Riemann Sums = let # of rectangles be n = 10, 100, 1,000 – exact area on a closed interval.

Ex: Find the area under $y = x^2$; $[0, 2]$, n = 10, n = 100, n = 1,000, exact area

1. Find the width: $2/n$

2. x's $x_0 = 0,\ x_1 = 2/n,\ x_2 = 2\left(\frac{2}{n}\right)\ x_3 = 3\cdot\left(\frac{2}{n}\right)\quad x_k = k\left(\frac{2}{n}\right)$

$x_n = n\left(\frac{2}{n}\right) = 2$

3. $A \sim \sum_{k=1}^{n} \frac{\textit{Area of}}{k+h\ rect} = \sum_{k=1}^{n} \textit{width}\ \frac{\textit{height of}}{k+h\ rect}$

function

$$\sum_{k=1}^{n} \frac{2}{n}\left(f(x_k)\right) = \sum_{k=1}^{n} \frac{2}{n}\left(\frac{2k}{n}\right)^2 = \sum_{k=1}^{n} \frac{8k^2}{n^3}$$

$$\frac{8}{n^3}\sum_{k=1}^{n} k^2 = \frac{8}{n^3}\cdot\frac{\overset{10}{n}(\overset{10}{n+1})(\overset{21}{2n+1})}{6}$$

$n = 10 \quad \approx 3.08$

$n = 100 \quad \approx$

$$\lim_{n\to\infty} \frac{8n(n+1)(2n+1)}{6n^3} = \frac{16}{6} = \frac{8}{3}$$

Ex: Find the area under $y = 2x^2 + 1$; $[0, 4]$, n = 10, n = 100, n = 1,000, exact area

1. Find the width: $\frac{4}{n}$,

$x_0 = 0 \quad x_1 = 4/n \quad x_2 = 2\left(\frac{4}{n}\right) \ldots x_k = k\left(\frac{4}{n}\right)$

$x_n = n\left(\frac{4}{n}\right) = 4$

$$\sum_{k=1}^{n} \frac{4}{n} f(x_k) = \sum_{k=1}^{n} \frac{4}{n} f\left(\frac{4k}{n}\right) = \sum_{k=1}^{n} \frac{4}{n}\left(2\left(\frac{4k}{n}\right)^2 + 1\right)$$

$$\sum \frac{4}{n}\left(\frac{32k^2}{n^2} + 1\right) = \sum \frac{128k^2}{n^3} + \sum \frac{4}{n}$$

$$\frac{128}{n^3}\sum_{k=1}^{n} k^2 + \frac{4}{n}\sum_{k=1}^{n} 1 = \frac{128}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{4}{n}\cdot n$$

$n = 10$

$n = 100$

$n = 1000$

$$\lim_{n\to\infty} = \left(\frac{128}{3} + 4\right)$$

Ex: Find the area under $y = 3x^2 + x$; $[1, 3]$, n = 10, n = 100, n = 1,000, exact area

1. Find the width: $\frac{2}{n}$

$x_0 = 0 \quad x_0 = 1 \quad x_1 = 1 + 2/n \quad x_k = 1 + 2k/n \Rightarrow \sum_{k=1}^{n} \frac{2}{n} f\left(1 + \frac{2k}{n}\right)$

$$\sum_{k=1}^{n} \frac{2}{n}\left[3\left(1+\frac{2k}{n}\right)^2 + 1 + \frac{2k}{n}\right] = \sum_{k=1}^{n} \frac{6}{n} + \frac{24k}{n^2} + \frac{24k^2}{n^3} + \frac{2}{n} + \frac{4k}{n^2}$$

$$\sum_{k=1}^{n}\left[\frac{8}{n} + \frac{28k}{n^2} + \frac{24k^2}{n^3}\right] = \frac{8}{n}\sum_{k=1}^{n} 1 + \frac{28}{n^2}\sum_{k=1}^{n} k + \frac{24}{n^3}\sum_{k=1}^{n} k^2$$

$$\frac{8}{n}\cdot n + \frac{28}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{24}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$n = 10 \quad \approx 32.64$

$n = 100 \quad \approx 30.2604$

$n = 1000$

$\lim_{n \to \infty} = 8 + 14 + 8 = 30$

Ex: Find the area under $y = 2x^3 + 5x$; $[0, 3]$, n = 10, n = 100, n = 1,000, exact area

1. Find the width: $\frac{3}{n}$

$x_0 = 0 \quad x_0 = 1 \quad x_1 = 3/n \quad x_k = 3k/n \quad \sum_{k=1}^{n} \frac{3}{n} f\left(\frac{3k}{n}\right)$

$$\sum_{k=1}^{n} \frac{3}{n}\left[2\left(\frac{3k}{n}\right)^3 + 5\left(\frac{3k}{n}\right)\right] = \sum_{k=1}^{n} \frac{162k^3}{n^4} + \sum_{k=1}^{n} \frac{45k}{n^2}$$

$$\frac{162}{n^4}\sum_{k=1}^{n} k^3 + \frac{45}{n^2}\sum_{k=1}^{n} k \Rightarrow \frac{162}{n^4}\left(\frac{n^2(n+1)^2}{4}\right) + \frac{45}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$n = 10 \quad \approx 73.755$

$n = 100$

$n = 1000$

$\lim_{n \to \infty} = \frac{162}{4} + \frac{45}{2} = 63$

Pg 263 #47, 49, 52, 53 Also:

$y = 2x + 1 \quad [0, 4]$

$y = 3x^2 \quad [2, 4]$

$y = x + x^2 \quad [0, 2]$

$y = x^2 - 3x \quad [1, 3]$