If we have a constant velocity then we can determine the distance travelled.

Ex: Travel 30 mph (one direction) for 2 hours = travel 60 miles

On the graph it looks like:

In this case, distance travelled = area under the velocity curve (because v > 0)

Can we find the total distance travelled?

But suppose the velocity curve doesn’t have a constant rate of change? (not linear)

Now we don’t know formulas for area? What to do?

**Estimate! What are our options?**

Ex 2: Coffee is poured into a big cup at a rate of *R(t)* measured in in3/sec.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t | 2 | 4 | 6 | 8 | 10 | 12 | 14 |
| *R(t)* | 1 | .5 | 2 | 3 | 2.5 | 1 | .5 |

How much coffee pours into cup over 14 seconds of time?

1. Using LRAM n = 6
2. Using RRAM n = 6
3. Using MRAM n = 3

Ex 3: Ship is travelling on the ocean with velocity each 15 minutes at a rate of *v(t)*.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t | 0 | 15 | 30 | 45 | 60 | 75 | 90 |
| *v(t)* | 10 | 12 | 11 | 20 | 16 | 17 | 14 |

How much coffee pours into cup over 14 seconds of time?

1. Using LRAM n = 6
2. Using RRAM n = 6
3. Using MRAM n = 3

These are called Riemann Sums:

Ex 4: ; , n = 5, RRAM, LRAM, MRAM Use Calc. table feature and sketch.

Upper Sums – Circumscribed Rectangles

Lower Suns – Inscribed Rectangles

Upper Sums:

Lower Sums:

Summations:

Ex:  Ex: 

Ex:  Ex: 

HW: pg. 261 #1,5,7,11,13,15,19,23,25 – 31 odd, 32 – 34 all, 35 – 43 odd.

Friday Homework:

1. The odometer on your car is broken. Estimate the distance driven over a 30 second time

interval, given the table below: (convert to ft./sec)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (sec) | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| *V(t) (mph)* | 17 | 21 | 24 | 29 | 32 | 31 | 28 |

1. Using LRAM n = 6
2. Using RRAM n = 6
3. Using MRAM n = 3

2. Oil is leaking from a tank at R(t) liters/ hr. Estimate the total amunt of oil that leaked out in

those 10 hours using the chart below:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t (hrs) | 0 | 2 | 4 | 5 | 8 | 10 | 12 |
| *R(t)* | 8.7 | 7.6 | 6.8 | 6.2 | 5.7 | 5.3 | 5 |

1. Using LRAM n = 6
2. Using RRAM n = 6
3. Using MRAM n = 3

3. The table below, provided by NASA, gives velocity data on the space shuttle Endeavour’s

Flight. Estimate height in miles above the earth at rocket booster separation.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Event | Launch | Begin Roll | End Roll | Throttle to 89% | Throttle to 67 % | Throttle to 104% | Max Dynamic Pressure | Rocket Booster Separate |
| t (sec) | 0 | 10 | 15 | 20 | 32 | 59 | 62 | 125 |
| *V(t) (mph)* | 0 | 185 | 319 | 447 | 742 | 1325 | 1445 | 4151 |

1. Using LRAM n = 7
2. Using RRAM n = 7

4. A car hits the brakes at t = 0. Estimate the distance it takes the car to stop.

1. Using LRAM n = 6
2. Using RRAM n = 6
3. Using MRAM n = 3

5. An ostrich runs at 20 kph for 2 minutes, 12 kph for 3 minutes, and 40 kph for a minute. How

far did the ostrich run? (Assuming it ran in the same direction the entire time)

Approximate the Upper and Lower Sums. (Using circumscribed and inscribed rectangles)

6. ; , n = 5

7. ; , n = 8

8. ; , n = 4

Approximate the Area Under the Curve using LRAM, RRAM, MRAM

9. ; , n = 4

10. ; , n = 4

11. ; , n = 4 Use a Calculator for this one.

Warm-up

1. Find 

2. A particle starts at x = 0 and moves along the x-axis with velocity of for time . Where is the particle at t = 3?

LRAM:

RRAM:

MRAM:

Upper Sum:

Lower Sum:

Riemann Sums = let # of rectangles be n = 10, 100, 1,000 – exact area on a closed interval.

Ex: Find the area under ; , n = 10, n = 100, n = 1,000, exact area

1. Find the width:

2. x’s

3. A ~ =

Ex: Find the area under ; , n = 10, n = 100, n = 1,000, exact area

1. Find the width:

Ex: Find the area under ; , n = 10, n = 100, n = 1,000, exact area

1. Find the width:

Ex: Find the area under ; , n = 10, n = 100, n = 1,000, exact area

1. Find the width:

Homework pg. 263: # 47, 49, 52, 53

Also , , , 

Riemann Sums:

Ex: Find the area under ; 

1. Find the width:

2. Set up the Area Formula:

Ex: Find the area under ; 

1. Find the width:

2. Set up the Area Formula:

Ex: Find the area under ; 

1. Find the width:

2. Set up the Area Formula:

Ex: Find the area under ; 

1. Find the width:

2. Set up the Area Formula:

Homework: , , , , , 

Go over examples from homework.

The Integrand: (Label parts)

Upper Limit

Integrand



Lower Limit

Width

Height

**Definite Integrands**: have the interval defined and will result in a numerical value related

to the Area under the curve.

**Indefinite Integrands**: gives us a family of functions whose derivative is *f(x)*.

Ex:  Ex: 

Ex:  Ex: 

Find:

Ex:  Ex: 

Ex:  Ex: 

Are there any integrals we want to know?

Examples:

Ex:  Ex: 

Ex:  Ex: 

What do we do in these cases?

Ex:  Ex: 

Ex:  Ex:  Ex: 

Ex:  Find: 

Integral and Area Relationship:

Ex:  Ex:  Ex: 

Ex: Given that  and 

A. Find 

B. Find 

C. Find 

 

 



2

1

d

c

b

a

7

Even Functions: =  Odd Functions: 

Also= 

The Fundamental Theorem of Calculus:

States that differentiation and (definite) integration are inverse operations.

If a function *f* is continuous on the closed interval and F is an antiderivative of *f* on the interval , then 

Ex:  Ex: 

Ex:  Ex: 

Ex: 

Warm Up

Evaluate each definite integral.

1. 

2. Ex:  Find: 

3. A car travelling at 75 miles per hour is brought to a stop, at a constant deceleration, 150 feet from where the brakes are applied.

1. How far has the car moved when its speed has been reduced to 40 mph?

Differential Equations

1. Find *f* given that and 

2. Find *f* given that and 

3. Find *f* given that and and

Homework: pg. 250 # 49 – 52, 55 – 62, 71 – 74, 77, 79 – 84, 87, 89 – 95

Let’s think of this from a test score stand point 90 + 88 + 94 + 80

Mean Value Theorem for Integrals:

If a function *f* is continuous on the closed interval then there exists a number c in the closed interval  such that 

Definition of the Average Value of a Function on an Interval

If *f* is integrable on the closed interval ,then the **average value** of *f* on the interval is 

Ex: Let’s think about on the interval .

Ex: Find the average value of on the interval .

Ex: Find the average value of on the interval .

Ex: Find the average value of on the interval .

The Second Fundamental Theorem of Calculus:

If *f* is continuous on an open interval *I* containing a, then, for every x in the interval.



Ex: 

 Ex: 

Ex:  Ex: 

Hmwk: pg. 284 – 285 # 1 -31 odd, 35, 37, 39, 45 – 51 odd, 54 – 61, 73 – 93 odd

Warm Up

Find the values of c that satisfy the Mean Value Theorem for Integrals.

1. 

Find the Average Value of the function over the given interval. Then, find the values of c that satisfy the Mean Value Theorem for Integrals.

2. 

Integration by Substitution:

1.  2. 

Let  Let 

3.  4. 

5.  6. 

7.  8. 

9.  10. 

11.  12. 

Evaluate

13.  14. 

pg. 297 # 1 – 38

Integration by Substitution: Changing the Variable

1.  2. 

3.  4. 

5.  6. 

pg. 297 # 41 -50, 55 – 62, 65 – 76, 83 – 87 odd, 90, 91

pg. 299 # 109, 110