Precalculus Notes Unit 1

Day 1

**Rules For Domain:** When the domain is not specified, it consists of (all real numbers) for which the corresponding values in the range are also real numbers.

1. If x is in the numerator and raised to a positive integral.

Ex. *f* (x) = x2 or *f* (x) =  Domain: **All reals **

2. If x is in the denominator, x **cannot** be any value that makes the **denominator zero.**

Ex.  Ex: *f* (x) = 

3. If x is inside a square root, values of x are restricted to ones that will make the

 **radicand  0.**

Ex. *f* (x) =  Ex: *f* (x) = 

4. If x is in the square root and in the denominator, values of x are restricted to the one

 that will make the **radicand > 0.**

Ex. *f* (x) =  Ex: *f*(x) = 

**Range of a Relation** – set of y values of a relation

**Function**: special type of relation in which each element of the domain is paired with

 **exactly one** element of the range.

**Testing For Functions Algebraically**– Solve for y in terms of x. If each value of x corresponds to exactly one value of y, then y is a function of x.

**Vertical Line Test**: tests a graph to see if it is a function.

**Horizontal Line Test**: tests a graph to see if the function’s inverse is also a function.

Ex: Ex: Ex:   

Function Notation: If the graph is a function we can use *f*(x) instead of y

Find  if .

Find  if .

**Piecewise Functions:** A functions that is defined by two (or more) equations over a

 specified domain.

Ex: *f*(x) =  Ex: *h*(x) = 

Find: *f*(-3) = *f*(6) = Find: *h*(0) = *h*(-6) = *h*(-3)=

The difference quotient, $\frac{f\left(x+h\right)-f(x)}{h}, h\ne 0$, plays an important role in

understanding the rate at which a function changes.

Using the difference quotient, find and simplify

1. *f(x)* = x2 – 4x + 3$f\left(x\right)= x^{2}-4x+3$ 2. $f\left(x\right)=\sqrt[2]{3-x}$

$ $

Day 2 *f*(x) = 

**Graph function by hand:**

 

Graph on calculator y1 = (x2 + 1) (x < 0) + (x – 1) (x ≥ 0)

 Ex: *f*(x) =  Ex: *f*(x) = 

  

Determine the intervals on which the function is:

Increasing: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Decreasing: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Constant: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Find the relative maximum: \_\_\_\_\_\_\_\_\_

 

Using a calc: Find the relative max/ and or min of *f*(x) = 2x3 + 3x2 – 12x + 1

Determine the intervals on which each is increasing, decreasing, or constant.

 a. $f\left(x\right)=x^{2}+4x-12$ b. $g\left(x\right)=x^{3}+1$

c.$ h\left(x\right)=x^{3}+x^{2}-x+2$ d. $p\left(x\right)=2$

**Even and Odd Functions:**

A graph has symmetry with respect to the y-axis if whenever (x, y) is on the graph then so is (-x, y).

A graph has symmetry with respect to the origin if (x, y) is on the graph then so is (-x, -y).

A graph has symmetry with respect to the x-axis if whenever (x, y) is on the graph then so is (x, -y).

**Even Function:** A function whose graph is symmetric to the y-axis.

 *f(-x)* = *f(x)*

 Ex: $f\left(x\right)=x^{2}$ Ex: $f\left(x\right)=3x^{2}+4x$

 Ex: $f\left(x\right)=4$ Ex: $f\left(x\right)=x^{6}-x^{3}+6x^{2}$

**Odd Function:** A function whose graph is symmetric to the origin.

 *f(-x)* = -*f(x)*

 Ex: $f\left(x\right)=3x^{3}+1$ Ex: $f\left(x\right)=\frac{2}{5}x^{}$

 Ex: $f\left(x\right)=4x-1$ Ex: $f\left(x\right)=x^{3}-x$

Determine whether each function is even, odd, or neither.

1. y = x2 + x b. y = x5 + x c. y = 

Day 3

**Toolkit Functions:**

1. y = c 2. y =  3. y = ax
2. y = x 5. y = x3 6. y = logax

7. y = x2 8. y= 1/x 9. y = sin x

10. y = |x| 11. y = [x] 13. y = cos x

Describe Domain & Range

Remember y = a(b(x – h )2 + k

 |a| < 1 h > 0

 |a| > 1 h < 0

 |b| > 1 k > 0

 |b| < 1 k < 0

Take a look at (Describe the transformations)

 y =  y = x3

y =  y = (x – 2)3

 y =  – 5 y = x3 + 4

 y = 3 + 2 y = ½ x3 – 6

 y = |x| y = sin x

 y = |x + 5| y = sin (x + 2)

 y = |x| – 3 y = sin x – 4

 y = 4|x| – 5 y = 3sin(x – 4) + 5

Describe transformations

 1: y = (x – 2)2 + 4 2: y = -|x + 6| – 2

 3: y = -5 4: y = sin(4x – 2) + 1

Day 4 – 5

Sum (*f* + *g*)(x) = *f*(x) + *g*(x)

Difference (*f* – *g*) (x) = *f*(x) – *g*(x)

Product (*fg*)(x) = *f*(x) • *g*(x)

Quotient () (x) =  *g*(x) ≠ 0

**Combination of Functions:**

**Given:** *f*(x) = 3x – 1 *g*(x) = x2 + 2x – 24

Find: *f*(x) + *g*(x) = *f*(x) – *g*(x) =

*f*(x) • *g*(x) = =

The domain of an arithmetic combination of two functions consists of all real numbers that are common to both functions.

 ***f*(x) = 3x – 1 *g*(x) = x2 + 2x – 24**

 Find: 1. (*f* + *g*) (2) = 2. (*f* – *g*) (-3) =

 3. *f*(4) • *g*(4) = 4. (-2) =

What is the domain:?

 5. *f*(x) =  6. *g*(x) = 

**Composition of Functions:** The composition of the functions f with g is denoted

 by $f∘g$ and is defined by the equation **(*f*  *g*)( x ) = *f*(*g*(x))**

 ***f*(x) = x2 – 4 *g*(x) = 2x – 5**

 1. (*f*  *g*) (4) = 2. (*g* *f*) (-2) =

3. (*g* (*f*(-3) )= 4. (*f* (*g*(3) )=

**The domain of the composite function** $f∘g$ **is the set of all x such that**

1. x is in the domain of g and

2. $g(x)$ is in the domain of f.

**The following values must be excluded from the input of x**:
 1. If x is not in the domain of g, it must not be in the domain of $f∘g$

2. Any x for which $g(x)$ is not in the domain of f must not be in the

 domain of $f∘g$

 *f*(x) =  *g*(x) = 2x2 + 3

1. *(f + g)(x)*$\left(f+g\right)(x)$ 2. *(g + f)(x)*$\left(f+g\right)(x)$ 3. $\left(\frac{f}{g}\right)(x)$

4. *f(x) • g(x)*$\left(f∙g\right)(x)$ 5. *(f  g)( x )*  6. *(g  f)( x )*

1. Find each of the following for *f*(x) = $f\left(x\right)=\frac{2}{x-1}$ and *g*(x) = $g\left(x\right)=\frac{3}{x}$.

 Find the domain of each.

a. *(f  g) (x)*  b. *(g  f) (x)*

2. For: *f*(x) = x2 + 3 *g*(x) = 

 Domain of (*f*  *g*) = Domain of (*g*  *f*) =

3. For : *f*(x) = x2 – 9 *g*(x) = 

 Domain of (*f*  *g*)(x) = Domain of (*g*  *f*)(x) =

4. For: *f*(x) = 2x + 3 *g*(x) = ½ (x – 3)

 What is (*f*  *g*) (x)? What is (*g*  *f*)(x)? What do we notice?

When you form a composite function, you “compose” two functions to form a new function. It is possible to reverse this process. You can “decompose” a given function and express it as a composition of two or more functions. Although there is more than one way to do this, there is often a “natural” selection that comes to mind. Consider *h(x)* = (3x2 – 4x + 1)5 $h\left(x\right)=(3x^{2}-4x+1)^{5}$.

Express the given functions h as a composition of two functions *f* and *g* so that *h(x)* = *(f  g)( x )* $h\left(x\right)=\left(f∘g\right)\left(x\right).$

a. *h(x)* = $h\left(x\right)=\sqrt[3]{x^{2}+1}$ b. *h(x)* = |3x – 4 |$h\left(x\right)=|3x-4|$ c. *h(x)* = $h\left(x\right)=\frac{1}{2x-3}$

**Inverses:**

Let *f* and *g* be two functions such that $f\left(g\left(x\right)\right)=x$ for every x in the domain of *g* and $g\left(f\left(x\right)\right)=x$ for every x in the domain of f.

The function g is the inverse of the function f, and is denoted by $f^{-1}$ (read “f-inverse”). Thus, $f\left(f^{-1}\left(x\right)\right)=x$ and $f^{-1}\left(f\left(x\right)\right)=x.$ The domain of *f* is equal to the range of $f^{-1},$ and vice versa.

 Find the inverse of f informally. Verify that $f\left(f^{-1}\left(x\right)\right)=x$ and $f^{-1}\left(f\left(x\right)\right)=x$.

 a. $f\left(x\right)=5x$ b. $f\left(x\right)=x^{2}$ c. $f\left(x\right)=x+2$

**The graph of an inverse is the reflection of the original function over the line** $y=x.$

To have an inverse function, a function must be one-to-one, which means no two elements in the domain correspond to the same element in the range of f. You can use the horizontal line test to determine if a function is one-to-one.

Algebraically find the inverse of each function. Then graph the function and the inverse.

a. $f\left(x\right)=x+3$ b. $f\left(x\right)=\frac{1}{x}$ c. $f\left(x\right)=(x+3)^{2}$ d. $f\left(x\right)=\sqrt{x+2}$

 Day 6

1. Minimum and maximum values are often referred to as \_\_\_\_\_\_\_\_\_\_\_\_\_ values. To approximate extreme values for a function.
	1. Sketch and label a diagram.
	2. Write a rule(equation) for the quantity to be minimized or maximized in terms of a single variable.
	3. Determine the domain for the equation.
	4. With a graphing calculator, graph the equation and use the function on the calculator to approximate the desired minimum or maximum value.
	5. Re-read the question and be sure to give the answer for the question that was asked.
2. Express the area A of a circle as a function of its circumference C, express C as a function of A.
3. P(x,y) is an arbitrary point on the line $x-3y=5.$
	1. Express the distance d from the origin to P as a function of the y-coordinate of P.
	2. Without graphing, find the minimum distance d and the point P associated with the minimum d.
	3. What are the domain and range of the distance function?
4. A power station and a factory are on the opposite sides of a river 60 m wide. A cable must be run from the power station to the factory. It costs $25 per meter to run the cable in the river and $20 per meter on land. Use a graphing calculator to find the minimum cost.



200 m

Factory

Power Station