Review Worksheet

Simplify each problem.

1. $2(-3.5)^0$

2. $-(10)^2$

3. $(5)^{-4}$

4. $7^{-1} \bullet 10^0$

5. $3^5 \bullet 3^{-8} \bullet 3^6$

6. $(2)^{-7}(2)^2$

7. $8m^{-4}n^{-5}$

8. $\dfrac{9}{a^5b^{-3}}$

9. $2^{-2} \bullet 8^3$

10. $7x^{-3} \bullet 4x^5$

11. $\left(-5x^5\right) \bullet 3y^5 \bullet 7x^5$

12. $\left(k^4\right)^3$

13. $4 \bullet 4^{3x} \bullet 4^{x+3}$

14. $\left(d^3\right)^{-2}$

15. $\left(x^8\right)^3\left(x^{-4}\right)^0$

16. $\left(-n^6\right)^9$

17. $\left(4g^8\right)^3$

18. $\left(-3a^3b^4\right)^3\left(a^5b^5\right)^5$

19. $\dfrac{6^5}{6^7}$

20. $\dfrac{c^{12}}{c^4}$

21. $\dfrac{t^5}{t^9}$

22. $\dfrac{3^9}{3^6}$

23. $\dfrac{m^7n^9}{m^{20}n^{-2}}$

24. $\left(\dfrac{a}{7}\right)^3$

25. $\left(-\dfrac{3}{5}\right)^4$

26. $\left(\dfrac{(-1)^7}{(-4)^{-2}}\right)^2$

11.1 Rational Exponents

1. Properties of Exponents:

Suppose m and n are positive integers, and a and b are real numbers.
Then the following properties hold.

> Product Property: $a^m a^n = a^{m+n}$
>
> Power of a Power Property: $(a^m)^n = a^{mn}$
>
> Power of a Quotient Property: $(\frac{a}{b})^m = \frac{a^m}{b^m}$
>
> Power of a Product Property: $(ab)^m = a^m b^m$

2. Evaluate each expression. $\frac{a^m}{a^n} = a^{m-n}$,

a. $3^2 3^3$ b. $(3^2)^3$ c. $\left(\frac{1}{2}\right)^3$ d. $(2a)^3$ e. $\frac{2^5}{2^3}$

3. Negative Exponents:

To get rid of a negative exponent, "move" the base that is being carried to the negative exponent.

Ex. $2^{-1} = \frac{2^{-1}}{1} = \frac{1}{2^1}$ or $\frac{3}{b^{-2}} = \frac{3b^2}{1}$

4. Definition of $b^{\frac{1}{n}}$:

For any real number $b \geq 0$ and any integer n > 1, $b^{\frac{1}{n}} = \sqrt[n]{b}$
This also holds when b<0 and n is odd.

5. Evaluate:

a. $625^{\frac{1}{4}}$ b. $3^{\frac{1}{2}} \cdot 21^{\frac{1}{2}}$ c. $8^{-\frac{2}{3}}$

6. Definition of Rational Exponents:

For any nonzero number b, and any integers m and n with n > 1, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$
Except when $\sqrt[n]{b}$ is not a real number.

7. Evaluate each.

a. $81^{\frac{5}{4}}$ b. $36^{\frac{3}{2}}$ c. $64^{\frac{5}{3}}$

8. Express using rational exponents.

a. $\sqrt{23}$ b. $\sqrt[3]{63}$ c. $\sqrt[4]{16z^2}$ d. $\sqrt[3]{5x^2y}$ e. $\sqrt[4]{27x^4y^3}$

9. Express using radicals.

a. $6^{\frac{1}{5}}$ b. $4^{\frac{1}{3}}$ c. $c^{\frac{2}{5}}$ d. $(x^2)^{\frac{4}{3}}$ e. $(5a)^{\frac{2}{3}}b^{\frac{5}{3}}$

10. Simplify each.

a. $y^{\frac{5}{3}}y^{\frac{7}{3}}$ b. $(b^{\frac{1}{3}})^{\frac{3}{5}}$ c. $\sqrt[3]{a^4b^8}$ d. $\sqrt{8m^5n^4}$ e. $\sqrt[4]{32a^9b^{11}}$

11.2 Exponential Functions

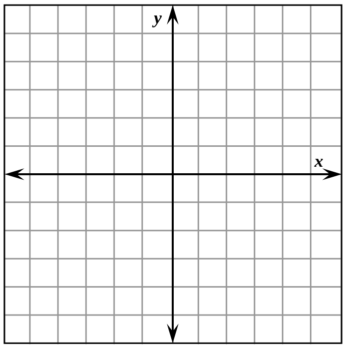
11. Evaluate to the nearest ten thousandth.

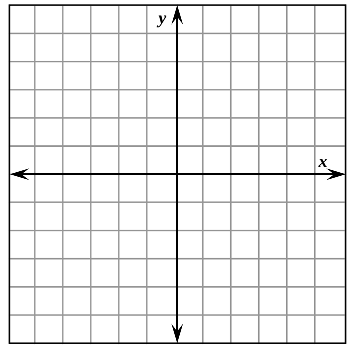
a. $5^{\sqrt{2}}$ b. 0.4^{π}

12. Exponential Function:

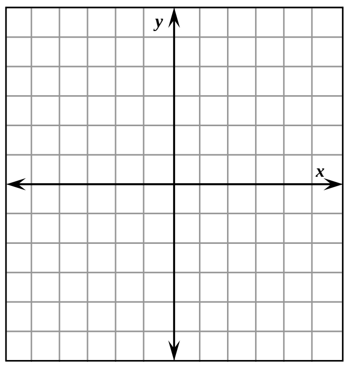
Has the form $y = a^x$, where a is a positive real number. Ex. 2^x, 3^x, $\left(\frac{1}{5}\right)^x$

13. Graph and compare the graphs of $y = 2^x$ and $y = 2^{-x}$.





14. Graph $y=\left(\frac{1}{2}\right)^x$.

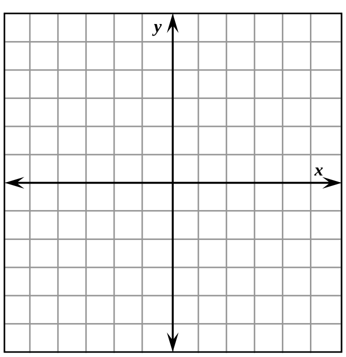


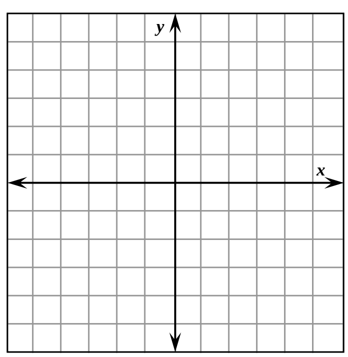
Describe the transformations of the parent graph $y=\left(\frac{1}{2}\right)^x$.

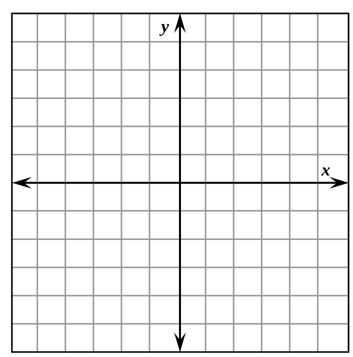
$y=\left(\frac{1}{2}\right)^x+1$

$y=\left(\frac{1}{2}\right)^x-2$

$y=3\left(\frac{1}{2}\right)^x$







Homework: Page 602 #1-51 odd and Page 612 #17-28 and Worksheet

DAY 1 HW
11.1 Worksheet

Simplify each problem.

1. $5^{\sqrt{2}} \bullet 5^{\sqrt{2}}$

2. $\left(5^{\sqrt{2}}\right)^{\sqrt{2}}$

3. $\dfrac{5^{\sqrt{2}+2}}{5^{\sqrt{2}-2}}$

4. $7^{\sqrt{3}} \bullet 7^{\sqrt{2}}$

5. $\left(7^{\sqrt{3}}\right)^{\sqrt{2}}$

6. $\dfrac{7^{\sqrt{3}+2}}{49}$

7. $\left(8^{\pi}\right)^{2}$

8. $\sqrt{6^{2\pi}}$

9. $\sqrt[3]{4^{6\pi}}$

10. $\dfrac{10^{\sqrt{3}-2}}{10^{\sqrt{3}+2}}$

11. $\dfrac{5^{\sqrt{2}}6^{\sqrt{8}}}{6^{3\sqrt{2}}}$

12. $\left(2^{\sqrt{2}}\right)^{-\frac{1}{\sqrt{2}}}$

13. $8^{1.2} \bullet 2^{-3.6}$

14. $\left(\sqrt{2}^{\,\pi}\right)^{0}$

15. $\dfrac{25^{2.4}}{5^{5.8}}$

16. $\dfrac{\left(1+\sqrt{3}\right)^{\pi-1}}{\left(1+\sqrt{3}\right)^{\pi+1}}$

17. $\dfrac{6^{\sqrt{2}}}{6^{-\sqrt{2}}}$

18. $4^{\pi} \bullet 2^{3-2\pi}$

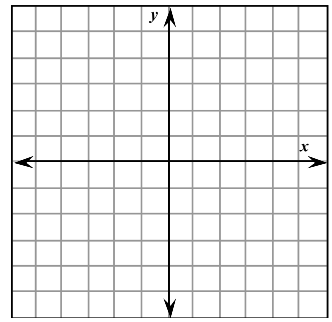
19. $\dfrac{5^{\sqrt{3}+1}}{5^{0} \bullet 5^{\sqrt{3}}}$

20. $\dfrac{7^{\sqrt{2}-1}}{7^{\sqrt{2}-2}}$

21. $\dfrac{\left(\sqrt{2}+\sqrt{3}\right)^{2+\pi}}{\left(\sqrt{2}+\sqrt{3}\right)^{\pi}}$

11.2 The Number *e*

1. An irrational number, symbolized by the letter e, appears as the base in many applied exponential functions. This irrational number is approximately equal to 2.72. More accurately, $e = 2.71828\ldots$



2. **Evaluate:**
   a. e b. e^3 c. e^{-2}

3. **Formulas for Compound Interest:** After t years, the balance A in an account with principal P and annual interest rate r (expressed as a decimal) is given by the following formulas.

   For n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

   For continuous compounding: $A = Pe^{rt}$

4. You invest $500 in a savings account that pays 3.5% annual interest. How much will be in the account after five years?

5. You invest $500 in a savings account that pays 3.5% annual interest compounded semi-annually. How much will be in the account after five years?

6. You invest $500 in a savings account that pays 3.5% annual interest compounded quarterly. How much will be in the account after five years?

7. You invest $500 in a savings account that pays 3.5% annual interest compounded monthly. How much will be in the account after five years?

8. You invest $500 in a savings account that pays 3.5% annual interest compounded daily. How much will be in the account after five years?

9. You invest $500 in a savings account that pays 3.5% annual interest compounded continuously. How much will be in the account after five years?

10. A population of insects is growing in such a way that the number in the population t days from now is given by the formula $P = 4000e^{0.02t}$. How large will the population be in one week?

Homework: Page 612 #17-28 and Page 617 #12-17, 25-28, 32, 33

DAY 2 HW
Page 612 #17-28 and Page 617 #12-17, 25-28, 32, 33

Graph each equation.

17. $y = 3^x$

18. $y = \left(\frac{1}{3}\right)^x$

19. $y = 3^{x-2}$

20. $y = 3^{-x}$

21. $y = -3^x$

22. $y = \left(\frac{1}{5}\right)^x$

23. $y = 2^{x+3}$

24. $y = -2^{x+3}$

25. $y = -2^{x-3}$

Use a calculator to evaluate each expression to the nearest thousandth.

12. $e^{1.6}$

13. $e^{4.3}$

14. $\sqrt[3]{e}$

15. $2\sqrt[4]{e^3}$

16. $4\sqrt[3]{e^2}$

17. e^0

Find the amount at the end of each investment.

25. P = $1,000, r = 10%, t = 4 years, monthly compounding

26. P = $3,200, r = 6%, t = 5 years and 6 months, quarterly compounding

27. P = $750, r = 5.5%, t = 3 years and two months, continuous compounding

28. P = $45,000, r = 7.2%, t = 30 years, daily compounding

32. The yield, y, in millions of cubic feet of trees per acre for a forest stand that is *t* years old is given by $y = 6.7e^{\frac{-48.1}{t}}$.

A. Find the yield after 15 years.

B. Find the yield after 50 years.

11.4 Logarithmic Functions

1. **Definition of a Logarithmic Function:**
   The logarithmic function $y = \log_a x$, where a > 0 and $a \neq 1$, is the inverse of the exponential function $y = a^x$.

2. Write in exponential form.

   a. $\log_5 125 = 3$ b. $\log_{13} 169 = 2$ c. $\log_4 \frac{1}{4} = -1$

   d. $\log_{\frac{1}{5}} 25 = -2$ e. $\log 100 = 2$ f. $\log \frac{1}{1000} = -3$

3. Write in logarithmic form.

   a. $8^3 = 512$ b. $3^3 = 27$ c. $5^{-3} = \frac{1}{125}$

4. Evaluate each expression.

   a. $\log_2 16$ b. $\log_{12} 144$ c. $\log_{16} 4$

   d. $\log_3 243$ e. $\log_2 \frac{1}{32}$ f. $\log_3 \frac{1}{81}$

5. **Properties of Logarithms:**
   Suppose m and n are positive numbers, b is a positive number other than 1, and p is any real number. Then the following properties hold.

   Product Property: $\log_b mn = \log_b m + \log_b n$

   Quotient Property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

   Power Property: $\log_b m^p = p \log_b m$

$\log_b m = \log_b n, \qquad m = n.$

6. Write the expression as the logarithm of a single quantity.

   a. $\log x + 3\log y$ b. $\log(2x+5) - \log x$ c. $\frac{1}{2}(\log_5 x + \log_5 y) - 2\log_5(x+1)$

7. Use the properties of logarithms to write the expression as a sum, difference, and/or constant multiple of logarithms. (Assume all variables are positive).

a. $\log_9 9x$ b. $\log_4\left(\dfrac{64}{y}\right)$ c. $\log_b\left(\dfrac{x^2 y}{z^2}\right)$

8. Evaluate each.

a. $\log_3(\log_4 64)$ b. $\log_2[\log_2(\log_3 81)]$

9. If $\log_7 6 = x$ and $\log_7 5 = y$, express the following in terms of x and y:

a. $\log_7 35$ b. $\log_7 7.2$

10. Inverse Properties of Logarithms:

For $b > 0$,

$\log_b b^x = x$ $\log_b b^x = x$ The logarithm with base b of b raised to a power equals that power.

$b^{\log_b x} = x$ $b^{\log_b x} = x$ b raised to the logarithm with base b of a number equals that number.

11. Evaluate each.

a. $\log_4 4^x$ b. $\log_7 7^8$ c. $6^{\log_6 9}$ d. $3^{\log_3 17}$

12. Solve each of the following.

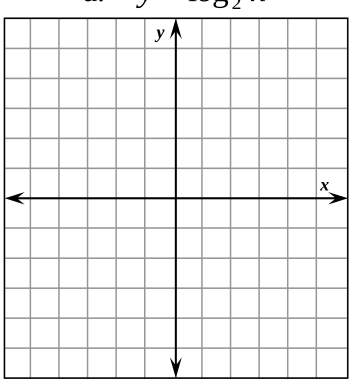
a. $\log_b 5 = -\dfrac{1}{3}$ b. $\log_3 5 + \log_3 x = \log_3 10$ c. $\log 16 - \log 2t = \log 2$

d. $\log(2x+5) = \log(5x-4)$ e. $\log_3(4x+5) - \log_3(3-2x) = 2$ f. $2\log 6 - \dfrac{1}{3}\log 27 = \log x$

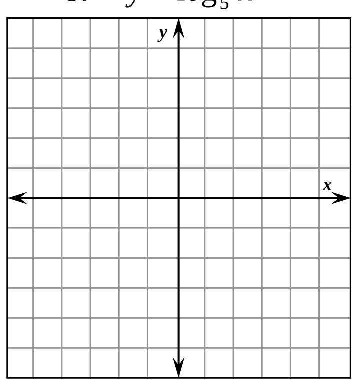
g. $\log_6(a^2+2) + \log_6 2 = 2$ h. $\log_3(x-5) + \log_3(x+3) = 2$ i. $\log x + \log(x-3) = 1$

13. Graph each.

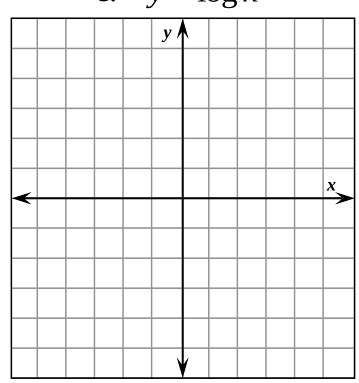
a. $y = \log_2 x$



b. $y = \log_5 x$



c. $y = \log x$



DAY 3 HW

11.4 Laws of Logarithm Worksheet

Write each equation in logarithmic form.

1. $2^5 = 32$ 2. $5^{-3} = \frac{1}{125}$ 3. $6^{-3} = \frac{1}{216}$

Write each equation in exponential form.

4. $\log_3 27 = 3$ 5. $\log_4 16 = 2$ 6. $\log \frac{1}{100} = -2$

Evaluate each expression without a calculator.

7. $\log_7 7^3$ 8. $\log 0.001$ 9. $3^{\log_3 6}$

10. $4^{2\log_4 5}$ 11. $\log_5 1$ 12. $\log_6 4 + 2\log_6 3$

13. $\frac{1}{2}\log_3 144 - 2\log_3 6$ 14. $\frac{1}{2}\log_5 16 - 2\log_5 10$ 15. $\frac{1}{3}(\log_2 12 + \log_2 16 - \log_2 3)$

Solve each equation.

16. $\log_2 24 - \log_2 2 = \log_2 x$ 17. $3\log_5 2 = \log_5 x$

18. $\log_6 x - \log_6 5 = \log_6 4$ 19. $\log x = \frac{1}{3}\log 8 + \frac{1}{2}\log 81$

20. $\log_3 x^2 = \log_3 8 + \log_3 10 - \log_3 5$ 21. $\log_3 x - \log_3 4 = 2\log_3 5$

22. $\frac{1}{2}\log_7 x = \log_7 20 - 2(\log_7 2 + \log_7 5)$ 23. $\log_6 5 + 2\log_6 x = \log_6 45$

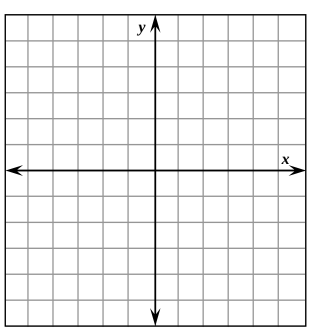
24. $\log_5 x = 3\log_5 4 - \frac{1}{3}\log_5 64$ 25. $\log_5 x + \log_5(x+1) = \log_5 20$

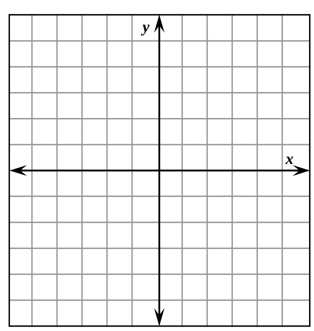
26. $\log_4(x-3) + \log_4(x+3) = 2$ 27. $\log_5(x+1) - \log_5(x-1) = 2$

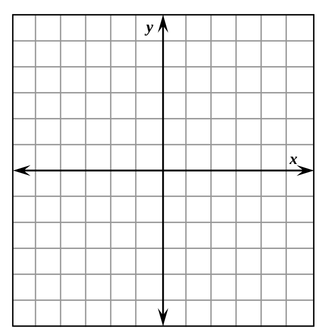
28. $\log x + \log(x-3) = 1$ 29. $\log_9((x-5) + \log_9(x+3) = 1$

Graph each equation.

30. $y = \log_3 x$ 31. $y = \log_3 x + 1$ 32. $y = \log_3(x+1)$







11.6 Exponential Equations

Solve over the set of real numbers. Express each solution to three significant digits.

1. $\left(\frac{1}{3}\right)^{x-6} = 3^x$

2. $\left(\frac{1}{8}\right)^{x} = 2^{x-6}$

3. $9^{3x} = 27^{x+2}$

4. $9^x = 45$

5. $5^x = 52$

6. $4^{3p} = 10$

7. $3^{n+2} = 14.5$

8. $9^{z-4} = 6.28$

9. $8.2^{n-3} = 42.5$

10. $2.1^{t-5} = 9.32$

11. $5^{x^2} = 68$

12. $5^x = 3^x$

13. $3^{x+1} = 7^{x-2}$

14. $2^{x-1} = 6^{2x+3}$

15. $x = \log_4 19.5$

16. $\log_{27} \frac{1}{3} = x$

17. $5a^{\frac{2}{5}} = 15.35$

18. $3x^{\frac{4}{3}} = 21.3$

Homework: page 639 #16, 17, 19-21, 23, 25-30, 32, 35, 36

DAY 4 Page 639 HW

11.6 Exponential Equations Homework

Solve each equation by using logarithms/exponents. Leave each answer in calculator ready form.

16. $6^x = 72$

17. $2^x = 27$

19. $2.2^{x-5} = 9.32$

20. $9^{x-4} = .713$

21. $6^{3x} = 81$

23. $x = \log_4 19.5$

25. $x^{\frac{2}{5}} = 17.3$

26. $5^{x-1} = 2^x$

27. $3^{2x} = 7^{x-1}$

28. $6^{x-2} = 4^x$

29. $12^{x-4} = 3^{x-2}$

30. $\log_2 x = -3$

32. $\log_{27} \frac{1}{3} = x$

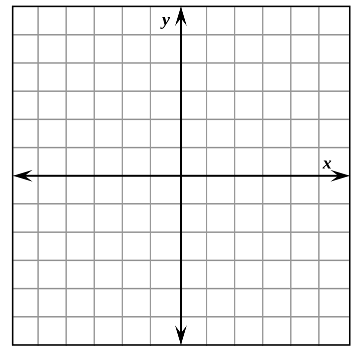
35. $\log_3 \sqrt[4]{5} = x$

36. $\sqrt[3]{4^{x-1}} = 6^{x-2}$

11.7 Natural Logarithms

1. **Natural Logarithms:**
   The natural log function is the inverse of $y = e^x$. Denoted $\ln\ x$, understood base e.

   All properties of logarithms also hold for natural logarithms.



2. Graph y $= e^x$ *and* $y = \ln x$

3. Write in logarithmic form.
   a. $e^{-x} = 5$ b. $e^2 = 6x$

4. Write in exponential form.
   a. $\ln e = 1$ b. $\ln 5.2 = x$

5. Evaluate each:
   a. $e^{\ln .2}$ b. $e^{\ln y}$ c. $\ln e^{-4x}$ d. $\ln e^{45}$

6. Use the properties of logarithms to write the expression in expanded form. (all variables are positive.)
   a. $\ln 4x^2$ b. $\ln\left(\frac{e^2}{5}\right)$

7. Write the expression as the logarithm of a single quantity.
   a. $\ln x + \ln 2$ b. $\frac{1}{2}\ln x + \ln y$ c. $4\ln x + 7\ln x - 3\ln x$

8. Solve each.
   a. $\ln N = 4.987$ b. $\ln N = 0.7831$ c. $e^x = 46$ d. $e^{4k} = 18$

   e. $3e^{.035x} = 519$ f. $\ln 19.8 = \ln e^{0.083t}$ g. $\ln 6.2 = \ln e^{0.55t}$

9. **Growth and Decay Formula:**
A general rule for growth and decay is $y = ne^{kt}$ $y = ne^{kt}$.

10. The half-life of Phosphorus-33 is about 25 days. When will a 20-gram sample of P-33 be reduced to 8 grams?

11. A major highway was constructed five years ago to accommodate a population of up to 40,000 commuters. It was estimated that the commuter population at that time was about 25,000. Today, there are 31,000 cars commuting on the highway each day.
    a. If the commuter population continues to grow at this rate, when will the highway need to be upgraded again?

    b. How many commuters will the upgraded highway have to accommodate to meet the demand for an additional 10 years?

12. If two languages have evolved separately from a common ancestral language, the number of years since the split, n(r), is given by the formula $n(r) = -5000 \ln r$ $n(r) = -5000 \ln r$, where r is the percentage of the words from the ancestral language that are common to both languages. Suppose two languages split off from a common language 1000 years ago. What portion of the words from the ancestral language would you expect to find in each of them today?

Page 643 #31-36 and Natural Log Worksheet

DAY 5 HW
11.7 Book pg. 643 #31-36

31. Radium 226, which is used for cancer treatment and as an ingredient in fluorescent paint, decomposes radioactively. Its half-life is 1800 years. Find the constant k you would use in the decay formula for radium. Use 1 gram as the original amount.

32. Mr. Cuthbert invested a sum of money in a certificate of deposit that pays 8% interest compounded continuously. Recall that the formula for the amount in an account earning interest compounded continuously is $A = Pe^{rt}$. If Mr. Cuthbert made the investment on January 1, 1986 and the account is worth $10,000 on January 1, 2005, what was the original amount in the account?

3. DDT is an insecticide that has been used by farmers. It decays slowly and is sometimes absorbed by plants that animals and humans eat. DDT absorbed in the mud at the bottom of a lake is degraded into harmless products by bacterial action. Experimental data shows that 10% of the initial amount is eliminated in 5 years.

a. Find the value of k in the decay formula.

b. How much of the original amount of DDT is left after 10 years?

c. The U. S. Environmental Protection Agency banned almost all use of DDT in the U. S. in 1972. If none has been used near the late since then, in what year will the concentration of DDT fall below 25%?

34. Sales of a product under relatively stable market conditions tend to decline at a constant annual rate in the absence of promotional activities. This sales decline can be expressed by the exponential function of the form $s = s_0e^{-at}$, where s is the sales time at time t, t is time in years, s_0 is the sales at time t = 0, and a is the sales decay constant. Suppose sales of On-Time Watches were 45,000 the first year and 37,000 the second year.

a. Find the value of a in the equation for this sales decline.

b. Find the projected sales for three years from now.

c. If the trend continues, when would you expect sales to be 15,000 units?

35. Mike Kallenberg deposited some money in a bank account that earns 5.6% interest compounded continuously.

a. How long would it take to double the amount of money in Mr. Kallenberg's account?

b. The Rule of 72 says that if you divide 72 by the interest rate of an account that compounds interest continuously, the result is the approximate number of years that it will take for the money in the account to double. Do you think that the rule of 72 is accurate? Explain.

36. The atmospheric pressure varies with the altitude above the surface of the Earth. Meteorologists have determined that for altitudes for up to 10 km, the pressure p in millimeters of mercury is given by $p = 760e^{-0.125a}$, where a is the altitude in kilometers.

a. What is the atmospheric pressure at an altitude of 3.3 km?

b. At what altitude will the atmospheric pressure be 450 m of mercury?

DAY 5 HW
Natural Logs Worksheet

Express in logarithmic form:

1. $e^{2.5} = 12.18$ 2. $e^{-2} = 0.13$ 3. $e^{\frac{1}{5}} = 1.221$ 4. $\sqrt{e} = 1.649$

Express in exponential form:

5. $\ln 8 = 2.079$ 6. $\ln 0.5 = -0.693$ 7. $\ln 0.1 = -2.303$ 8. $\ln 0.01 = -4.605$

Simplify:

9. $\ln e^4$ 10. $e^{\ln 2}$ 11. $e^{\ln 6+\ln 7}$ 12. $e^{2\ln 7}$

13. $\ln 48 - 4\ln 2$ 14. $\frac{1}{2}\ln 9 + \ln 12 - 2\ln 3$ 15. $\frac{1}{2}(\ln 45 + \ln 5) - 2\ln 3$

16. $e^{\ln 8-\ln 6}$ 17. $e^{\frac{1}{2}\ln 3}$ 18. $\ln 6 + \ln 30 - (\ln 5 + 3\ln 2)$

19. $\frac{1}{2}\ln 4 + \ln 8 - (5\ln 2 + \ln 3)$ 20. $3\ln 4 - (\ln 2 + \ln 8)$

Application Notes:

1. **Compound Interest:**
   The initial principal P is invested at r percent (expressed in decimal form).
   The accumulated amount after t years is given by

   $A = P\left(1+\frac{r}{n}\right)^{nt}$ when interest is compounded n times per year

   $A = Pe^{rt}$ when interest in compounded continually

Ex: If $750 is invested at 8% annual interest that is compounded monthly, when will the investment be worth $1600?

Ex: A piece of office equipment worth $8000 depreciates at 7.5% per year for the first ten years. At this rate, when will the piece of equipment be worth $5000?

Ex: If $100 is invested at 12% annual interest that is compounded continuously, when will the investment be worth $250?

2. **Half Life:** $A = P\left(\frac{1}{2}\right)^{\frac{time}{half\ life\ rate}}$

Ex: Technetium-99 has a half-life of 6 hours. Suppose a lab has 80 mg of technetium-99. How much technetium-99 is left after 24 hours?

Ex: Phosphorus-32 is a radioactive substance with a half-life of 14.3 days. How long would it take to reduce a 100-gram sample of P-32 to 15 grams?

Ex: Cesium-137 has a half-life of 30 years. Suppose a lab stored a 30-mCi sample in 1973. How much of the sample will be left in 2003? In 2063?

3. Under ideal conditions, the population of a certain bacterial colony will double in 45 minutes. How much time will it take for the population to increase by 5 fold?

4. A population of insects is growing in such a way that the number in the population t days from now is given by the formula $A = 4000e^{0.02t}$. How large will the population be in one week?

5. In 1990, the population of Washington, D.C., was about 604,000 people. Since then the population has decreased about 1.8% per year.
    a. Write an equation to model the population of Washington, D.C., since 1990.

    b. Suppose the current trend in population change continues. Predict the population of Washington, D.C., in 2010.

6. Suppose your community has 4512 students this year. The student population is growing 2.5% each year.
    a. Write an equation to model the student population.

    b. What will the student population be in 3 years?

7. Logistic Growth Models: $y = \dfrac{a}{1 + be^{-rt}}$

Ex: The function $f(t) = \dfrac{30,000}{1 + 20e^{-1.5t}}$ $f(t) = \frac{30,000}{1+20e^{-1.5t}}$ describes the number of people, $f(t)$ f(t), who have become ill with influenza t weeks after its initial outbreak in a town with 30,000 inhabitants.
    a. How many people became ill with the flu when the epidemic began?

    b. When will the 5000th person be infected at the current rate of the outbreak?

Homework: Application Worksheet

DAY 6 HW
Application Problems Unit 5

Round to the hundredths place when necessary and do not forget units.

1. A bacterial culture doubles every 2 hours. If the culture started with 24,000 bacteria, how many bacteria will be present in 5 hours?

2. The half-life of a radioactive sample is 4 hours. If 60 g of the sample was initially present, how much will remain after 7 hours?

3. The population of a town triples every 6 years. If 4000 people are present in 2006, how many people will be in the town in 2016?

4. The half-life of a radioactive sample is 6.2 hours. If 2000 g of the sample is present after 7 hours, how much was initially present?

5. A bacterial culture doubles every 5 hours. How long will it take for a culture to quadruple?

6. A radioactive sample has a half-life of 3 days. How long will it take for only 1/8 of the sample to remain?

7. The population of a town triples every 8 years. How many years will it take for the population to double?

8. The population of a town halves every 15 years. In how many years will 20% of the population have fled?

9. A radioactive sample has a half-life of 3 years, and has an initial mass of 68 g. How many **months** will it take for the sample to lose 8 g?

10. $5000 is invested at 7.2% compounded annually for 4 years. What is the amount of money at the end of the 4 years?

11. $2300 is invested at 6% compounded monthly for 7 years. How much interest is earned?

12. $300 is invested at 10% compounded quarterly. How many years must it stay in the bank to double?

13. An investment is invested at 5% compounded continuously. How many years will it take to triple in value?

14. A small tremor of magnitude 3.4 is followed by a stronger one of magnitude 4.1. How much stronger is the second tremor than the first?

15. An earthquake has a magnitude of 6.5, and the following day a stronger earthquake occurs with double the intensity. What is the magnitude of the stronger earthquake?

16. A strong earthquake with a magnitude of 6.7 is three times more intense than a weaker earthquake. What is the magnitude of the weaker earthquake?

17. A small tremor of magnitude 2.3 is then followed by a stronger one of magnitude 5.3. How much stronger is the second tremor than the first?

18. An earthquake has a magnitude of 6.7, and the following week a stronger earthquake occurs with six times the intensity. What is the magnitude of the stronger earthquake?

19. An earthquake with a magnitude of 5.9 is one thousand times more intense than a weaker earthquake. What is the magnitude of the weaker earthquake?

20. The formula for acid strength is: $pH = -\log[H^+]$, where PH is the acid strength and H^+ is the concentration of hydrogen ion. A beaker of acid has a hydrogen concentration of 3.5×10^{-6} mol/L. Calculate the pH of the acid.

Log Review

1) From 1971 to 1995, the average number *n* of transistors on a computer chip can be modeled by:

$n = 2300(1.59)^t$, where *t* is the number of years since 1971.

a) Identify the initial amount, the growth factor, and annual percent increase.

b) Estimate the number of transistors on a computer chip in 1998.

2) You have inherited land that was purchased for $30,000 in 1960. The value of the land increased by approximately 5% per year.

a) Write a model for the value of the land *t* years after 1960.

b) What is the approximate value of the land in the year 2015?

c) At what year would the land be valued at about half a million dollars?

3) Write as a single logarithm: $(2\log_x 3 + \log_x p) - 2\log_x(4y)$

4) Solve: $\log_7 x = 4\log_7 2 + (\log_7 3 - \log_7 6)$

5) Solve: $\log_2(x+3) + \log_2(x-3) = 4$

6) Simplify: $\log_8(\log_2(\log_2 16))$

7) Solve: $3^{\frac{x}{7}} = 2^x$

8) John's new house in Apex is valued at $105,000. The area he lives in has had a steady rate of appreciation for homes of 12% per year. At this steady rate, when will his house be worth ½ million dollars?

9) A radioactive element has a half-life of 10 hours. If you have 300 g of the element initially, how much remains after 25 hours?

10) Solve: $\ln e^{1.32x} = 5.8$

11) $\log_9 7 + \frac{1}{2}\log_9 24 = \log_9 x$

12) Solve: $e^{4x} - 5e^{2x} - 24 = 0$

13) $Solve: 7^{2x+4} = \left(\frac{1}{49}\right)^{x-3}$

14) Solve: $\frac{1}{3^x} = 12$

15) $Solve: 10^x = 3.91$

16) Solve: $8^{x-1} = \left(\frac{1}{4}\right)^{1-x}$

17) Solve: $2\log_3(x+4) = \log_3 9 + 2$

18) An earthquake has a magnitude of 4.6 and the following day there is a stronger one that is fifteen times more intense. What was the magnitude of the stronger one?

19) Solve: $5e^x = 23$

20) $2400 is invested at 5% compounded quarterly for 7 years. How much interest is earned?

Laws of Logarithms: Solve for x.

1. $\log_8 4 + \log_8 16 = x$
2. $\log_3 405 - \log_3 5 = x$
3. $\log_7 x = 3\log_7 5 - \log_7 5$
4. $\log_2 x = 4\log_2 3 + (\log_2 2 - \log_2 6)$
5. $\log_3 5x = (\log_3 15 + \log_3 8) - \log_3 4$
6. $\log_5 x = 3\log_5 7$
7. $\log_2 x = \frac{1}{2}\log_2 81$
8. $\log_{10} x = \frac{1}{2}\log_{10} 144 - \frac{1}{3}\log_{10} 8$
9. $\log_6 (x+5) = \log_6 7$
10. $\log_3 5 + \log_3 2 = \log_3 x$
11. $3\log_5 2 = \log_5 x$
12. $\log_3 x - \log_3 7 = \log_3 2$
13. $\log_2 x = \frac{1}{3}\log_2 27$
14. $\log_7 x = 2\log_7 3 + \log_7 5$
15. $\log_5 x = \frac{3}{2}\log_5 9 + \log_5 2$
16. $\log_{10} x + \log_{10} 5 = 3$
17. $\log_2 x^3 - \log_2 27 = 3$
18. $\log_{10} x + \log_{10}(x-3) = 1$
19. $\log_2 (x-5) + \log_2 (x-1) = 5$
20. $\log_5 2x^2 - \log_5 (x+5) = 1$
21. $\log_3 (x^2 - 1) - \log_3 (5x+5) = 0$
22. $\log_{11}(x^2 + 7) = \frac{2}{3}\log_{11} 64$
23. $\log_2 x - \log_2 (x-5) = \log_2 6$
24. $2\log_3 x - \log_3 (x-2) = 2$
25. $\log_2 (x+6) - \log_2 (x-1) = 3$
26. $\log_2 x + \log_2 (x-2) = 3$
27. $\log_3 (x+5) - \log_3 (x-1) = 2$
28. $\log_3 (x+4) + \log_3 (x-4) = 2$
29. $\log_7 (x+1) + \log_7 (x-5) = 1$
30. $\log_{10}(x+3) - \log_{10}(x-1) = 1$
31. $\log_{10} x + \log_{10}(x+21) = 2$
32. $\log_{10}(10x+5) - \log_{10}(x-4) = \log_{10} 2$
33. $\log_2 (x+2) = \log_2 (x-2) + 1$
34. $\log_{10}(4x-3) = \log_{10}(x+1) + \log_{10} 3$
35. $2\log_3 (2x-1) = \log_3 (x-1) + 2$
36. $\log_5 (\log_3 x) = 0$
37. $\log_4 [\log_3 (\log_2 x)] = 0$
38. $\log_{x+1}(3x+7) = 2$
39. $\log_4 (1-5x) = -1$
40. $\log_x (6-5x) = 2$

Answers

1. 2	2. 4	3. 25	4. 27	5. 6	6. 343	7. 9
8. 6	9. 2	10. 10	11. 8	12. 14	13. 3	14. 45
15. 54	16. 200	17. 6	18. 5	19. 9	20. 5, -5/2	21. 6
22. 3, -3	23. 6	24. 3, 6	25. 2	26. 4	27. $\frac{7}{4}$	
28. 5	29. 6	30. $\frac{13}{9}$	31. 4	32. **No Solution**		33. 6
34. 6	35. $\frac{5}{4}, 2$	36. 3	37. 8	38. 3	39. $\frac{3}{20}$	**40. No Solution**

AFM Unit 5 Review

Simplify:

1. $\left(2^{\sqrt{2}}\right)^{\sqrt{24}}$

2. $13^{\sqrt{6}} \cdot 13^{\sqrt{24}}$

3. $125^{\sqrt{11}} \div 5^{\sqrt{11}}$

4. $\left(n^{\sqrt{3}} m^{-\sqrt{5}}\right)^{\sqrt{5}}$

5. $b^{3m-5} \cdot b^{4-m}$

6. $\left(\frac{8a^{-2}b^{3}}{-3b^{-5}}\right)^{-2} \left(\frac{-2b^{-1}}{3a^{14}}\right)^{0}$

7. $\left(a^{\sqrt{2}} b^{\sqrt{32}}\right)^{\sqrt{2}}$

8. $5^{1+\sqrt{7}} \div 5^{1-\sqrt{7}}$

9. $\left(7^{\sqrt{3}+2}\right)\left(7^{\sqrt{3}-2}\right)$

10. $\left(5^{1+\sqrt{7}}\right)^{1-\sqrt{7}}$

11. $\sqrt[3]{-125x^{4}y^{17}}$

12. $\left(m^{\frac{2}{3}} n^{\frac{3}{4}}\right)^{2}$

Evaluate:

13. $8^{\log_8 3}$

14. $\log_a b^4 \cdot \log_b a^5$

15. $2^{3\log 5 + 3\log 2}$

16. $\log_3\left(\log_3(\log_3 27)\right)$

17. $\frac{4}{5}\log_2 32 - \log_2 8$

18. $e^{4\ln 2}$

19. $\ln\sqrt[5]{e^2}$

20. $\ln e$

True or False:

21. $\ln(x+y) = \ln x + \ln y$

22. $\log_3\left(\frac{m^2}{3n}\right) = 2\log_3 m - 1 - \log_3 n$

23. $e^{2\ln 3 - 4\ln 2} = \frac{9}{16}$

24. $If \log_5(2x-3) = 5\ then\ x = 4$

25. $\ln 0 = 1$

26. $\log_3 \frac{1}{243} = -5$

Graph the following on graph paper and give the domain and range and show asymptotes with dotted lines.

27. $y = 3^{x-2} + 4$

28. $y = \log_3 x$

29. $y = 2^{-x} - 3$

Solve the following. Exact value or round to 4 places.

30. logx + log(x + 21) = 2

31. $log_3(x^2 - 1) - log_3(5x + 5) = 0$

32. $log_{x+1}(3x + 7) = 2$

33. $2log_6 4 - \frac{1}{4}log_6 16 = log_6 x$

34. $log_{12}3 = x$

35. $3^{4x} = 42$

36. $4^{x+3} = 25.8$

37. $2^{x+1} = 3^{2-2x}$

38. $250 = 35e^{-.03x}$

39. $5 - 6e^{4x} = 17$

40. $6x^{\frac{2}{3}} = 22$

41. $e^{2x} - 6e^x - 27 = 0$

42. $log_x 5 = -2$

43. $log_{15}(x + 2) + log_{15}x = 1$

44. Write the equation that transforms y = lnx with the following transformations:
reflect over the y axis, shift left 8 units and shift down 10 units.

For numbers 44 – 47, let *m* = $\log_7 2$, and n = $\log_7 10$. Express the following in terms of *m* and *n*:

45. $\log_7 40$

46. $\log_7 200$

47. $\log_7 5$

48. $\log_7 0.8$

Study all word problems and know the formulas!