4.2 Quadratic Equations Notes Day 1

1. **Solve by factoring:**

   a. $3x^2 = 16x + 12$ b. $x^2 - 2x = 3$ c. $x^2 + 8x = 0$

   d. $x^2 + 9x + 18 = 0$

2. **Quadratic Formula:**

   The roots of a quadratic equation of the form $Ax^2 + Bx + C = 0$ with $a \neq 0$ are given by the following formula:

3. Solve by using the quadratic formula.

   a. $3x^2 + x - 2 = 0$ b. $2x^2 + 8x + 12 = 0$ c. $8x^2 + x + 2 = 0$

4. **Discriminant:**

   Expression under the radical in the quadratic formula: $b^2 - 4ac$. It helps to determine what kind of roots that you have.

   - If $b^2 - 4ac > 0$ perfect there are 2 real, rational solutions.
   - If $b^2 - 4ac > 0$ not perfect there are 2 real, irrational solutions.
   - If $b^2 - 4ac = 0$ there are 1real rational solution.
   - If $b^2 - 4ac < 0$ there are 2 complex conjugate roots or 2 unequal imaginary roots.

5. Determine the discriminant for each and describe the nature of the roots.

   a. $x^2 - 4x - 5 = 0$ b. $4x^2 + 20x + 25 = 0$ c. $2x^2 + x + 28 = 0$

6. Sum and Product Rule:

The sum of the roots of a quadratic are equal to $-\frac{b}{a}$.

The product of the roots of a quadratic are equal to $\frac{c}{a}$.

7. Write a quadratic equation for each set of roots given.

a. 2, 3 b. 4, 7 c. $\frac{2}{3}, -\frac{3}{5}$ d. $\frac{1}{2}, -\frac{5}{4}$

8. State a quadratic equation whose roots have the given **sum and product**.

a. $2 \pm \sqrt{3}$ b. $-3 \pm \sqrt{7}$ c. $4 \pm 3i$ d. $5 \pm 2i\sqrt{3}$

Quadratic Worksheet 1

I. For each problem, find the discriminant and the nature of the roots as to number, real or imaginary and rational or irrational.

1. $4x^2 - 2x + 2 = 0$

2. $36x^2 - 12x + 1 = 0$

3. $20x^2 + 17x = 3$

4. $x^2 + 13 = 4x$

5. $x^2 = 13x$

6. $7x^2 + 6x + 2 = 0$

II. Give the sum and product of the roots of each equation:

5. $2x^2 - 6x + 5 = 0$

8. $x^2 - 6x = 8$

9. $2x^2 + 3x = 0$

10. $4x^2 = 2 - 5x$

III. State the quadratic equation whose roots have the given sum and product:

6. sum: 3 product: 2

12. sum: 0 product: 3

13. sum: $\frac{2}{3}$ product: $\frac{5}{2}$

IV. Solve by factoring.

14. $3x^2 = 16x + 12$

15. $7x^2 - 8x = 12$

16. $5x^2 = 23x - 26$

V. Solve by using the quadratic formula.

17. $3x^2 + 2x - 1 = 0$

18. $6x + 5 = -2x^2$

19. $x^2 + 8x = 16$

9. **Completing the Square:**

You can change an expression like $x^2 + bx$ into a perfect square trinomial by adding $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$.

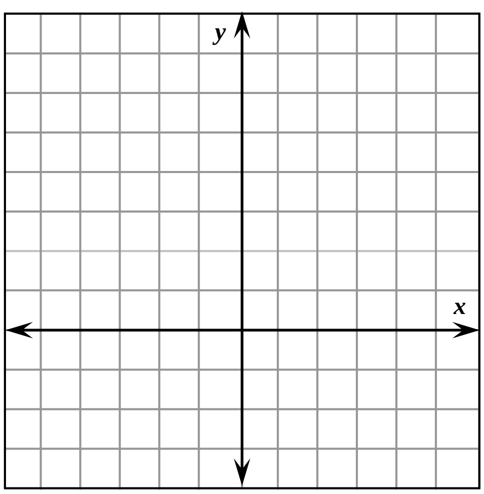
10. Solve by completing the square.
    a. $x^2 - 8x - 36 = 0$
    b. $x^2 + 6x - 41 = 0$
    c. $2x^2 = 6x + 5$

11. To graph a quadratic, the best form to have the equation is in vertex form: $y = a(x-h)^2 + k$

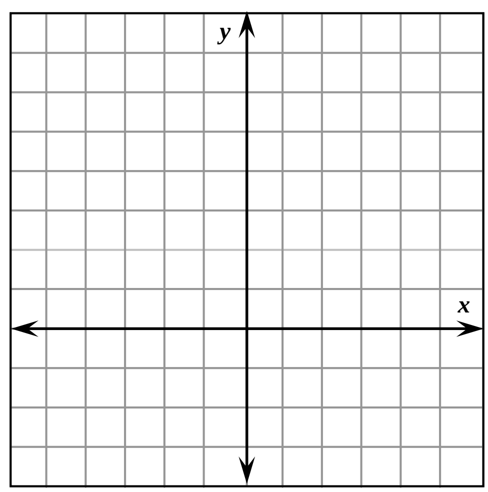
12. If the equation is not in vertex form then either complete the square or use $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

13. Once you have the vertex form, then you can plot the vertex and use the patterns for parabolas to complete your sketch. If you are asked to also find the roots (x-intercepts), you can include those on the sketch as well. It is also helpful to find the y-intercept and use that to help sketch as well.

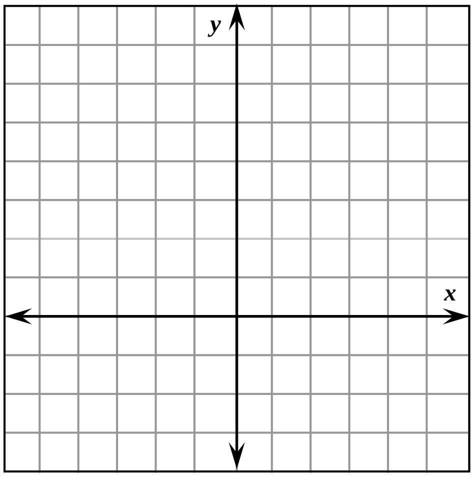
14. Sketch each quadratic.
    a. $y = 2(x-3)^2 - 4$



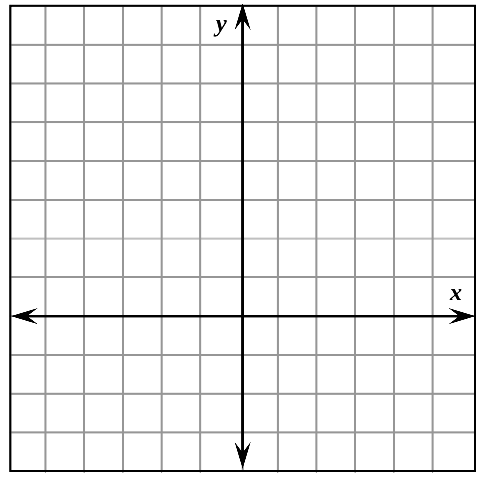
    b. $y = x^2 + 6x + 8$



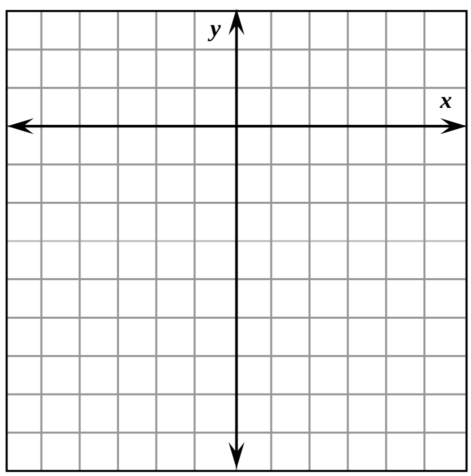
c. $y = x^2 - 3x - 1$



d. $y = 2x^2 + 4x + 1$

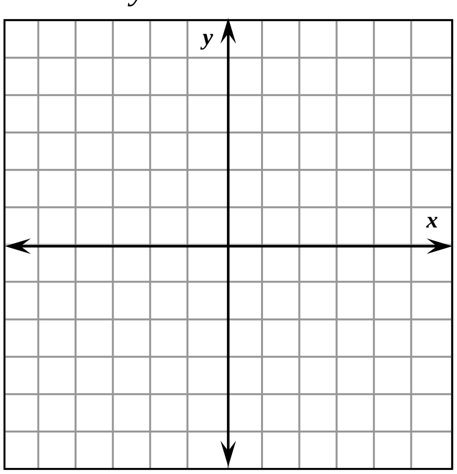


e. $y = -3x^2 + 12x - 11$

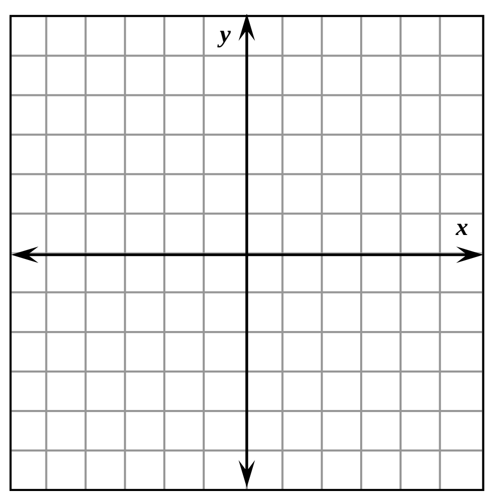


15. Sketch each quadratic inequality.

a. $y > x^2 - 2x - 4$



b. $y \leq 2x^2 - 4x + 5$



Solving Quadratic Inequalities

1. Interval Notation: All answers to quadratic inequalities will be in interval notation.

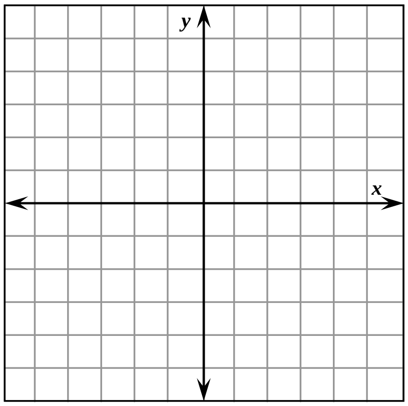
2. Solve each quadratic inequality. Put answers in interval notation

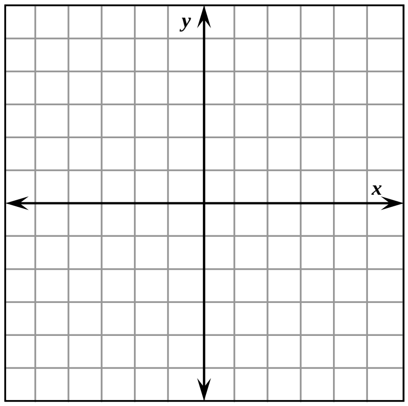
.

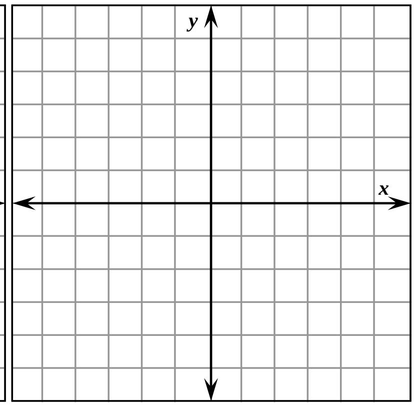
a. $x^2 - 4x - 12 > 0$

b. $3x^2 + 14x - 5 \leq 0$

c. $2x^2 + x \geq 15$







d. $6x^2 + x > 1$

e. $5x < 2 - 3x^2$

f. $x^4 - 9 > 0$

g. $x^3 + 27 < 0$

h. $12x^3 - 10x \leq 12$

i. $(x-3)^2(x+1) < 0$

Homework Quadratic Worksheet 2

Quadratics Worksheet 2

I. Put each quadratic in vertex form.

1. $y = x^2 - 6x + 11$

2. $y = 15 - 4x - 2x^2$

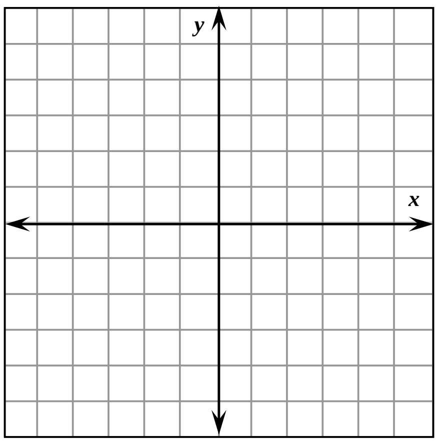
3. $y = 3x^2 - 9x + 17$

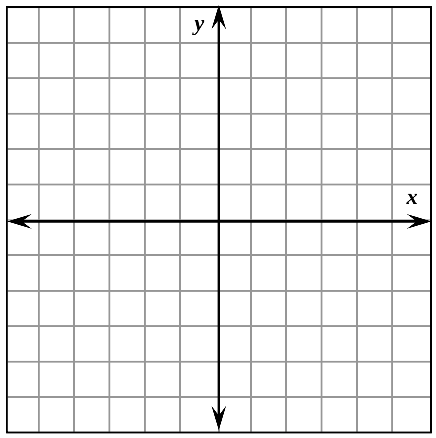
II. Put each quadratic in vertex form, then sketch the graph.

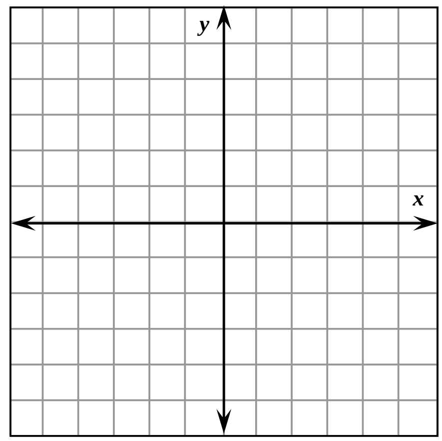
4. $y = 2x^2 - 16x + 33$

5. $y \le -x^2 + 8x - 13$

6. $y = 7 - 3x - x^2$



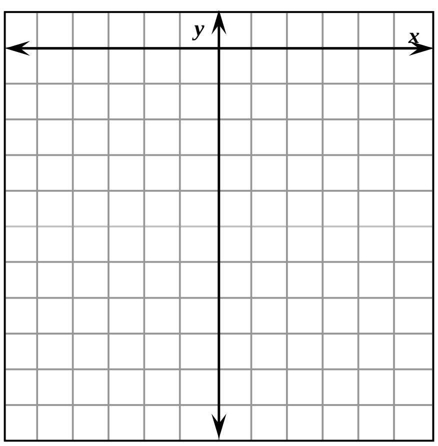


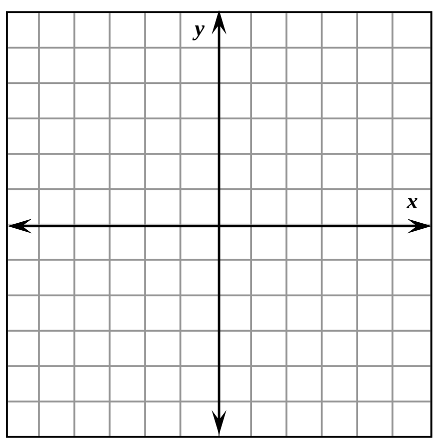


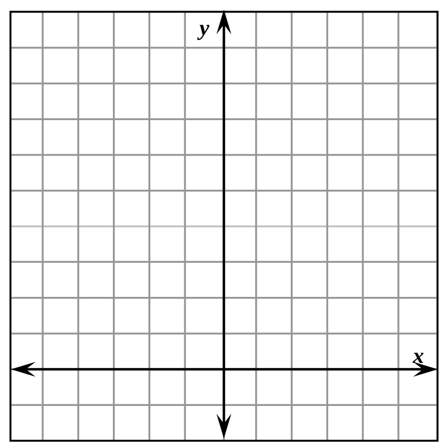
7. $y = 2x^2 - 10x + 1$

8. $y > x^2 + 6x + 5$

9. $y \le 4 - 6x - 2x^2$







Solve each inequality and put in Interval Notation.

10. $x^2 + 3x - 10 < 0$

11. $5x^3 - 9x^2 \le -4x$

12. $(x+2)^2 (x-1)(x-4) < 0$

4.1 Polynomial Functions

1. **Zeros of Polynomial Functions:**
If f is a polynomial function, then the values of x for which *f*(x) is equal to 0 are called the zeros of f. These values of x are the roots, or solutions, of the polynomial equation *f*(x) = 0. Each real root of the polynomial equation appears as an x-intercept of the graph of the polynomial function.

2. Determine whether each number is a root of $x^4 - 4x^3 - x^2 + 4x = 0$.
   a. 2 b. 0 c. -1 d. -2

3. **Degrees of a Polynomial:**

Degree	Name	Example
0	Constant	
1	Linear	
2	Quadratic	
3	Cubic	
4	Quartic	
5	Quintic	

4. **Fundamental Theorem of Algebra:**
Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.

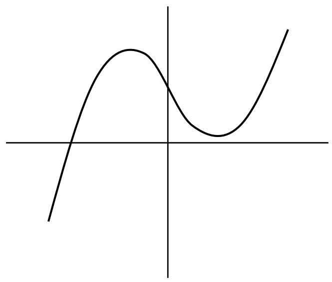
5. **Complex Numbers:** Any number that can be written in the form a + b*i*, where a and b are real numbers and *i* is the imaginary unit.

6. State the number of complex roots of the equation $x^3 - 2x^2 - 15x = 0$, then find the roots.

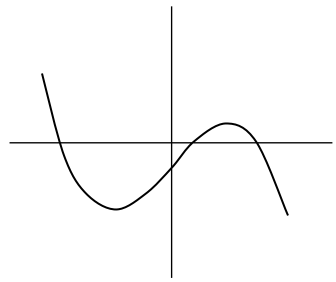
7. If -3 is a root of $2x^2 + bx - 9 = 0$ find the other roots and the value of b.

8. Find the values of k for which the equation has roots of the specified nature:
   a. $2x^2 + 5x + k = 0; 2\,real$ b. $kx^2 + 5x - 4 = 0; 2\,complex$

9. The shape of the graph of a cubic function $f(x) = ax^3 + bx^2 + cx + d$ is similar to a "sideways S."

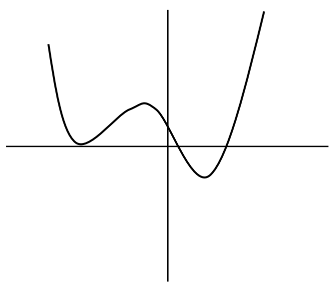


***a* > 0**

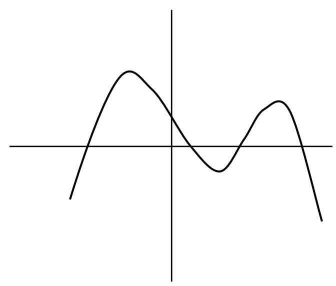


***a* < 0**

10. The shape of the graph of a quartic function $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ has a "*W*-shape" or "*M*-shape."

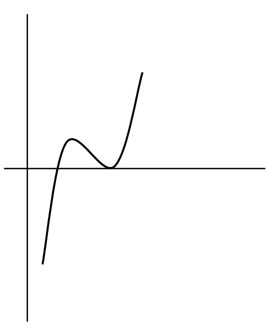


***a* > 0**



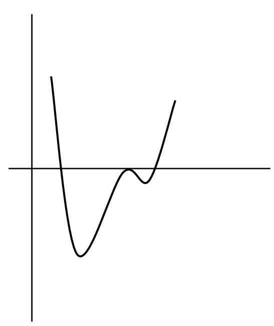
***a* < 0**

11. If a polynomial *P*(x) has a squared factor such as $(x-c)^2$, then $x = c$ is a **double root** of *P*(x) = 0 and the graph of y = *P*(x) is tangent to the *x*-axis at $x = c$, as illustrated below.



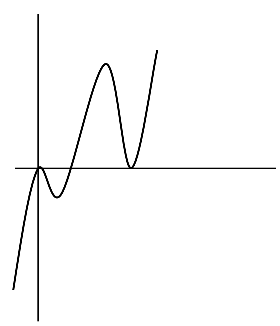
Cubic

$y = (x-1)(x-3)^2$



Quartic

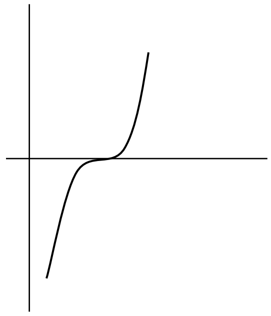
$y = (x-1)(x-3)^2(x-4)$



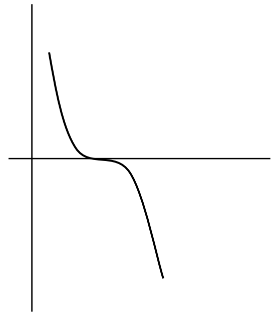
Quintic

$y = x^2(x-1)(x-3)^2$

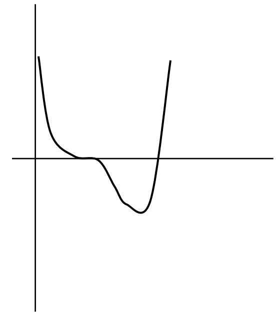
12. If a polynomial $P(x)$ has a cubed factor such as $(x-c)^3$, then $x = c$ is a **triple root** of $P(x) = 0$ and the graph of $y = P(x)$ flattens out around $(c, 0)$ and crosses the *x*-axis at this point.



Cubic
$y = (x-2)^3$



Cubic
$y = -(x-2)^3$



Quartic
$y = (x-1)^3(x-3)$

13. Find the roots, then sketch each function.

a. $f(x) = 3x - 3x^3$

b. $g(x) = (x-1)^2(x+2)$

c. $h(x) = (x-2)^3(x+3)^2$

4.1 Day 3 Homework Worksheet

I. Find the real values of k for which the equation has roots of the specified nature:

2. $x^2 - 10x + k = 0$, two real roots

2: $kx^2 + 8x - 4 = 0$, imaginary conjugate roots

3. $k\,x^2 - 4x + 8 = 0$, a double root

4. $kx^2 + 6x + k = 0$, imaginary conjugate roots

II. Find the solution for each.

5. If 2 is one root of $5x^2 + bx - 8 = 0$. find the other roots and the value of b.

6. $2 + 2i$ is a root of $x^2 + bx + 8 = 0$, find the other root and the value of b.

7. If $\frac{3}{2}$ is one root of $3x^2 + 6x + c = 0$. Find the other root and c.

8. If $2x^2 - 8x + c = 0$ has a double root, find the root and the value of c.

9. Write a cubic equation in general form with roots 5, $1+\sqrt{2}$, $1-\sqrt{2}$.

III. Find the roots and sketch.

10. $y = x^3 + x^2 - 2x$

11. $y = x^3 - 1$

12. $y = x^4 - 16$

4.3 Remainder and Factor Theorem and 4.4 Descartes Rule Notes

1. **Synthetic Division:**
   a. $\left(x^2-5x-12\right)\div(x-3)$
   b. $\left(3x^2+4x-12\right)\div(x-5)$

2. **The Remainder Theorem:**
   If a polynomial $P(x)$ is divided by $x-r$, the remainder is a constant, $P(r)$, and $P(x)=(x-r)\cdot Q(x)+P(r)$ where $Q(x)$ is a polynomial with degree one less than the degree of P(x).

3. Let $P(x)=2x^3+x^2-4x+3$ Show that $P(-1)$ is the remainder when $P(x)$ is divided by x + 1.

4. **The Factor Theorem:**
   The binomial x – r is a factor of the polynomial $P(x)$ if and only if $P(r)=0$.

5. Let $P(x)=x^3-x^2-5x-3$ Determine if x – 3 is a factor of $P(x)$.

6. Find the value of k so that each remainder is zero.
   a. $\left(2x^3+kx^2+7x-3\right)\div(x-3)$
   b. $\left(x^3+9x^2+kx-12\right)\div(x+4)$

7. Show that 1 is a zero of multiplicity of 2 of the polynomial function $P(x)=x^4-2x^2+1$ and express $P(x)$ as a product of linear factors.

8. Descartes' Rule of Signs:

Suppose $P(x)$ is a polynomial whose terms are arranged in descending powers of a variable. Then the number of positive real zeros of $P(x)$ is the same as the number of changes in sign of the coefficients of the terms, or is less than this by an EVEN number.

The number of negative real zeros of $P(x)$ is the same as the number of changes in sign of the coefficients of the terms of $P(-x)$ or is less than this by an **EVEN** number.

9. State the number of complex zeros, the number of possible positive real zeros, the number of possible negative real zeros, and the number of possible imaginary zeros.

a. $P(x)=2x^4-x^3-2x^2+5x+1$

b. $P(x)=2x^3+3x^2+5x+2$

10. Rational Root Theorem:

If the polynomial $f(x)=a_nx^n+a_{n-1}x^{n-1}+....+a_1x+a_0$ has integer coefficients, every rational zero of f has the form of $\frac{p}{q}$ (where $\frac{p}{q}$ is reduced), p is a factor of the constant term, a_0, and q is a factor of the leading coefficient, a_n.

You use the rational zero test to list all possible rational roots of a polynomial.

You use this list to find all rational roots of a polynomial, without a graphing calculator.

11. List all possible rational zeros of $f(x)=-x^4+3x^2+4$.

12. List all possible rational zeros of $f(x)=4x^5+12x^4-x-3$.

Homework: 4.3 and 4.4 Worksheet

4.3 and 4.4 Worksheet

I. Synthetic Division, Remainder Theorem, Factor Theorem

1. Determine k so that $f(x) = x^3 + kx^2 - kx + 10$ is divisible by $x + 3$.

2. Determine all values of k so that $f(x) = k^2x^2 - 4kx + 3$ is divisible by $x - 1$.

3. Find k so that when $x^3 - kx^2 - kx + 1$ is divided by $x - 2$, the remainder = 0.

4. Find k so that when $x^3 - x^2 - kx + 10$ is divided $x - 3$, the remainder is -2.

5. Find the remainder if the polynomial $3x^{100} + 5x^{85} - 4x^{38} + 2x^{17} - 6$ is divided by $x + 1$.

II. Use synthetic division to show that c is a zero of *f*(x).

6. $f(x) = 3x^4 + 8x^3 - 2x^2 - 10x + 4$; $c = -2$

7. $f(x) = 4x^3 - 9x^2 - 8x - 3$; $c = 3$

8. $f(x) = 4x^3 - 6x^2 + 8x - 3$; $c = ½$

9. Show that -3 is a zero of multiplicity 2 of the polynomial function $P(x) = x^4 + 7x^3 + 13x^2 - 3x - 18$ and express P(x) as a product of linear factors.

10. Show that -1 is a zero of $f(x)$ with multiplicity of 4 if $f(x) = x^5 + x^4 - 6x^3 - 14x^2 - 11x - 3$. Express $f(x)$ as a product of linear factors.

11. Find a polynomial function of degree 4 such that both -2 and 3 are zeros of multiplicity 2.

12. Find a polynomial function of degree 5 such that -2 is a zero of multiplicity 3 and 4 is a zero of multiplicity 2.

III. List all possible rational roots for each equation.

13. $f(x) = 2x^4 + 7x^3 + 5x - 4$

14. $g(x) = 3x^3 - 8x^2 + 9x - 6$

IV. Descartes Rule of Signs:

Complete the chart:

14. $y = -x^3 + x^2 - x + 1$
15. $y = 3x^4 + 2x^3 - 3x^2 - 4x + 1$
16. $y = -7x^3 - 6x + 1$
17. $y = x^{10} - x^8 + x^6 - x^4 + x^2 - 1$
18. $y = x^5 - x^3 - x + 1$
19. $y = 3x^4 - x^2 + x - 1$

Total Complex Roots	Number of possible +	Number of possible -	Number of possible imaginary
3	3 or 1	0	0 or 2

V. Find all rational roots using the rational root theorem and Descartes Rule to help.

20. $f(x) = x^3 - 7x^2 + 7x + 15$

4.5 Locating the Zeros of a Function and Graphing Polynomials

1. Find the roots for each function on the graphing calculator.

   a. $y = 4x^4 - 6x^2 - 2x + 1$ b. $y = -x^3 + 4x^2 - 6x + 8$ c. $y = 2x^5 + 3x^4 - 12x + 4$

2. To sketch a polynomial function:
   a. Find the zeros of the function. Remember zeros = x-intercepts so graph these points on the x-axis.
   b. Find the y-intercept
   c. Determine the end behavior of the function based on the degree and the leading coefficient.
   d. Using the end behavior and the intercepts to make a smooth curve.
   e. We will be not be finding max and min points by hand so "estimate" in your sketch.

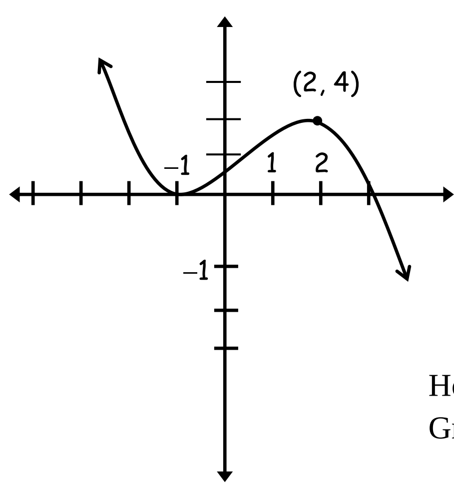
   f. y = -2(x^2 – 9)(x + 4)

4. $y = x^3 + 3x^2 - 4x - 12$

8. $y = -x^4 + 13x^2 - 36$

6. $y = \frac{1}{4}(x+2)(x-1)^2$

7. Write the equation in factored form. Find "a".



Homework pg. 213 #15-20 and Graphing Polynomials Worksheet

Graphing Polynomial Functions Worksheet

Remember: If it isn't factorable, then use the rational root theorem and Descartes Rule to find the roots the long way, without a graphing calculator. Finding extra points makes your graph more accurate!

1. $y = \frac{1}{5}(x-3)^2(x+1)^2$

2. $y = (x+1)^3(x-4)$

3. $y = x^5 - 6x^3 + 9x$

4. $y = x^3 - 21x + 20$

5. $y = -x^4 + 9x^2 - 20$

6. $y = x^3 + 4x^2 + x - 6$

Applications of Polynomial Functions Notes

1. Find two numbers whose difference is 8 and whose product is a minimum.

2. Find the dimensions and maximum area of a rectangle if its perimeter is 48 inches.

3. Julio has 48 feet of fencing to make a rectangular dog pen. If a house is used for one side of the pen, what would be the length and width for maximum area?

4. A clothing store sells 40 pairs of jeans daily at \$30 each. The owner figures that for each \$3 increase in price, 2 fewer pairs will be sold each day. What price should be charged to maximize profit?

5. An object is thrown upward into the air with an initial velocity of 128 feet per second. The formula $h(t) = 128t - 16t^2$ gives its height after "t" seconds.

   a) What is the height after 2 seconds?

   b) What is the maximum height reached?

   c) For how many seconds will the object be in the air?

6. A ball is thrown vertically upward with an initial speed of 80 ft/sec. Its height after t seconds is given by, $h(t) = 80t - 16t^2$.

   a) How high does the ball go?

   b) When does the ball hit the ground?

7. A square, which is 2 in. by 2 in. , is cut from each corner of a rectangular piece of metal. The sides are folded up to make a box. If the bottom must have a perimeter of 32 in., what would be the length and width for maximum volume? (find max V also)

8. Mr. Greene has 8.5-by-11 in. cardboard sheets. As a class project, Mr. Greene asked each of his students to make an open-top box under these conditions:

   I) Each box must be made by cutting small squares from each corner of a cardboard sheet.

   II) The box must have a volume of 48 in3.

   III) The amount of cardboard waste must be minimized.

   What is the ***approximate*** side length for the small squares that would be cut from the cardboard sheet?

HW: Polynomial Applications Homework

Solve each problem algebraically. MUST SHOW WORK FOR CREDIT.

1. Find two numbers whose difference is 40 and whose product is a minimum.

2. A rectangle has a perimeter of 24 meters. Find the dimensions of the rectangle with the maximum area.

3. Melissa plans to put a fence around her rectangle garden. She has 150 feet of fencing material to make the fence. If there is to be a 10 foot opening left for an entrance on one side of the garden, what dimensions should the garden be for the maximum area?

4. Kyle has 120 feet of fence to make a rectangular kennel for his dogs. If the house is to be used as one side of the kennel, what dimensions should the garden be for the maximum area?

5. The Center Stage Community Theater can seat 500 people. They sold out for every performance last season. They intend to raise the \$3 admission price for the upcoming season. They estimate that for every \$.20 increase in price, 25 fewer people will attend a performance. What ticket price will maximize the theater's income?

6. An open box is made from an 8-by-10-inch rectangular piece of cardboard by cutting squares from each corner and folding up the sides. If x represents the side length of the squares, which of the following is a function giving the volume $V(x)$ of the box in terms of x?

7. The volume of a fudge tin must be 120 cubic centimeters. The tin is 7 centimeters longer than it is wide and six times longer than it is tall. Find the dimensions of the tin.

8. The volume of a milk carton is 200 cubic inches. The base of the carton is square and the height is 3 inches more than the length of the base. What are the dimensions of the carton?

9. A rectangular solid has a volume of 14 cubic units. The width is twice the height and the length is 2 units more than the width. Find the dimensions of the solid.

10. A ball is thrown straight up with an initial velocity of 64 ft per second. The height of the ball t seconds after it is thrown is given by: $h(t) = 64t - 16t^2$.

a) What is the height of the ball after 1.5 seconds?

b) What is the maximum height?

c) After how many seconds will the ball return to the ground?