Precalculus Notes Unit 1
Day 1

Rules For Domain: When the domain is not specified, it consists of $\Re$ (all real numbers) for which the corresponding values in the range are also real numbers.

1. If x is in the numerator and raised to a positive integral.

Ex. $f(x) = x^2$ or $f(x) = \frac{x^3}{4} + 8$ Domain: **All reals** $(-\infty, \infty)$

2. If x is in the denominator, x **cannot** be any value that makes the **denominator zero.**

Ex. $f(x) = \frac{x^2}{x+1}$ Ex: $f(x) = \frac{x+1}{x^2+4x-21}$

3. If x is inside a square root, values of x are restricted to ones that will make the **radicand $\geq$ 0.**

Ex. $f(x) = \sqrt{x-3}$ Ex: $f(x) = \sqrt{3x+7}$

4. If x is in the square root and in the denominator, values of x are restricted to the one that will make the **radicand > 0.**

Ex. $f(x) = \frac{1}{\sqrt{x}}$ Ex: $f(x) = \frac{x}{\sqrt{x-5}}$

Range of a Relation – set of y values of a relation

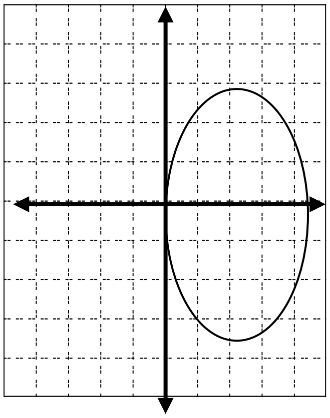
Function: special type of relation in which each element of the domain is paired with **exactly one** element of the range.

Testing For Functions Algebraically– Solve for y in terms of x. If each value of x corresponds to exactly one value of y, then y is a function of x.

Vertical Line Test: tests a graph to see if it is a function.

Horizontal Line Test: tests a graph to see if the function's inverse is also a function.

Ex:



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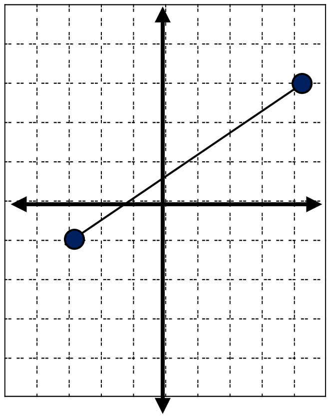

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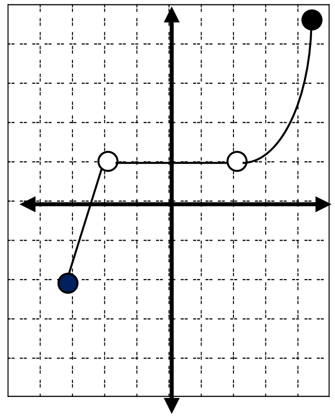


Function Notation: If the graph is a function we can use *f*(x) instead of y

Find $f(-1)$ if $f(x) = -x^3 - 1$.

Find $f(-2)$ if $f(x) = \frac{x-1}{x^2}$.

Piecewise Functions: A functions that is defined by two (or more) equations over a specified domain.

Ex: $f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ x - 1 & \text{if } x \geq 0 \end{cases}$

Ex: $h(x) = \begin{cases} x + 3 & \text{if } x < -3 \\ -(x-1) & \text{if } x \geq -3 \end{cases}$

Find: *f*(-3) = *f*(6) =

Find: *h*(0) = *h*(-6) = *h*(-3)=

The difference quotient, $\frac{f(x+h) - f(x)}{h}$ $h \neq 0$, plays an important role in understanding the rate at which a function changes.

Using the difference quotient, find and simplify

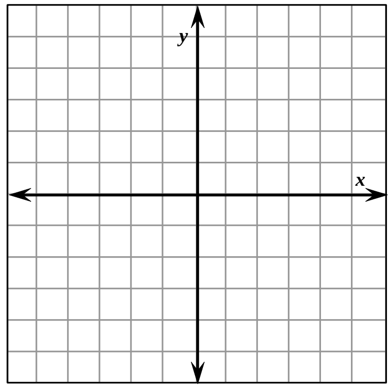
1. $f(x) = x^2 - 4x + 3$

2. $f(x) = \sqrt[2]{3 - x}$

Day 2

$$f(x) = \begin{cases} x^2 + 1 & x < 0 \\ x - 1 & x \geq 0 \end{cases}$$

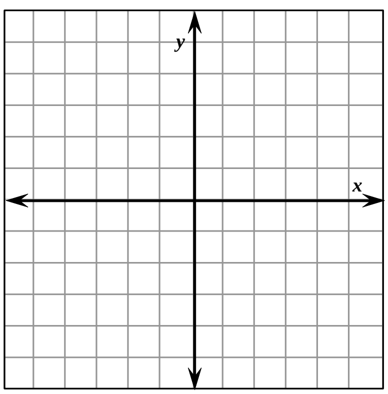
Graph function by hand:

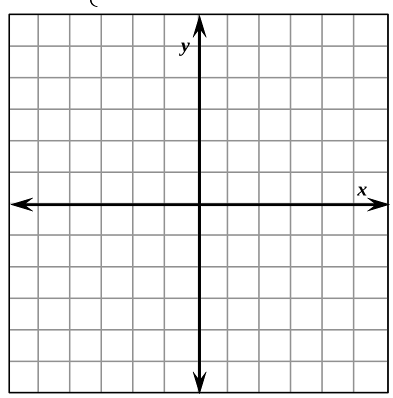


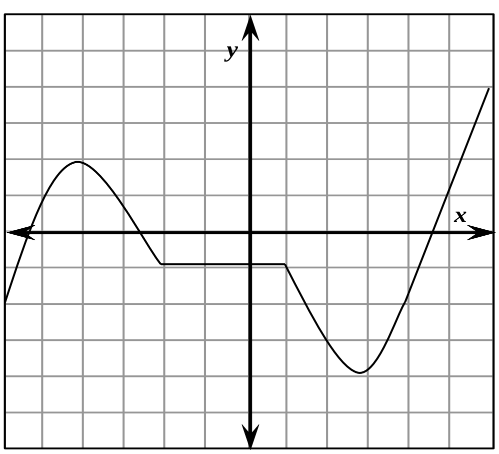
Graph on calculator $\quad y_1 = (x^2 + 1)\ (x < 0) + (x – 1)\ (x \geq 0)$

Ex: $f(x) = \begin{cases} 1 - x^2 & x \leq 2 \\ x & x > 2 \end{cases}$

Ex: $f(x) = \begin{cases} (x+4)^2 & x \leq -2 \\ 2 & -2 < x < 0 \\ -x^2 + 2x & x \geq 0 \end{cases}$







Determine the intervals on which the function is:

Increasing: ______________________

Decreasing: ______________________

Constant: ________________________

Find the relative maximum: _________

Using a calc: Find the relative max/ and or min of $f(x) = 2x^3 + 3x^2 – 12x + 1$
Determine the intervals on which each is increasing, decreasing, or constant.

a. $f(x) = x^2 + 4x - 12$

b. $g(x) = x^3 + 1$

c. $h(x) = x^3 + x^2 - x + 2$

d. $p(x) = 2$

Even and Odd Functions:
A graph has symmetry with respect to the y-axis if whenever (x, y) is on the graph then so is (-x, y).

A graph has symmetry with respect to the origin if (x, y) is on the graph then so is (-x, -y).

A graph has symmetry with respect to the x-axis if whenever (x, y) is on the graph then so is (x, -y).

Even Function: A function whose graph is symmetric to the y-axis.

$$f(-x) = f(x)$$

Ex: $f(x) = x^2$

Ex: $f(x) = 3x^2 + 4x$

Ex: $f(x) = 4$

Ex: $f(x) = x^6 - x^3 + 6x^2$

Odd Function: A function whose graph is symmetric to the origin.

$$f(-x) = -f(x)$$

Ex: $f(x) = 3x^3 + 1$

Ex: $f(x) = \frac{2}{5}x$

Ex: $f(x) = 4x - 1$

Ex: $f(x) = x^3 - x$

Determine whether each function is even, odd, or neither.

a. $y = x^2 + x$

b. $y = x^5 + x$

c. $y = x\sqrt{1-x^2}$

Day 3

Toolkit Functions:

1. $y = c$	2. $y = \sqrt{x}$	3. $y = a^x$
4. $y = x$	5. $y = x^3$	6. $y = \log_a x$
7. $y = x^2$	8. $y = 1/x$	9. $y = \sin x$
10. $y = \|x\|$	11. $y = [x]$	13. $y = \cos x$

Describe Domain & Range

Remember $y = a(b(x - h)^2 + k$

$\|a\| < 1$	$h > 0$
$\|a\| > 1$	$h < 0$
$\|b\| > 1$	$k > 0$
$\|b\| < 1$	$k < 0$

Take a look at (Describe the transformations)

$y = \sqrt{x}$
$y = \sqrt{x+3}$
$y = \sqrt{x} - 5$
$y = 3\sqrt{x-4} + 2$

$y = x^3$
$y = (x - 2)^3$
$y = x^3 + 4$
$y = \frac{1}{2} x^3 - 6$

$y = |x|$
$y = |x + 5|$
$y = |x| - 3$
$y = 4|x| - 5$

$y = \sin x$
$y = \sin (x + 2)$
$y = \sin x - 4$
$y = 3\sin(x - 4) + 5$

Describe transformations

1: $y = (x - 2)^2 + 4$

2: $y = -|x + 6| - 2$

3: $y = -5\sqrt{2x+3}$

4: $y = \frac{1}{2}\sin(4x - 2) + 1$

Day 4 – 5

Combination of Functions:

Sum $(f + g)(x) = f(x) + g(x)$
Difference $(f - g)(x) = f(x) - g(x)$

Product $(fg)(x) = f(x) \cdot g(x)$

Quotient $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ $g(x) \neq 0$

Given: $f(x) = 3x - 1$ $g(x) = x^2 + 2x - 24$

Find: $f(x) + g(x) =$ $f(x) - g(x) =$

$f(x) \cdot g(x) =$ $\frac{f(x)}{g(x)} =$

The domain of an arithmetic combination of two functions consists of all real numbers that are common to both functions.

$\boldsymbol{f(x) = 3x - 1}$ $\boldsymbol{g(x) = x^2 + 2x - 24}$

Find: 1. $(f + g)(2) =$ 2. $(f - g)(-3) =$

3. $f(4) \cdot g(4) =$ 4. $\frac{f}{g}(-2) =$

What is the domain:?

5. $f(x) = \sqrt{x}$ 6. $g(x) = \sqrt{4 - x^2}$

Composition of Functions: The composition of the functions f with g is denoted by $f \circ g$ and is defined by the equation $\boldsymbol{(f \circ g)(x) = f(g(x))}$

$\boldsymbol{f(x) = x^2 - 4}$ $\boldsymbol{g(x) = 2x - 5}$

1. $(f \circ g)(4) =$ 2. $(g \circ f)(-2) =$

3. $(g(f(-3)) =$ 4. $(f(g(3)) =$

The domain of the composite function $f \circ g$ is the set of all x such that

1. x is in the domain of g and
2. $g(x)$ is in the domain of f.

The following values must be excluded from the input of x:

1. If x is not in the domain of g, it must not be in the domain of $f \circ g$
2. Any x for which $g(x)$ is not in the domain of f must not be in the domain of $f \circ g$

$f(x) = \sqrt{x}$ $g(x) = 2x^2 + 3$

1. $(f + g)(x)$ 2. $(g + f)(x)$ 3. $\frac{f(x)}{g(x)}$

4. $f(x) \bullet g(x)$ 5. $(f \circ g)(x)$ 6. $(g \circ f)(x)$

1. Find each of the following for $f(x) = \frac{2}{x-1}$ and $g(x) = \frac{3}{x}$.

   Find the domain of each.

   a. $(f \circ g)(x)$ b. $(g \circ f)(x)$

2. For: $f(x) = x^2 + 3$ $g(x) = \sqrt{x}$

   Domain of $(f \circ g)$ = Domain of $(g \circ f)$ =

3. For: $f(x) = x^2 – 9$ $g(x) = \sqrt{9 - x^2}$

   Domain of $(f \circ g)(x)$ = Domain of $(g \circ f)(x)$ =

4. For: $f(x) = 2x + 3$ $g(x) = \frac{1}{2}(x – 3)$

   What is $(f \circ g)(x)$? What is $(g \circ f)(x)$? What do we notice?

When you form a composite function, you "compose" two functions to form a new function. It is possible to reverse this process. You can "decompose" a given function and express it as a composition of two or more functions. Although there is more than one way to do this, there is often a "natural" selection that comes to mind. Consider $h(x) = (3x^2 - 4x + 1)^5$.

Express the given functions h as a composition of two functions f and g so that $h(x) = (f \circ g)(x)$

a. $h(x) = \sqrt[3]{x^2 + 1}$ b. $h(x) = |3x - 4|$ c. $h(x) = \frac{1}{2x-3}$

Inverses:

Let f and g be two functions such that $f(g(x)) = x$ for every x in the domain of g and $g(f(x)) = x$ for every x in the domain of f.
The function g is the inverse of the function f, and is denoted by f^{-1} (read "f-inverse"). Thus, $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The domain of f is equal to the range of f^{-1}, and vice versa.

Find the inverse of f informally. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

a. $f(x) = 5x$ b. $f(x) = x^2$ c. $f(x) = x + 2$

The graph of an inverse is the reflection of the original function over the line $y = x$.

To have an inverse function, a function must be one-to-one, which means no two elements in the domain correspond to the same element in the range of f. You can use the horizontal line test to determine if a function is one-to-one.

Algebraically find the inverse of each function. Then graph the function and the inverse.

a. $f(x) = x + 3$ b. $f(x) = \frac{1}{x}$ c. $f(x) = (x + 3)^2$ d. $f(x) = \sqrt{x + 2}$

Day 6

1. Minimum and maximum values are often referred to as _____________ values. To approximate extreme values for a function.
    a. Sketch and label a diagram.
    b. Write a rule(equation) for the quantity to be minimized or maximized in terms of a single variable.
    c. Determine the domain for the equation.
    d. With a graphing calculator, graph the equation and use the function on the calculator to approximate the desired minimum or maximum value.
    e. Re-read the question and be sure to give the answer for the question that was asked.
2. Express the area A of a circle as a function of its circumference C, express C as a function of A.

3. P(x,y) is an arbitrary point on the line $x - 3y = 5$.
    a. Express the distance d from the origin to P as a function of the y-coordinate of P.

    b. Without graphing, find the minimum distance d and the point P associated with the minimum d.

    c. What are the domain and range of the distance function?

4. A power station and a factory are on the opposite sides of a river 60 m wide. A cable must be run from the power station to the factory. It costs $25 per meter to run the cable in the river and $20 per meter on land. Use a graphing calculator to find the minimum cost.