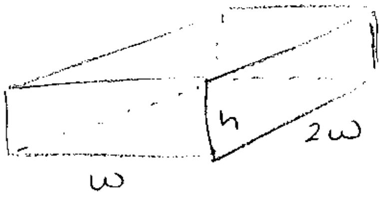
Unit 5 Review

1. What are the dimensions of the base of the rectangular box of greatest volume that can be constructed from 100 square inches of cardboard if the base is to be twice as long as it is wide? (Assume the box has a top)



$$S.A = 2(2w^2) + 2(2wh) + 2(w \cdot h)$$

$$100 = 4w^2 + 6wh$$

$$\frac{100-4w^2}{6w} = h$$

$$V = 2w^2 \cdot h \Rightarrow 2w^2\left(\frac{100-4w^2}{6w}\right)$$

$$V = \frac{w}{3}(100-4w^2) = \frac{100w}{3} - \frac{4w^3}{3}$$

$$\frac{dV}{dt} = \frac{100}{3} - 4w^2 \qquad 4w^2 - \frac{100}{3} = 0 \quad w^2 = \frac{100}{12}$$

$$w = \frac{10}{2\sqrt{3}} \text{ or } \frac{5}{\sqrt{3}} \quad l = \frac{10}{\sqrt{3}} \quad h = \frac{20\sqrt{3}}{9}$$

2. You sling a shovelful of dirt up from the bottom of a 17 foot deep hole with the velocity of 32 feet per second. There is a wind that blows all the dirt that reaches the top of the hole over to the side on the ground. Should you duck? (Give a mathematical reason)

$$h(t) = -16t^2 + 32t$$

$$h'(t) = -32t + 32 \qquad t = 1$$

$$h(1) = 16 \text{ ft.}$$

Duck The dirt will never reach the height of the top of the hole.

3. Rectilinear Motion: $x(t) = 1 - \cos(t) - \sin(t)$; $[0, 2\pi]$

A. Find where the particle is at rest. $\sin(t) = \cos(t) \quad t = \pi/4, 5\pi/4$

$\sin(t) - \cos(t) = 0$

$- \quad + \quad -$ on a number line marked $\pi/4$, $5\pi/4$

B. Find where the particle is moving left.

$(0, \pi/4) \cup (5\pi/4, 2\pi)$

C. Where is the particle speeding up?

$x'(t) = \; - \;|_{\pi/4}\; + \;|_{5\pi/4}\; -$

$\cos(t) + \sin(t) = 0$

$x''(t) \; {}_0\; + \;|\; + \;|_{3\pi/4}\; - \;|\; - \;|_{7\pi/4}\; + \; {}_{2\pi}$

$(\pi/4, 3\pi/4) \cup (5\pi/4, 7\pi/4)$

D. Find the Average Velocity over the interval $[0, \pi]$.

$$\frac{x(\pi) - x(0)}{\pi - 0} = \frac{2}{\pi}$$

E. What is the distance travelled of the particle over the interval $[0, 2\pi]$?

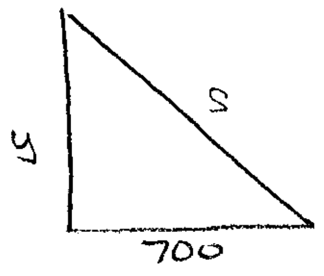
$x(0) = 0$
$x(\pi/4) = 1 - \sqrt{2}$ $> 1 - \sqrt{2}$

$x(\pi/4) = 1 - \sqrt{2}$
$x(5\pi/4) = 1 + \sqrt{2}$ $= 2\sqrt{2}$

$x(5\pi/4) = 1 + \sqrt{2}$
$x(2\pi) = 0$ $> 1 + \sqrt{2}$

Distance $= 4\sqrt{2}$

4. An observer stands 700 ft. away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft. from the ground?



$y^2 + 700^2 = S^2$ when $y = 2400$ $S = 2500$

$2y \frac{dy}{dt} = 2S \frac{dS}{dt}$

$2(2400)(900) = 2(2500) \frac{dS}{dt}$

$864 \text{ ft/sec} = \frac{dS}{dt}$

5. A cube whose edge is x is contracting. When its surface area is changing at a rate which is equal to 6 times the rate of its edge, then the length of the edge is?

$S.A. = 6x^2$

$SA' = 12x \frac{dx}{dt}$ $\quad S.A' = 6 \cdot \frac{dx}{dt}$

$6 \frac{dx}{dt} = 12x \frac{dx}{dt}$

$6 = 12x$

$\frac{1}{2} = x$