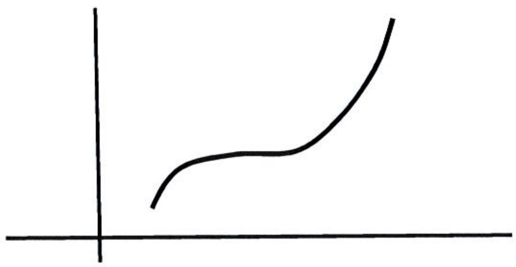
Let's think of this from a test score stand point 90 + 88 + 94 + 80

Mean Value Theorem for Integrals:

If a function f is continuous on the closed interval $[a,b]$ then there exists a number c in the closed interval $[a,b]$ such that $\int_a^b f(x)\,dx = f(c)(b-a)$



Definition of the Average Value of a Function on an Interval

If f is integrable on the closed interval $[a,b]$, then the **average value** of f on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Ex: Let's think about $f(x) = x^2$ on the interval $[0,4]$.

$\frac{1}{4}\int_0^4 x^2\,dx \quad \frac{1}{3}x^3$

$x^2 = \frac{64}{12} \quad x = \frac{8}{2\sqrt{3}} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

$\frac{64}{12} = 5\frac{1}{3} \quad c = \frac{4\sqrt{3}}{3}$

Ex: Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[1,4]$.

$\frac{1}{3}\int_1^4 3x^2 - 2x\,dx \quad 3x^2 - 2x - 16 = 0$

$x^3 - x^2 \Big|_1^4 \quad (3c-8)(c+2) = 0$

$\frac{1}{3}\cdot 48 = 16 \quad c = 8/3 \quad c = -2$

$c = 8/3 \quad \cancel{c = -2}$

Ex: Find the average value of $f(x) = \sin(x)$ on the interval $[0,\pi]$.

$\frac{1}{\pi}\int_0^\pi \sin x\,dx \quad \sin c = \frac{2}{\pi}$

$-\cos x \Big|_0^\pi \quad c = \arcsin\frac{2}{\pi}$

$\frac{1}{\pi}[1 - -1 =] = \frac{2}{\pi} \quad c = \arcsin\frac{2}{\pi}$

$c = \pi - \arcsin 2/\pi$

Ex: Find the average value of $f(x) = \sqrt{x}$ on the interval $[0,9]$.

$$\frac{1}{9}\int_0^9 x^{1/2}\,dx$$

$$\frac{2}{3}x^{3/2}\Big|_0^9$$

$$\frac{1}{9}\cdot 18 = 2$$

$\sqrt{x} = 2$

$x = 4$

$c = 4$

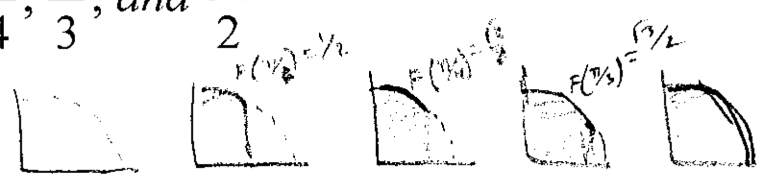
The Second Fundamental Theorem of Calculus:

If f is continuous on an open interval I containing a, then, for every x in the interval.

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$$

Ex: $F(x) = \int_0^x \sin(t)\,dt$ at $x = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, and $\frac{\pi}{2}$

$$-\cos(t)\Big|_0^x$$



$$F(x) = -\cos(x) - 1$$

$$F'(x) = \sin(x)$$

$F(x) = \int_0^x t^3\,dt$

$$F(x) = \frac{1}{4}t^4\Big|_0^x$$

$$= \frac{1}{4}x^4$$

$$F'(x) = x^3$$

Ex: $F(x) = \int_0^x \sqrt{t}\,dt$

$$F(x) = \frac{2}{3}t^{3/2}\Big|_0^x$$

$$F(x) = \frac{2}{3}x^{3/2}$$

$$F'(x) = x^{1/2} = \sqrt{x}$$

Ex: $F(x) = \int_{\pi/2}^{x^3} \cos(t)\,dt$

$$F(x) = \sin(x^3) + 1\Big|_{\pi/2}^{x^3}$$

$$f(x) = F'(x) = \cos(x^3)\cdot 3x^2$$

Ex: $F(x) = \int_{\sqrt{x}}^{\tan(x)} \ln(t)\,dt$

$$F'(x) = \sec^2 x\,\ln(\tan x) - \frac{\ln\sqrt{x}}{2\sqrt{x}}$$

Hmwk: pg. 284 – 285 # 1 -31 odd, 35, 37, 39, 45 – 51 odd, 54 – 61, 73 – 93 odd

Warm Up

Find the values of c that satisfy the Mean Value Theorem for Integrals.

$\int_a^b f(x) = f(c)\cdot(b-a)$

1. $f(x) = \int_{-3}^{1} -\frac{x^2}{2} + x + \frac{3}{2}$ $[-3, 1]$

$\frac{1}{4}\left[-\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x\right]\Big|_{-3}^{1} = \frac{-2}{3}$

$f(c) = -2/3$

$6\left[-\frac{c^2}{2} + c + \frac{3}{2} = -2/3\right]$

$-3c^2 + 6c + 9 = -4$

$-3c^2 + 6c - 13 = 0$

$c = \frac{3-4\sqrt{3}}{3}$

Find the Average Value of the function over the given interval. Then, find the values of c that satisfy the Mean Value Theorem for Integrals.

2. $f(x) = -x + 2$ $[-2, 2]$

$\frac{1}{4}\int_{-2}^{2} -x + 2$

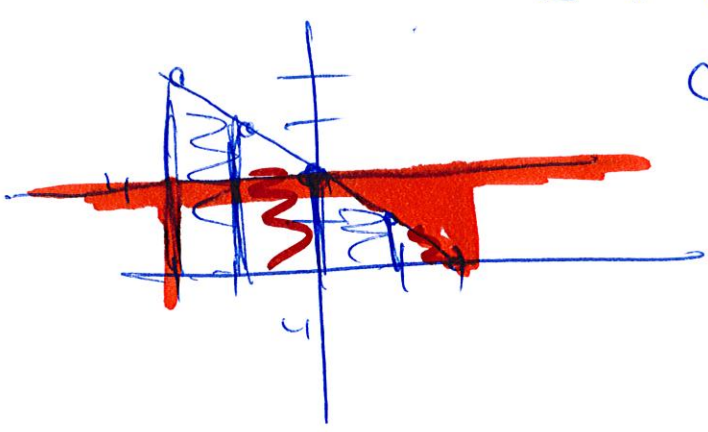
$\frac{1}{4}\left[-\frac{1}{2}x^2 + 2x \Big|_{-2}^{2}\right] = 2$

$f(c) = 2$

$\frac{1}{2}\cdot 16 = 8$

$-\cancel{c} + 2 = 2$

$c = 0$



Integration by Substitution:

1. $\int \sqrt{3x+2}\,dx =$

$\int 3\sqrt{3x+2}\;dx$

Let $u = 3x+2$ $\quad du = 3\,dx$

$\int 3\sqrt{u}\;du$ $\quad \frac{1}{3}du = dx$

$\frac{1}{3}\int u^{1/2}\,du$

$\frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C \Rightarrow \frac{2}{9}(3x+2)^{3/2} + C$

2. $\int x(x^2+1)^2\,dx =$ $\quad \frac{du}{dx} = 2x$

Let $u = x^2+1$ $\quad du = 2x\,dx$

$\frac{1}{2}\int u^2\,du$

$\frac{1}{3}u^3 + C$

$F(x) = \frac{1}{6}(x^2+1)^3 + C$

3. $\int \cancel{16x^3}\sec^2(4x^4-2)\cancel{dx} =$

$u = 4x^4 - 2$

$du = 16x^3\,dx$

$\int \sec^2(u)\,du$

$\tan(4x^4-2) + C$

4. $\int (5x^4+5)^{\frac{2}{3}} \bullet 20x^3\,dx =$

$u = 5x^4 + 5$

$du = 20x^3\,dx$

$\int u^{2/3}\,du$

$3/5\,u^{5/3} + C$

$3/5\,(5x^4+5)^{5/3} + C$

5. $\int x\sqrt{3x^2+4}\,dx =$

$u = 3x^2+4 \quad du = 6x\,dx$

$\frac{1}{6}\int u^{1/2}$

$\frac{1}{6}\cdot 2/3\,u^{3/2} + C$

$\frac{1}{9}(3x^2+4)^{3/2} + C$

6. $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx =$

$u = \sqrt{x}$

$du = \frac{1}{2}x^{-1/2}dx$

$2\int \sin u$

$-2\cos(\sqrt{x}) + C$

7. $\int \frac{-8x^3}{(-2x^4+5)^5}\,dx =$

$u = -2x^4 + 5$

$du = -8x^3\,dx$

$\int u^{-5}\,du$

$\frac{-1}{4(-2x^4+5)^4} + C$

8. $\int 36x^3(3x^4+3)^5\,dx =$

$u = 3x^4 + 3$

$du = 12x^3\,dx$

$3\int u^5\,du$

$\frac{1}{2}(3x^4+3)^6 + C$