Practice on the Log Function ICMAB 2016

1. Given $f(x)=\frac{1}{x}+\ln x$, defined only on the closed interval $\frac{1}{e}\le x<e$.

   a. Showing your reasoning, determine the value of x at which f has its

   (i) absolute maximum

   (ii) absolute minimum

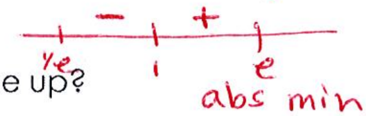
$f(x) = x^{-1} + \ln x$

$f'(x) = -x^{-2} + \frac{1}{x} = \frac{x-1}{x^2}$

$f(1/e) = e-1$

$f(1) = 1$

$f(e) = 1/e + 1$



abs min at $x = 1$

max at $x = 1/e$

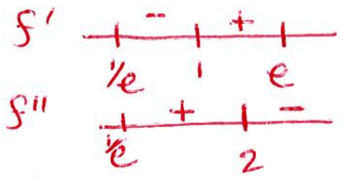
   b. For what values of x is the curve concave up?

$f''(x) = 2x^{-3} - x^{-2}$

$= 2/x^3 - \frac{1}{x^2} = \frac{2-x}{x^3}$

ccu $(1/e, 2)$

   c. On the coordinate axis provided, sketch the graph of f over the interval $\frac{1}{e}\le x<e$.



2. Let f(x) = $(1 - x)^2$ for all real numbers x, and let g(x) = ln(x) for all x > 0. Let h(x) = $(1 - \ln(x))^2$.

$f(g(x))$

   a. Determine whether h(x) is the composition f(g(x)) or the composition g(f(x)).

   b. Find h' (x). Where is the tangent line horizontal? Does h have a relative max or min there?

$h'(x) = 2(1-\ln x)' \cdot \frac{-1}{x}$

$= \frac{-2}{x}(1-\ln x)$

$x = e$

minimum

   c. Find h"(x). Where is h concave up?

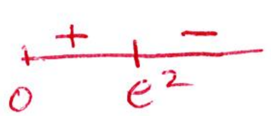
$\frac{-2}{x} \cdot \frac{-1}{x} + +2x^{-2}(1-\ln x)$

$\frac{2+2(1-\ln x)}{x^2}$

$4 - 2\ln x = 0$

$\ln x = +2$

$x = e^2$



ccu $(0, e^2)$

3. Let f be the function defined by $f(x) = -2 + \ln(x^2)$.

a. For what real numbers x is f defined? $(0, \infty)$

b. Find the zeros of f. $\ln(x^2) = 2$

$e^2 = x^2$ $\quad x = e$

c. Write an equation for the line tangent to the graph of f at x = 1.

$(1, -2)$

$$f'(x) = \frac{2x}{x^2} = \frac{2}{x}$$

$$y + 2 = 2(x - 1)$$

4. Let f be the function given by $f(x) = 2\ln(x^2 + 3) - x$ with domain $-3 \le x \le 5$.

a. Find the x-coordinate of each relative maximum point and each relative minimum point of f. Justify your answer.

$x = 1$ min $\quad x = 3$ max

$$f'(x) = 2\left(\frac{2x}{x^2+3}\right) - 1$$

$$= \frac{4x}{x^2+3} - \frac{x^2+3}{x^2+3} =$$

$$\frac{-x^2 + 4x - 3}{x^2+3}$$

$$\frac{-(x-3)(x-1)}{x^2+3}$$

$$\begin{array}{ccccccc} & - & & + & & - & \\ -3 & & 1 & & 3 & & 5 \end{array}$$

b. Find the x-coordinate of each inflection point of f.

$$f'' = \frac{(x^2+3)(-2x+4) - (-x^2+4x-3)(2x)}{(x^2+3)^2}$$

$x = \pm\sqrt{3}$

$$\begin{array}{ccccc} - & & + & & - \\ & -\sqrt{3} & & \sqrt{3} & \end{array}$$

c. Find the absolute maximum value of f(x).

$f(-3) = 2\ln 12 + 3$

$f(3) = 2\ln 12 - 3$

$f(5) =$ not a max (decreasing).

$\therefore$ Abs max $= 2\ln 12 + 3$.