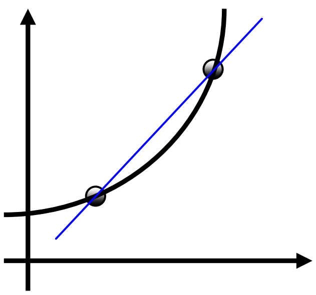
Unit 3 Notes

Average Rate of Change -

Instantaneous Rate of Change –



You can approximate the slope by using the secant line through the point of tangency and a second point on the curve. If $(c, f(c))$ is the point of tangency and $(c+\Delta x, f(c+\Delta x))$ is a second point on the graph of f.

Slope =

The right hand side of this equation is the difference quotient. The denominator Δx is the change in x, and the numerator $\Delta y = f(c+\Delta x - f(c))$ is the change in y.

Approximating the slope:

Definition of Tangent Line with Slope *m*

If *f* is defined on an open interval containing c, and if the limit $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = m$ exists, then the line passing through $(c, f(c))$ with slope *m* is the tangent line to the graph of *f* at the point $(c, f(c))$.

Ex: Find the slope of the tangent line of $f(x) = x^2$ at x = -2. x = 0. x = 2.

*** Notice that the slope varies with x's when the function is nonlinear.**

Ex: Find the slope of the tangent line of $f(x)=\sqrt{x}$ at x = 4, x = 9.

The Derivative of a Function: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

provided the limit exists. For all x for which this limit exists , f' is a function of x.

Other notation for the derivative: $\frac{dy}{dx}$ "*the derivative of y with respect to x*", y' $D_x[y]$, $\frac{d}{dx}[f(x)]$

Ex: Find the derivative of $f(x)=3x-1$

Ex: Find the derivative of $f(x)=x^3+2x$

Alternative Definition of a Derivative: $f'(x)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$

This is useful to show the relationship between differentiability and continuity.

This requires that both one-sided limits exist.

Ex: Write the equation of the tangent line and normal line of $f(x)=\frac{1}{x}$ at x = 3

Differentiability – what does this mean? When is a function not differentiable?

Ex: $f(x)=|x+3|$

Ex: $f(x)=\sqrt[3]{x}$

Ex: $f(x)=\frac{1}{x}$

If *f* is differentiable at x = c, then *f* is continuous at x = c.

Vertical Tangents:

Day 2 Notes: Power Rule:

Ex: What is the derivative of $f(x) = -2$?

Ex: What is the derivative of $f(x) = 3x$?

Ex: Use the limit definition to find the derivative of $f(x) = x^2$?

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

Ex: Use the limit definition to find the derivative of $f(x) = x^3$?

Ex: Use the limit definition to find the derivative of $f(x) = x^4$?

Ex: Use the limit definition to find the derivative of $f(x) = x^5 + 3x^2 - 6x$?

Ex: Use the power rule to find the derivative of $f(x) = \sqrt{x}$?

Ex: Use the power rule to find the derivative of $f(x) = \frac{1}{x}$?

Ex: How do we find the derivative of $f(x) = \sin(x)$?

Ex: Write the equation of the tangent line of $f(x) = \sqrt[3]{x}$ at x = 2.

Ex: Use the limit definition to find the derivative of $f(x) = \cos(x)$?

Ex: Use the limit definition to find the derivative of $f(x) = \tan(x)$?

Ex: Use the limit definition to find the derivative of $f(x) = \sec(x)$?

Ex: Use the limit definition to find the derivative of $f(x) = \csc(x)$?

Day 3 Notes:

Practice the Power Rule: Find the derivative of each of the following.

Ex 1: $f(x) = 4x^{12}$

Ex 2: $f(x) = 3x^{\frac{7}{4}}$

Ex 3: $f(x) = \dfrac{5}{x^3}$

Ex 4: $f(x) = -3\sqrt[4]{x} + 2x^{-3}$

Ex 5: $f(x) = \dfrac{2}{3}x^{\frac{3}{2}} - \dfrac{3}{4}x^{\frac{3}{5}}$

Ex 6: $f(x) = \sqrt{2}\,\sqrt[3]{x} + \sqrt{2}\,\sqrt[5]{x}$

The Product Rule: $\dfrac{d}{dx}\left[f(x) \bullet g(x)\right] = f(x)g'(x) + g(x)f'(x)$

Ex 7: $y = (2x - 3x^3)(5x - 4)$

Ex 8: $y = 3\sin(x)\cos(x)$

Ex 9: $y = \dfrac{2}{x^4}\tan(x)$

What would be the second derivative ($f''(x)$ or $\dfrac{d^2y}{dx^2}$) of example # 7?

The Quotient Rule: $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ **(Low D High – High D Low)**

Ex 10: $y = \dfrac{x^2 + 3x}{2x + 1}$

Ex 11: $y = \cot(x)$

Ex 12: $y = \dfrac{\sqrt{x} + 3}{\sqrt{x} - 3}$

Practice: Find the derivatives of each.

Ex 13: $y = \sqrt[3]{x^2}\sec(x)$

Ex 14: $y = \dfrac{\sqrt{x+2}}{x^3}$

Ex 15: $y = \dfrac{3 - \frac{1}{x}}{x+5}$

Ex 16: $y = \dfrac{1-\cos(x)}{\sin(x)}$ (Use both forms)

What would you do here? Find the derivative of $f(x) = \dfrac{x\sqrt{1+x}}{x^3\sin(x)}$

Higher Order Derivatives:

Ex 17: $y = -x^2$ Find $\dfrac{d^2y}{dx^2}$

Ex 18: $y = -2x^3 - 4x^{-3}$ Find $\dfrac{d^3y}{dx^3}$

Ex 19: $y = -x^4 + 2\sqrt[5]{x^3}$ Find y'''

Ex 20: $y = \cos(x) - 3x^{-2}$ Find y'''

Ex 21: $y = -x^2\cos(x) + \dfrac{1}{2x^3}$ Find $\dfrac{d^3y}{dx^3}$

Day 5 Notes:

The Chain Rule: : $If\ \ h(x) = f(g(x))$ then $h'(x) = f'(g(x)) \bullet g(x)$

Ex 1: $y=(2-5x^2)^3$

Ex 2: $y=(2x^3)^4$

Ex 3: $y=\sqrt{4x+5}$

Ex 4: $y=\sqrt[3]{2x^2-5x}$

Ex 5: $y=(6x+1)^5+\sqrt{2x}$

Ex 6: $y=\sin(x^3)$

Ex 7: $y=\tan(\sqrt{x})$

Ex 8: $y=x^3\cos\left(\frac{1}{x}\right)$

Ex 9: $y=\frac{(2x+1)^3}{\sqrt{3x-5}}$

Ex 10: $y=3x^2\csc(x^4)$

Ex 11: $y=\sqrt{4x}\cos(2x)$

Ex 12: $y=3x\sec(5x)$

Find the 2nd derivative for each.

Ex 13: $y=(\sin(5x^3))^2$

Ex 14: $y=-7(5-3x^2)^5$

Ex 15: $y=\sqrt{\sin(2x)+\cos(x^5)}$

Day 7 Notes: Implicit Differentiation:

Ex: $x^2 - 4x + y^2 = 21$ is a circle and has a well-defined slope at nearly every point because it is the union of two graphs $y = \pm\sqrt{-x^2 + 4x + 21}$ which are both differentiable.

The question is how do we find the slopes ? We need to treat y as a differentiable function of x and differentiate both sides with respect to x. Then solve for $\frac{dy}{dx}$.

Ex: $x^2 - 4x + y^2 = 21$ to find the derivative: $2x - 4 + 2y\frac{dy}{dx} = 0$

$$2y\frac{dy}{dx} = -2x + 4$$

$$\frac{dy}{dx} = \frac{-2x + 4}{2y}$$

Ex: $y^2 = x$ at x = 4 we differentiate the left side using the chain rule. $2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y}$ the y is useful because it shows we have two tangent lines. Write the equations of both:

Ex: Find the slope of the circle at $x^2 + y^2 = 25$ at the point (3, -4).

Find the tangent lines at the given point of each curve.

Ex: $y^2 - 2x - 4y = 1$ at (-2, 1)

Ex: $2xy + \pi\sin(y) = 2\pi$ at $\left(1, \frac{\pi}{2}\right)$

Ex: $x\sin(2y) = y\cos(2x)$ at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Ex: $x^2\cos^2(y) - \sin(y) = 0$ at $(0, \pi)$

Ex: $x + \tan(xy) = 0$