**Unit 4 Notes**

**Extrema On An Interval:**

Let *f* be defined on an Interval ***I*** containing c.

1. *f*(c) is the **minimum of *f* on *I*** if for all x in *I*.

2. *f*(c) is the **maximum of *f* on *I*** if for all x in *I*.

The minimum and maximum of a function on an interval are the extreme values, or extrema, of the function on the interval. The minimum and maximum are also called the **absolute minimum** and **absolute maximum** on the interval.

  

**Max (3, 4) Max none Max (3, 4)**

**Min (0, -3) Min (0, -3)**  Min none

**Extreme Value Theorem**: ( This is an existence theorem)

If *f* is continuous on the closed interval , then *f* has both a minimum and a maximum on the interval.

Label Parts:



**Critical Points:** Let *f* be defined at c. If or if *f* is not differentiable at c,

then c is a critical number of *f*.

*Relative Extrema occur only at critical numbers. If f has a relative minimum or maximum at x = c, then c is a critical number.*

**Guidelines for Finding Extrema on a Closed Interval**

1. Find the critical numbers of *f* in (a, b)
2. Evaluate *f* at each critical number in (a, b)
3. Evaluate *f* at each endpoint of [a, b]
4. The least of these values is the minimum and the greatest is the maximum.

Locate the absolute extrema of the function on the interval.

Ex 1:  Ex 2: 

Ex 3:  Ex 4: 

Ex 5:  Ex 6: 

Ex 7:  Ex 8: 

Rolle’s Theorem:

If *f* is continuous on the closed interval , and differentiable on the open interval 

If then then there is at least one number c in  such that .

***(The graph has to turn around somewhere in between )***

I. Find the intercepts of the given function and show that 

at some point between the intercepts.

Ex:  Ex: 

II. Given an interval, find the c value(s) that satisfy Rolle’s Theorem:

Ex:  Ex: 

III. For each problem, determine if Roll’s Theorem can be applied. If so, find all c values.

Ex:  Ex: 

Ex: 

Mean Value Theorem:

If *f* is continuous on the closed interval , and differentiable on the open interval ,

then there exists a number c in  such that



**The average rate of change will equal the instantaneous rate at some point on the interval.**

**In other words, the slope of the secant line equals the slope of the tangent line.**

Ex: Cooking a hamburger: A hamburger is pulled out of the refrigerator at 50 degrees.

The burger is cooked on a grill set at 500 degrees for three minutes. At some point, the

temperature of the hamburger must be 150 degrees.

For the following, find the c value(s) that satisfy the Mean Value Theorem.

Ex:  Ex: 

Ex:  Ex: 

Day 3:

Definition of Increasing and Decreasing Functions

A function *f* is increasing on an interval if for any two numbers a and b in the interval, a < b implies .

A function *f* is decreasing on an interval if for any two numbers a and b in the interval, a < b implies .

Let *f* be a function that is continuous on the closed  and differentiable on the open interval .

1. If for all x in , then *f* is increasing on  .

2. If for all x in , then *f* is decreasing on  .

3. If for all x in , then *f* is constant on  .

Ex 1: Find the open intervals on which is increasing or decreasing. (Graph)

Ex 2: Find the open intervals on which is increasing or decreasing.

Guidelines for Finding Intervals on Which a Function is Increasing or Decreasing

1. Locate the Critical Points on the interval , use these numbers to determine test intervals.

2. Determine the sign of at one test value in each of the intervals.

3. Determine whether *f* is increasing or decreasing.

The First Derivative Test:

Let c be a critical number of a function *f* that is continuous on an open interval *I* containing c.

If *f* is differentiable on the interval, except possibly at c, then can be classified as follows.

1. If changes from negative to positive at c, thenis a relative minimum of *f*.

2. If changes from positive to negative at c, thenis a relative maximum of *f*.

3. If does not change sign at c, then is neither a relative minimum nor a relative maximum.

Find the relative extrema of the functions below, if they exist.

Ex 3:  Ex 4: 

Ex 5:  Ex 6: 

Ex 7:  Ex 8: 

Day 4: Concavity and the Second Derivative Test

Let *f* be differentiable on an open interval *I*. The graph of *f* is concave upward on *I* if is increasing on the interval and concave downward on *I* if is decreasing on the interval.

Let *f* be a function whose second derivative exists on an open interval *I*.

1. If for all x in *I*, then the graph of *f* is concave upward in *I*.

2. If for all x in *I*, then the graph of *f* is concave downward in *I*.

**Points of Inflection: If  is a point of inflection of the graph of *f*, then either or does not exist at x = c.**

I. For each of the following find all x coordinates of all points of inflection, find all discontinuities, and

find all open intervals in which the function is concave up or concave down.

Ex:  Ex: 

Ex:  Ex: 

Ex:  Ex: 

What does tell us about ?

1.  and  3.  and 
2.  and  4.  and 

Match with the Graph?

1. B.

C. D.

Second Derivative Test:

Let *f* be a function such that and the second derivative of *f* exists on an open interval containing c

1. If then  is a relative minimum.

2. If then  is a relative maximum.

If , the test fails. In such cases, you can use the First Derivative Test.

Ex:  Ex:  Ex: 

Find the relative minimum and maximum using the Second Derivative Test.

Ex:  Ex: 

Ex:  (Sketch a graph.)

Ex: Let’s look at what we know. Sketch a graph given the following information.



-

-

+

-

+



1

5

-2

What does this tell us about *f*? Let *f*(-4) = 0 Sketch:

Let’s sketch a possible graph for *f* given the following graph of on the closed interval 

-8

-4

-2

5

2

9

12

-6

Let 



Ex: *f* is continuous on  , Find the absolute extrema, point(s) of infection, and sketch.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| x | 0 < x < 1 | 1 < x < 2 | 2 < x < 3 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Find each of the following, then sketch the graph. Show work/logic used.

NO GRAPHING CALCULATORS

1. 

A: Intercepts: B: Symmetry:

x-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ y-axis: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

y-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ origin: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

C: Domain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D: Vertical Asymptote(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

End Behavior: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Critical Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

F:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Inflection Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



x

y

Label Axis

Find each of the following, then sketch the graph. Show work/logic used.

NO GRAPHING CALCULATORS

1. 

A: Intercepts: B: Symmetry:

x-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ y-axis: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

y-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ origin: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

C: Domain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D: Vertical Asymptote(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

End Behavior: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Critical Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

F:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Inflection Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



x

y

Label Axis

Find each of the following, then sketch the graph. Show work/logic used.

NO GRAPHING CALCULATORS

1. 

A: Intercepts: B: Symmetry:

x-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ y-axis: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

y-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ origin: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

C: Domain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D: Vertical Asymptote(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

End Behavior: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Critical Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

F:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Inflection Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



x

y

Label Axis

Find each of the following, then sketch the graph. Show work/logic used.

NO GRAPHING CALCULATORS

1. 

A: Intercepts: B: Symmetry:

x-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ y-axis: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

y-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ origin: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

C: Domain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D: Vertical Asymptote(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

End Behavior: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Critical Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

F:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Inflection Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



x

y

Label Axis

Find each of the following, then sketch the graph. Show work/logic used.

NO GRAPHING CALCULATORS

1. 

A: Intercepts: B: Symmetry:

x-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ y-axis: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

y-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ origin: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

C: Domain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D: Vertical Asymptote(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

End Behavior: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Critical Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

F:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Inflection Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



x

y

Label Axis

Find each of the following, then sketch the graph. Show work/logic used.

NO GRAPHING CALCULATORS

1.  

A: Intercepts: B: Symmetry:

x-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ y-axis: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

y-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ origin: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

C: Domain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D: Vertical Asymptote(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

End Behavior: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Critical Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

F:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Inflection Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



x

y

Label Axis

Find each of the following, then sketch the graph. Show work/logic used.

NO GRAPHING CALCULATORS

1. 

A: Intercepts: B: Symmetry:

x-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ y-axis: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

y-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ origin: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

C: Domain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D: Vertical Asymptote(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

End Behavior: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Critical Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

F:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Inflection Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



x

y

Label Axis

Find each of the following, then sketch the graph. Show work/logic used.

NO GRAPHING CALCULATORS

1. 

A: Intercepts: B: Symmetry:

x-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ y-axis: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

y-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ origin: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

C: Domain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D: Vertical Asymptote(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

End Behavior: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Critical Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

F:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Inflection Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



x

y

Label Axis

Find each of the following, then sketch the graph. Show work/logic used.

NO GRAPHING CALCULATORS

1.  

A: Intercepts: B: Symmetry:

x-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ y-axis: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

y-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ origin: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

C: Domain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D: Vertical Asymptote(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

End Behavior: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Critical Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

F:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Inflection Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



x

y

Label Axis

Find each of the following, then sketch the graph. Show work/logic used.

NO GRAPHING CALCULATORS

1.  

A: Intercepts: B: Symmetry:

x-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ y-axis: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

y-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ origin: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

C: Domain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D: Vertical Asymptote(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

End Behavior: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Critical Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

F:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Inflection Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



x

y

Label Axis

Find each of the following, then sketch the graph. Show work/logic used.

NO GRAPHING CALCULATORS **This One is Hard!**

1.  

A: Intercepts: B: Symmetry:

x-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ y-axis: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

y-intercept(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ origin: Y/N \_\_\_\_\_\_\_\_\_\_\_\_\_\_

C: Domain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D: Vertical Asymptote(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

End Behavior: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Critical Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

F:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Inflection Point(s): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



x

y

Label Axis