^2+4\right)dx =$

Evaluate

13. $\int_{-\pi}^{\pi} \cos\left(\frac{x}{4}\right)dx =$

14. $\int_{-2}^{2} \left(x\sin^4(x)+x^3-x^4\right)dx =$

pg. 297 # 1 – 38

Integration by Substitution: Changing the Variable

1. $\int \sin^2(3x)\cos(3x)dx =$

2. $\int x\sqrt{2x-1}\,dx =$

3. $\int_0^1 x(x^2+1)^3\,dx =$

4. $\int_0^{\pi/4} \tan(x)\sec^2(x)dx =$

5. $\int_1^5 \frac{x}{\sqrt{2x-1}}dx =$

6. $\int_1^5 \frac{2x+1}{\sqrt{x+4}}dx =$

pg. 297 # 41 -50, 55 – 62, 65 – 76, 83 – 87 odd, 90, 91

pg. 299 # 109, 110