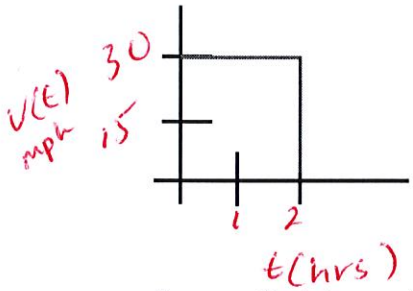
NOTES

If we have a constant velocity then we can determine the distance travelled.

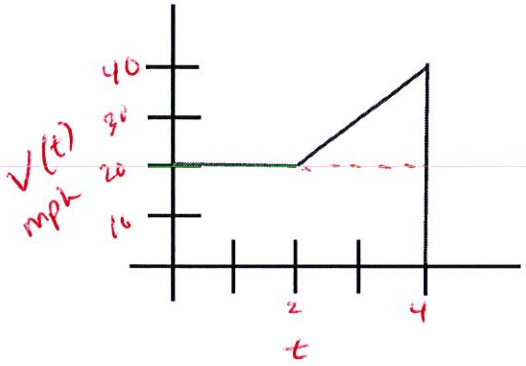
Ex: Travel 30 mph (one direction) for 2 hours = travel 60 miles

On the graph it looks like:



In this case, distance travelled = area under the velocity curve (because $v > 0$)

Can we find the total distance travelled?

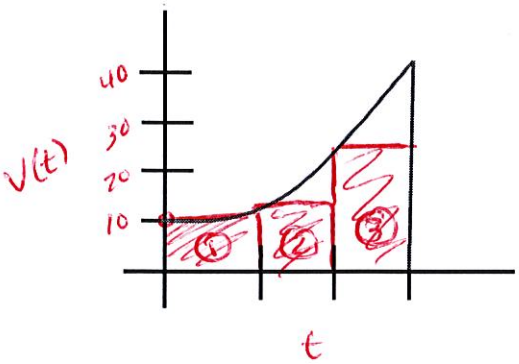


$4(20) + \frac{1}{2}(2)(20) = 100$ miles

But suppose the velocity curve doesn't have a constant rate of change? (not linear)

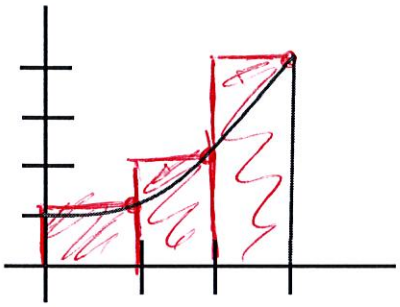
Now we don't know formulas for area? What to do?

Estimate! What are our options?



(LRAM) Left Rectangular Approx Method (start w/ left point on the curve)

$1(10) + 1(12) + 1(25) \approx 47$ miles.



(RRAM) Right Rectangular Approx Method)

$1(12) + 1(22) + 1(40) \approx 74$ miles.

(MRAM)
Middle Rectangular Approx Method.
(use middle point on the curve)

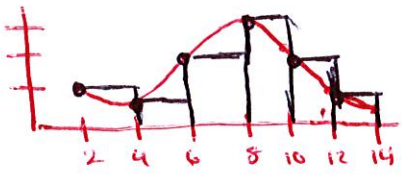
$1(11) + 1(18) + 1(32) \approx 61$ miles

Ex 2: Coffee is poured into a big cup at a rate of $R(t)$ measured in in^3/sec.

t	2	4	6	8	10	12	14
$R(t)$	1	.5	2	3	2.5	1	.5

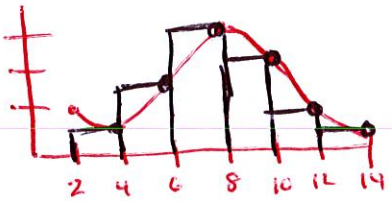
How much coffee pours into cup over 14 seconds of time?

A. Using LRAM n = 6



$2(1) + 2(1/2) + 2(2) + 2(3) + 2(2.5) + 2(1) \approx 20$

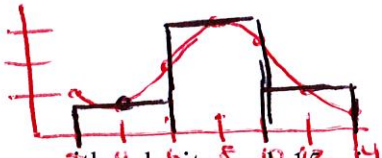
B. Using RRAM n = 6



$2(1/2) + 2(2) + 2(3) + 2(2.5) + 2(1) + 2(1/2) \approx 19$

C. Using MRAM n = 3

↑ 3 divisions



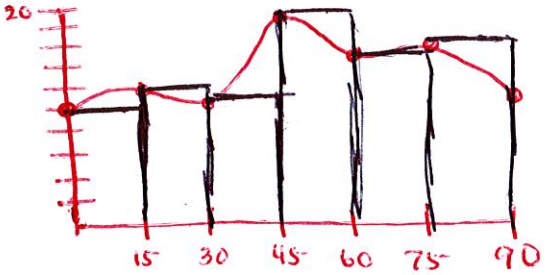
$4(1/2) + 4(3) + 4(1) \approx 18$

Ex 3: Ship is travelling on the ocean with velocity each 15 minutes at a rate of $v(t)$.

t	0	15	30	45	60	75	90
$v(t)$	10	12	11	20	16	17	14

How much coffee pours into cup over 14 seconds of time?

A. Using LRAM n = 6



$15(10) + 15(12) + 15(11) + 15(20) + 15(16) + 15(17) = 1290$

B. Using RRAM n = 6

$15(12) + 15(11) + 15(20) + 15(16) + 15(17) + 15(14) = 1350$

C. Using MRAM n = 3

$30(12) + 30(20) + 30(17) = 1470$

15 45 75

↑ middle value

These are called Riemann Sums:

Ex 4: $y = x^2 + 3$; $[0, 5]$, n = 5, RRAM, LRAM, MRAM Use Calc. table feature and sketch.

0	1	2	3	4	5
3	4	7	12	19	28

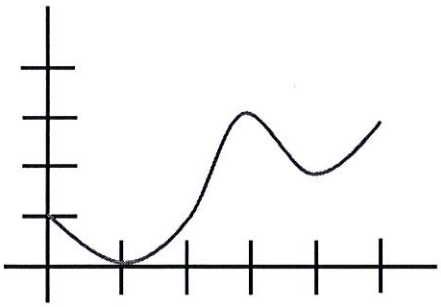
LRAM $1(3) + 1(4) + 1(7) + 1(12) + 1(19) \approx 45$

RRAM $1(4) + 1(7) + 1(12) + 1(19) + 1(28) \approx 70$

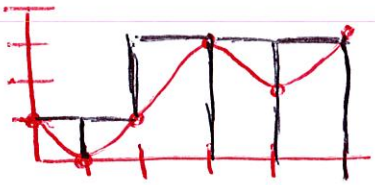
MRAM [Need $(1/2, 3.25), (1.5, 5.25), (2.5, 9.25), (3.5, 15.25), (4.5, 23.25)$]

Upper Sums – Circumscribed Rectangles $1(3.25) + 1(5.25) + 1(9.25) + 1(15.25) + 1(23.25)$

Lower Suns – Inscribed Rectangles ≈ 56.25

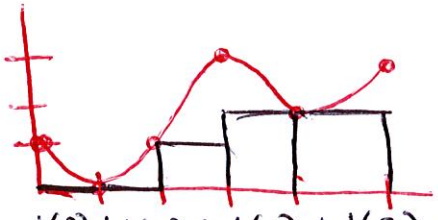


Upper Sums:



$1(1) + 1(1) + 1(3) + 1(3) + 1(3) = 11$

Lower Sums:



$1(0) + 1(0) + 1(1) + 1(2) + 1(2) = 5$

Summations:

Ex: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ Ex: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{2}$ Ex: $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

HW: pg. 261 #1,5,7,11,13,15,19,23,25 – 31 odd, 32 – 34 all, 35 – 43 odd.