Differentation & Integration – Bases other than e

$2^{\log_2 5} = 5$

Definition of Exponential Function to Base a: If a is a positive real number ($a \neq 1$) and x is any number, then the **exponential function to the base a** is denoted by a^x and is defined as $a^x = e^{(\ln a)x}$, if a = 1, then y = 1 (the constant function).

Definition of Logarithmic Function to Base a: If a is a positive real number ($a \neq 1$) and x is any number, then the **logarithmic function to the base a** is denoted by $\log_a x$ and is defined as $\log_a x$ $\log_a x = \dfrac{\ln x}{\ln a}$.

Derivative for Bases Other than e.

$$\frac{d}{dx}[a^x] = (\ln a)a^x \qquad \frac{d}{dx}[a^u] = (\ln a)a^u \frac{du}{dx}$$

$$\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x} \qquad \frac{d}{dx}[\log_a u] = \frac{1}{(\ln a)u}\frac{du}{dx}$$

Ex: $f(x) = 4^x$

$\ln 4 \cdot 4^x$

Ex: $f(x) = 4^{2x-3}$

$2\ln 4 \cdot 4^{2x-3}$

$\ln 16 \cdot 4^{2x-3}$

Ex: $f(x) = 4^{x^3}$

$3x^2 \ln 4 \cdot 4^{x^3}$

Ex: $f(x) = \log(\cos x) = \dfrac{\ln(\cos x)}{\ln 10}$

$\dfrac{1}{\ln 10} \cdot \dfrac{-\sin x}{\cos x} = \dfrac{-\tan x}{\ln 10}$

Ex: $f(x) = \log_3 \dfrac{\sqrt{x}}{x+3}$

$\frac{1}{2}\log_3 x - \log_3(x+3)$

$\dfrac{1 \cdot \ln(x)}{2\ln 3} - \dfrac{\ln(x+3)}{\ln 3} = \dfrac{1}{2x\ln 3} - \dfrac{1}{(x+3)\ln 3}$

Ex: $f(x) = 3^{-x}\tan(x^2)$

$3^{-x}\sec^2(x^2)\cdot 2x - \ln 3 \cdot 3^{-x}\tan(x^2)$

Ex: $f(x) = \log_a a^{\sin x}$

$f(x) = \sin x$

$f'(x) = \cos x$

$$\int a^x\,dx = \left(\frac{1}{\ln a}\right)a^x + C \qquad \int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$$

Ex: $\int 5^{-x+3}\,dx$

$\dfrac{-1}{\ln 5} \cdot 5^{-x+3} + C$

$u = -x+3$

$du = -1\,dx$

Ex: $\int 3^{2x+5}\,dx$

$\dfrac{1}{2\ln 3} \cdot 3^{2x+5} + C$

$u = 2x+5$

$du = 2\,dx$

Ex: $\int \dfrac{2^{3x}}{4+2^{3x}}\,dx$

$u = 4 + 2^{3x}$

$du = 2^{3x} \cdot \ln 2 \cdot 3$

$\dfrac{1}{3\ln 2}\int \dfrac{1}{u}\,du$

$\dfrac{1}{3\ln 2} \cdot \ln(4+2^{3x}) + C$

$\log_a a^{\sin x} = x$

$a^x = a^{\sin x}$