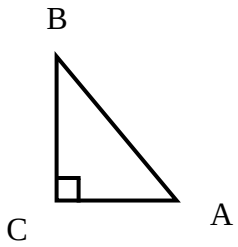
5.5 Right Triangles

1. For an acute angle A in right triangle ABC, the trigonometric functions are as follow:

   $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}}$

   $\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}}$

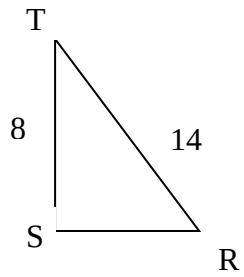
   $\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$



SOH-CAH-TOA

2. A right triangle has sides whose lengths are 8 cm, 15 cm, and 17 cm. Find the values of missing angles.

3. In triangle RST, find the measure of $\angle$ R to the nearest minute.



4. Solve each right triangle. Round angles to the nearest minute and sides to the nearest tenth.
   a. A = 20°, c = 35
   b. A = 49° 13', a = 10

5. **Angle of Elevation:** Paul views the top of a water tower at an angle of elevation of 36°. If he estimates that he is 120 ft from the base of the tower, how tall is the tower?

6. **Angle of Depression:** A plane is flying at an altitude of 1800 ft. and there is a 42° angle of depression to the landing strip. How far does the plane need to travel to land?

7. From a point on level ground 125 feet from the base of a tower, the angle of elevation is 57.2°. Approximate the height of the tower to the nearest foot.

8. A kite flies at a height of 30 feet when 65 feet of string is out. If the string is in a straight line, find the angle that it makes with the ground. Round to the nearest tenth of a degree.

9. You are standing on level ground 800 feet from Mt. Rushmore, looking at the sculpture of Abraham Lincoln's face. The angle of elevation to the bottom of the sculpture is 32° and the angle of elevation to the top is 35°. Find the height of the sculpture of Lincoln's face to the nearest tenth of a foot

5.5 Word Problem Worksheet **Draw the right triangle and solve the problem.**

1. A 13.5 meter ladder is leaning against a wall. Find the distance the ladder goes up the wall if it makes an angle of $43^\circ 50'$ with the ground.

2. A guy wire that is 77.4 meters long is attached to the top of an antenna mast that is 71.3 meters high. Find the angle that the wire makes with the ground.

3. Find the length of a guy wire that makes an angle of $45^\circ 30'$ with the ground if the wire is attached to the top of the tower 63 meters high.

4. Suppose the angle of elevation of the sun is 23.4°. Find the length of the shadow cast by Cindy, who is 5.75 feet tall.

5. The shadow of a tower is 40.6 meters long when the angle of elevation of the sun is 34.6°. Find the height of the tower.

6. Find the angle of elevation of the sun if a 48.6 ft flagpole casts a shadow 63.1 ft. long.

7. The angle of depression from the top of a building to a point on the ground is $32^\circ 30'$. How far is the point on the ground from the top of the building if the building is 252 meters high?

8. An airplane is flying 10,500 feet above the level ground. The angle of depression from the plane to the base of a tree is $13^\circ 50'$. How far horizontally must the plane fly to be directly over the tree?

9. The angle of elevation from the top of a small building to the top of a nearby taller building is $46^\circ 40'$, while the angle of depression to the bottom is $14^\circ 10'$. If the smaller building is 28 meters high, find the height of the taller building.

10. Shelly McCarthy knows that when she stands 123 feet from the base of a flagpole, the angle of elevation is $26^\circ 40'$. If her eyes are 5.3 feet above ground, find the height of the flagpole.

5.5 Practice Worksheet

Solve each triangle described, given the triangle below. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. $A = 39^\circ 12', b = 2.1$

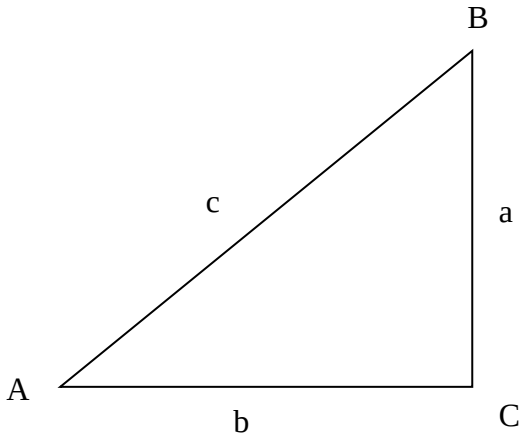
2. $B = 49^\circ, a = 9$

3. $B = 64^\circ, b = 19.2$

4. $B = 56^\circ 48', c = 63.1$

5. $a = 0.4, c = 0.5$

6. $A =$



8. A blimp is hovering over a landing pad 150 m from where you are standing. The blimp's angle of elevation with the ground is 36°. What is the altitude of the blimp?

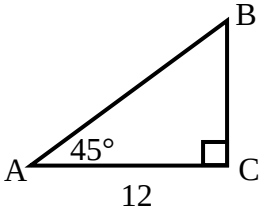
9. A 20-foot ladder leans against a wall so that the base of the ladder is 9 feet from the base of the building. What angle (nearest minute) does the ladder make with the ground?

10. A 55-meter vertical tower is braced with a cable secured at the top of the tower and tied 32 meters from the base. What angle does the cable form with the ground (nearest degree)?

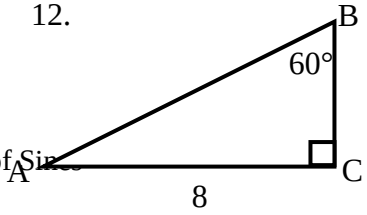
11. From the top of a lighthouse 210 feet high, the angle of depression of a boat is 27°. Find the distance from the boat to the foot of the lighthouse.

Solve each right triangle.

11.



12.



5.6 Law of Sines

1. **Law of Sines:**

   Let triangle ABC be any triangle with a, be and c representing the measures of the sides opposite the angle with measurements A, B, and C respectively. Then the following is true.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

2. Solve triangle ABC if $A = 32°14', B = 57°40',$ and $c = 14.3.$ Round angle measures to the nearest minute and side measures to the nearest tenth.

3. **Recall the following from Geometry:**

   The sum of the lengths of two sides of a triangle is greater than the length of the third side.
   SSS, SAS, AAS, ASA, and HL guarantee a unique triangle

   SSA (two sides and the angle opposite one of them) *does not guarantee* a unique triangle (or even that a triangle exists). Two angle measures, one, or none may satisfy the value of the sine ratio when you use the law of sines given SSA. When two angle measures satisfy the sine ratio given SSA, two triangles can be determined. This is called the _____ case.

4. When the measures of two sides of a triangle and the measures of the angles opposite one of them are given (SSA), there may not always be one solution. However, one of the following will be true.
   a. No triangle exists. (no solution)
   b. Exactly one triangle exists. (one solution)
   c. Two triangles exist. (two solutions)

   If angle A is acute, there are four possible outcomes:

Number of Possibilites	Sketch	Condition Necessary for Case to Hold
0		$a < b\sin A$
1		$a = b\sin A$

1		$a > b\sin A$ and $a \geq b$
2		$b > a > b\sin A$

If angle A is obtuse, there are two possible outcomes:

Number of Possible Triangles	Sketch	Condition Necessary for Case to Hold
0		$a \leq b$
1		$a > b$

5. Determine whether each triangle has no solution, one solution, or two solutions. Then solve each triangle.

a. $A = 72°12', b = 22, a = 21$

b. $A = 58°, b = 14, a = 14$

c. $B = 33°, b = 2, a = 3.5$

d. $B = 68°, b = 3, a = 5$

e. $A = 124°, a = 1, b = 2$

f. $A = 99°, a = 2.5, b = 1.5$

Homework page 280 #14-24 all

Page 280 Bookwork:

Determine the number of possible solutions. If a solution exists, solve the triangle. Round angle measures to the nearest minute and side measures to the nearest tenth.

14. $a = 8, A = 49°, B = 57°$

15. $a = 6, b = 8, A = 150°$

16. $a = 26, b = 29, A = 58°$

17. $A = 40°, B = 60°, c = 20$

18. $B = 70°, C = 58°, a = 84$

19. $a = 12, b = 14, A = 90°$

20. $A = 25°, a = 125, b = 150$

21. $A = 76°, a = 5, b = 20$

22. $A = 37°20', B = 51°30', c = 125$

23. $b = 40, a = 32, A = 125°20'$

24. $A = 107°13', a = 17.2, c = 12.2$

29. Suppose the angle measures of ΔABC are equal to the measures of ΔXYZ. Use the Law of Sines to show that the triangles are similar, but not necessarily congruent.

31. ***Landscaping***: A corner of Alston Park occupies a triangular area that faces two streets that meet at an angle measuring 85°. The sides of the area facing the streets are each 60 ft. in length. The park's landscaper wants to plant flowers around the edges of the triangular area.

Find the perimeter of the triangular area.

5.5 and 5.6 Worksheet

Read each problem carefully. Draw a picture then solve the problem.

1. In a sightseeing boat near the base of Horseshoe Falls at Niagara Falls, a passenger estimates the angle of elevation to the top of the falls to be 30°. If the Horseshoe Falls are 173 feet high, what is the distance from the boat to the base of the falls?

2. A surveyor stands 100 feet from a building and sights the top of the building at a 55° angle of elevation. Find the height of the building.

3. In order to construct a bridge across a river, the width of the river at that location must be determined. Suppose a stake is planted on one side of the river directly across from a second stake on the opposite side. Another stake is put 50 meters to the north, and an angle of 82° is formed between it and the one across the river. Find the width of the river.

4. The Aerial run in Snowbird, Utah, has an angle of elevation of 20.2°. Its vertical drop is 2900 ft. Estimate the length of the ski slope.

Determine whether each triangle has one, two, or zero solutions. Then solve each triangle.
Round sides to the nearest tenth and angles to the nearest minute.

5. $A = 124^\circ, a = 1, b = 2$

6. $A = 99^\circ, a = 2.5, b = 1.5$

7. $A = 33^\circ, a = 2, b = 3.5$

8. $A = 68^\circ, a = 3, b = 5$

9. $B = 61^\circ, a = 8, b = 23$

10. $A = 52^\circ, a = 200, b = 190$

5.7 Law of Cosines

1. **Law of Cosines:**

   Let triangle ABC be any triangle with a, b, c representing the measures of the sides opposite the angle with measurements A, B, and C respectively. Then the following is true.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

2. Suppose you want to fence a triangular lot as shown below. If two sides measure 84 feet and 78 feet and the angle between the two sides is $102°$, what is the length of the fence to the nearest foot?

3. Solve triangle ABC if $a = 19.b = 24.3,$ and $c = 21.8.$ Round angle measures to the nearest minute.

4. Determine whether each triangle should be solved by beginning with the Law of Sines or the Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measure of angles to the nearest degree.

   a. $a = 20, c = 24, B = 47°$

   b. $a = 345, b = 648, c = 442$

   c. $A = 36°, a = 10, b = 19$

   d. $A = 25°, B = 78°, a = 13.7$

5.8 Area of Triangles

1. **Area of a Triangle with a Known Height:** **Formulas for finding area of Oblique Triangle:**

$$A = \frac{1}{2}bh$$

2. Find the area of each triangle described below. Round answers to the nearest tenth.

A. $a = 8.4, b = 10,\ C = 108°$ B. $a = 3, c = 4, B = 120°$ C. $a = 14.2, A = 18°50',\ B = 69°18'$

3. Heron's Formula:

If the measures of the sides of a triangle are a, b, c, then the area K, of the triangle is as follows:

$$K = \sqrt{S(s-a)(s-b)(s-c)}, \text{ where } s = \frac{1}{2}(a+b+c)$$ **(s = semi perimeter)**

4. Use Heron's formula to find the area of each triangle.

A. $a = 30, b = 50, c = 56$ B. $a = 5, b = 12, c = 13$

HW Practice 5.7 Worksheet

5.7 Practice Worksheet

Solve each triangle. Round angles measures to the nearest minute and side measures to the nearest tenth.

1. $a = 1.5, b = 2.3, c = 1.9$

2. $b = 40, c = 45, A = 51°$

3. $A = 52°, b = 120, c = 160$

4. $a = 15, b = 18, c = 17$

5. $A = 42°, b = 120, c = 120$

6. $a = 15, b = 18, c = 20$

7. $b = 12, a = 20, c = 28$

8. $a = 12.5, b = 15.1, c = 10.3$

9. $c = 49, b = 40, A = 53°$

10. $a = 10, c = 8, B = 100°$

Practice 5-7 Answers

1. $A = 40^\circ 28'$, $B = 84^\circ 16'$, $C = 55^\circ 17'$

2. $a = 36.9$, $B = 57^\circ 28'$, $C = 71^\circ 32'$

3. $a = 127.9$, $B = 47^\circ 40'$, $C = 80^\circ 20'$

4. $A = 50^\circ 39'$, $B = 68^\circ 8'$, $C = 61^\circ 13'$

5. $a = 86.0$, $B = 69^\circ$, $C = 69^\circ$

6. $A = 46^\circ 8'$, $B = 59^\circ 53'$, $C = 73^\circ 59'$

7. $A = 38^\circ 13'$, $B = 21^\circ 47'$, $C = 120^\circ$

8. $A = 55^\circ 8'$, $B = 82^\circ 20'$, $C = 42^\circ 32'$

9. $a = 40.5$, $B = 52^\circ 2'$, $C = 74^\circ 58'$

10. $A = 45^\circ 20'$, $b = 13.8$, $C = 34^\circ 40'$

5.8 Practice Worksheet

Find the area of each triangle to the nearest hundredth.

1. $c = 3.58, A = 37°40', B = 69°20'$

2. $a = 5, b = 12, c = 13$

3. $a = 11, b = 13, c = 16$

4. $C = 85°, a = 2, B = 19°$

5. $A = 50°, b = 12, c = 14$

6. $b = 14, C = 110°, B = 25°$

7. $b = 15, c = 20, A = 115°$

8. $a = 68, c = 110, C = 100°$

Practice 5-8 Answers

1. 3.8 *square units*

2. 30 *square units*

3. 71.0 *square units*

4. 0.7 *square units*

5. 64.3 *square units*

6. 154.1 *square units*

7. 135.9 *square units*

8. 2526.8 *square units*

Law of Sines and Cosines Applications Worksheet
Round all lengths to the nearest tenths and angles to the nearest minute.

1. A 98 ft. extension ladder rests on top of a hook ladder truck with its base 11 ft. from the ground. When the angle of elevation of the ladder is 73°, how high up the building will it reach?

2. The angle of depression of an airplane to the airport is 30°21' when the plane is 10.3 km from the airport. Determine the altitude of the plane.

3. The Gateway Arch in St. Louis is 630 ft. high. If a person is walking away from the Arch until the angle of elevation to the top is 55°, how far from the Arch is the person?

4. A ranger's tower is located 44m from a tall tree. From the top of the tower, the angle of elevation to the top of the tree is 28°, and the angle of depression to the base of the tree is 36°. How tall is the tree?

5. From a point A, the angle of elevation to the top of a building is 50°. From point B which is 11 m closer to the building the angle of elevation to the top is 63°. How far is point B from the top of the building? How tall is the building?

6. If a pole has one 62 ft. guy wire that makes an angle of 39° with the ground, and a second 50 ft guy wire is available for the opposite side of the pole, what angle measure will the second wire make with the ground?

7. A field is triangular shaped with sides of lengths 83 m, 120 m, and 165 m. What is the measure of the angle formed by the two longest sides?

8. The base paths in baseball are 90 ft. The angles formed by the base paths are 90°. If a center fielder stands 300 ft. from second base in straight center field, how far is he from third base?

9. Two roads intersect at an angle of 102°10'. Your friend's mailbox is 476 ft. from the intersection. Your mailbox is on the other road and is 615 ft. from the intersection. How far is it from your mailbox to your friend's?

10. The sides of a parallelogram are 12 ft. and 8 ft., and each of the larger angles measures 125°. Find the length of the shorter diagonal of the parallelogram.

11. How many triangles exist with an angle of 51°, adjacent side of length 27 cm, and opposite side of length 11 cm?

12. A jet took off at a rate of 260 ft./sec and climbed in a straight path for 3.8 minutes. What was the angle of elevation of its path when its altitude was 17,820 ft.?

AFM Test Review Trig Unit 4 Name_____

I. State the number of solutions each triangle will have.

1. A = 70°, b = 12, a = 8
2. a = 15, b = 10, B = 35°
3. a = 8, C = 65°, c = 4
4. B = 33°, a = 1, b = 1.2
5. a = 16, b = 8, c = 20

II. Tell if you would use Law of Sines or Law of Cosines to solve each triangle.

1. C = 25°, c = 11, A = 30°
2. b = 6, c = 10, A = 70°
3. a = 2, b = 5, A = 63°
4. a = 4, b = 15, c = 6
5. a = 12, b = 15, C = 52°

III. Solve the triangle. Round angles to nearest minute and sides to nearest tenth.

1. A = 38°, a = 172, b = 203
2. A = 51°, b= 7, c= 10
3. A = 58°, b = 29, a = 26
4. a= 4, b = 5, c= 7

IV. Find the Area.

1. a = 5, b = 6, c = 7
2. A = 37°, B = 84°, and c = 5
3. a = 4, b = 5, c = 7
4. C = 28°, a = 14, b = 9

V. Draw the triangle and show all work. Round answers to the nearest tenth.

1. From the top of a lighthouse $163\,ft$ above sea level the angle of depression of a ship at sea is $31^\circ 20'$. Find the distance of the ship from the base of the lighthouse.

2. A tree casts a shadow on the ground because of the sun's rays.
The length of the shadow is $75\,ft$. The angle of elevation is 32°. Find the height of the tree.

3. The measure of angle B is 56°. The measure of angle C is 90° and side c measures 20.
Solve the triangle.

Test Review Trig Unit 4 - ANSWERS

I: 1. 0 2. 2 3. 0 4. 1 5. 1

II: 1. sines 2. cosines 3. sines
4. cosines 5. Cosines

III: 1. B=46°36’ , C=95°24’ , c=278.1
or B=133°24’, C=8°36’, c=41.8

2. a=7.8, B=44°13’, C=84°47’

3. B=71°4’, C=50°56’, c=23.8
or B=108°56’, C=13°4’, c=6.9

5.A=34°3’, B=44°25’, C=101°32’

IV: 1. 14.7 units^2 2. 8.7 units^2

3. 9.8 units^2 4. 29.6 units^2

V: 1. 267.7 ft
2. 46.9 ft
3. A = 34° a = 11.2 b = 16.6