Unit 5 ICM/AB Applications of the Derivative Fall 2016

Nov 4 Learn Optimization, New PS up on Optimization,

HW pg. 216 3,5,17,19,21,23,25,27,29,33,39,41,49,50 a,b,54

Nov 7 Continue on HW from Nov 4 and work on PS

Nov 8 Examples on Optimization

Nov 9 **Quiz** Optimization, PS due, new PS up

Nov 10 Learn Velocity and Acceleration: HW p. 115 91-98 P. 126 101,103 p. 153 34-36

Nov 14 Work on PS

Nov 15 PS Velocity and Acceleration due, new PS up

Nov 16 **Quiz** VA

Nov 17 Half test on Opt and VA

Nov 18 Related Rates HW p. 149 1,3,7,9-11,13-27 odd, 28-30 all, 33,35,36,43,45

p. 155 108,109

p. 157 #9 PS Related Rates up

Nov 21 go over HW and work on PS

Nov 28 Work on PS and HW RR

Nov 29 PS RR due and **Quiz** on RR

Nov 30 Wkst of all 3 mixed together

Dec 1,2 Test Applications of the Derivative, MC and FR

Applications of the Derivative: Optimization

There are the same types of word problems that you encountered in Algebra but they may involve more complicated functions, and our method for solving is much easier.

Strategy:

1. READ THE WHOLE PROBLEM
2. Draw a picture and/ or collect the given info
3. Write down the equation of the quantity to be maximized or minimized.
4. Express *f*(x) in terms of just one variable
5. Find the domain from the physical restrictions of the problem that is appropriate
6. Find $f'(x)$ and find the critical points from it. (Be sure to check where $f'(x)$ DNE)

   Use either the first or second derivative test to locate the max/min and remember to check the endpoints where appropriate.

1) Find the dimensions of the rectangle with perimeter of 100 cm. and area as large as possible.

2) An open top box is to be made from a piece of cardboard 16 in. by 30 in., by cutting out squares from each corner and bending up the edges. What size should the squares be in order to produce the box with a maximum volume?

3) Find the radius and height of the largest right circular cylinder that can be inscribed in a right circular cone with base radius 6 in. and height 10 in.

4) A metal cylindrical container with an open top is to hold one cubic foot. If there is no waste in construction, find the dimensions which require the least amount of material.

5) A wire, 36 cm long, is to be cut into two pieces (or not). One of the pieces is to be bent into an equilateral triangle, and the other into a rectangle whose length is twice its width. Where should the wire be cut so the combined area is maximum? And minimum?

6) A package can be sent by UPS if the sum of its length and its girth (the perimeter of its base) is not more than 96 inches. Find the dimensions of the box of maximum volume that can be sent, if the base of the box is a square.

Notes Optimization Day 2: More Examples 2016

1) You are designing a rectangular poster to contain 50 sq. inches of print with a 4 inch margin at the top and bottom and a 2 inch margin at each side. What overall dimensions will minimize the total amount of paper used?

2) Lisa is 2 miles offshore in a boat and wishes to reach a coastal village 6 miles down a straight shoreline from the point on the shore nearest to the boat. She can row 2 mph and walk 5 mph. Where should she land her boat on shore in order to reach the village in the least amount of time?

3) There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees ?

4) What angle $\theta,$ between two edges of length 3, will result in an isosceles triangle with the largest area?

5) Find the dimensions of the rectangle of largest area which can be inscribed in the closed region bounded by the x-axis, y-axis, and graph of $y = 8 - x^3$.

Warm Up

1. Two vertical poles, one 12 ft. high and the other 28 ft. high, stand 30 feet apart. They are to be stayed by two wires, attached to a single stake, running from the ground level to the top of each post. Where should the stake be placed to use the least amount of wire?

2. A cylindrical container, open on the top and having a capacity of 24π cubic inches, is to be manufactured. If the cost of the material used for the bottom of the container is three times that used for the curved part and if there is no waste of the material, find the dimensions which will minimize the cost.

Warm Up

3. A cylindrical can is to hold $20m^3$. The material for the top and bottom costs $10 per square meter and the material for the side costs $8 per square meter. Find the radius and height of the most economical can.

4. An observatory with cylindrical walls and a hemisphere roof is to be constructed so that its total volume of the cylindrical part will be 40,000 cubic feet. What dimensions will result in the minimum total surface area for the wall and the roof?

Motion

Free Fall: Gravity is the main source.

(g: Gravitational Acceleration Constant -32 ft/s^2 or -9.8 m/s^2)

Ex: A diver is on a 32 ft. platform and jumps at an initial velocity of 32 ft/s^2. We will use the equation $h(t) = \frac{1}{2}gt^2 + v_0 t + h_0$ (v_0 and h_0 are initial velocity and initial height)

Questions:

1. How high does the diver go? Set the first derivative of the position function equal to 0.

2. When will the diver hit the water? Set the position function equal to zero.

3. What velocity will the diver hit the water? (Things hit the ground with negative velocity)

Ex: A dynamite blasts propels a heavy rock straight up with a launch velocity of 160 ft/sec (about 100 mph). It reaches a height of $h(t) = 160t - 16t^2$ $ft.$ after t seconds.

1. How high does the rock go?

2. What is the velocity of the rock when it is 256 ft. above the ground on the way up?

3. What is the acceleration of the rock at any time *t* during the flight, after the blast?

4. When does the rock hit the ground?

Know: Acceleration Constants

s(t) is often representing position (measured in ft., inches, km, miles)

h(t) is often representing height

v(t) is often representing velocity = *s'(t)* or *h'(t)* (ft./s, in./s, kph, mph)

a(t) is often representing acceleration = *s''(t)* or *h''(t)* (ft./s^2, in./s^2, kph^2, mph^2)

Stop or at rest means $v = 0$.

When $v > 0$ and $a = v' < 0$ (opposite signs) slows down

When $v < 0$ and $a = v' < 0$ (same signs) speeds up

(Baseball or chalk example)

Speed is the absolute value of velocity. (Scalar quantity, no direction)

Ex: Drop out of the window. What would *h(t)* look like here if we are 30 ft. in the air?

1. When will he hit the ground?

2. What will his velocity be when he hits the ground?

Rectilinear Motion: on a straight line

s(t) or *x(t)* position

$v(t) = s'(t)$

$a(t) = v'(t) = s''(t)$

Speed equals $|v(t)|$ if $v(t) > 0$ up or right, if $v(t) < 0$ down or left

Speed up: v, a have same signs

Slow down: v, a have opposite signs

Displacement: Change in position (not necessarily the same as distance travelled)

$$*** \; Average\ Velocity = \frac{Change\ in\ Position}{Change\ in\ Time}$$

Distance: going back and forth.

Max or Min of *v(t)* possibly the max speed

Ex: $s(t) = -t^3 + 18t^2 - 81t \; [2, 7]$ When will it reached its maximum speed?

Ex: $s(t) = t^3 - 23t^2 + 120t \; ; \; t \geq 0$ Find the times that the particle changes direction, the intervals when it is moving left or right, when acceleration is equal to zero, and the intervals that the particle is slowing down or speeding up.

Analyze Motion

Ex: $s(t) = 2t^3 - 21t^2 + 60t + 3\ [0, 9]$

What we know: *v(t)* positive, right *v(t)* negative, left *a(t)* Speed up v, a have same sign

	velocity	direction	acceleration	Speeding up/down
(0,2)				
t = 2				
(2, 3.5)				
t = 3.5				
(3.5, 5)				
t = 5				
(5, 9)				

Notes on Related Rates AB 2016

Derivatives tell us how fast the amount of something is changing. In other words, its rate of change.

When you are working these problems take notice of which quantities are changing and which are constant. **(Be sure to treat changing quantities like variable and constant like constants.**)

Also take the derivatives BEFORE you plug in temporary constants.

Strategy: 1. Read the problem and write down knowns and unknowns. Draw pic if appropriate.

2. Write the equation that relates the variables representing the quantity that you know, the rate of change of, to the quantity that you are looking for the rate of change of.

3. Eliminate any variable that you don’t know the rate of change of and aren’t looking for the rate of change of. (Not by erasing it ☺)

4. Differentiate both sides with respect to time.

5. Plug in knowns and solve for unknowns, answer the question with correct units.

EXAMPLES:

1. If you know that $y = x^2 + 3$ and we know $\frac{dx}{dt} = 2$ when x = 1, find $\frac{dy}{dt}$.

   What if you knew that $\frac{dy}{dt} = 8$, what would $\frac{dx}{dt} = ?$

2. A stone is dropped into a pond, forming concentric circles. We know that $\frac{dr}{dt} = 1\, ft/\sec$. Find the change in the area $\frac{dA}{dt}$ when the radius is 4 feet .

3. Air is pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius of the balloon when the radius is 6 feet.

4. A hot air balloon, rising straight up from a level field, is tracked by a range finder 500 feet from its lift off spot. At the moment that the angle of elevation of the range finder is $\frac{\pi}{4}$, the angle is increasing at the rate of 0.14 radians/minute. How fast is the balloon rising then?

5. Coffee drains into a very large conical coffee cup at a rate of 9 cubic feet per minute. (The cone is inverted or vertex down ☺) The cup has a height of 10 feet and a base radius of 5 feet. How fast is the coffee level changing when it is 6 feet deep, again ☺.

6. How fast does the water level drop when a cylindrical tank is drained at 3 liters per second?

   (1 ml = 1 cubic cm)

7. Two trucks convoys leave a truck stop, at the same time, with convoy A traveling east at 40 mph and convoy B traveling north at 30 mph. How fast are the convoys separating 6 minutes later?

8.