WHY WAS POLY NOMIAL AFRAID OF FINDING HER SECOND DERIVATIVE?

For each polynomial function find requested derivatives or values.

$f(x) = x^2 - 4x + 3$	$f(x) = 4 - x^2$	$f(x) = 3x^2 - x^3$
1) $f'(x) =$ 2x − 4	7) $f'(x) =$ −2x	13) $f'(x) =$ 6x − 3x²
2) min: x = 2 (2,−1)	8) min: x = none	14) min: x = 0 (0,0)
3) max: x = none	9) max: x = 0 (0,4)	15) max: x = 2 (2,4)
4) $f''(x) =$ 2	10) $f''(x) =$ −2	16) $f''(x) =$ 6 − 6x
5) inflection point: x = none	11) inflection point: x = none	17) inflection point: x = (1,2)
6) graph:	12) graph:	18) graph:

$f(x) = x^3 - 6x^2 + 9x - 9$	$f(x) = \frac{1}{4}x^4 - 6x^2$
19) $f'(x) =$ 3x² − 12x + 9	25) $f'(x) =$ x³ − 12x x(x² − 12)
20) min: x = 3 (3,−9)	26) min: x = ±√12 (−√12, 36)(√12, 36)
21) max: x = 1 (1,−5)	27) max: x = 0 (0,0)
22) $f''(x) =$ 6x − 12	28) $f''(x) =$ 3x² − 12 3(x+2)(x−2)
23) inflection point: x = 2 (2,−7)	29) inflection points: x = (−2,−20)(2,−20)
24) graph:	30) graph:

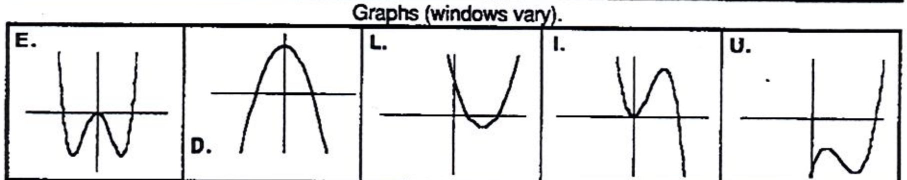
First or second derivatives.

C. −2	E. 6(1 − x)	F. −2x	G. 2(x − 2)	H. 6(x − 2)
N. 2	O. 3x(2 − x)	P. 3(x − 3)(x − 1)	U. x(x² − 12)	W. 3(x − 2)(x + 2)

x–coordinate of relative maxes, mins, and points of inflection.

E. −2	G. 3	H. 1	I. 0	J. 6	K. −6
M. 12	N. ±2	O. ±2√3	R. 4	S. 2	T. none

Graphs (windows vary).



S	H	E		T	H	O	U	G	H	T		S	H	E		W	O	U	L	D
15	21	30		5	22	13	25	20	22	3		2	17	16		28	13	24	6	12

G	E	T		P	O	I	N	T	S		O	F		I	N	F	L	I	C	T	I	O	N
1	30	11		19	26	14	29	8	23		26	7		18	29	7	6	27	10	3	9	26	4

f′ − +
2
x³ − 12x
− + − +
−√12 0 √12

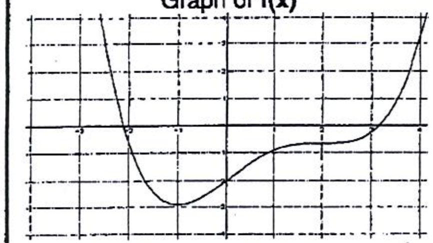
−3x(x − 2)
− + −
0 2

3(x² − 4x + 3)
(x − 3)(x − 1)
+ − +
1 3

HOW DID THE PRISON DENTIST HELP THE INMATE TO GO STRAIGHT?

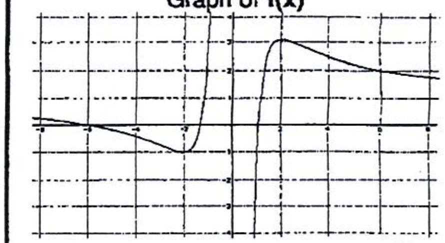
Find the x-coordinates and intervals described: Window [−8,8] X [−4,4]

Graph of **f(x)**



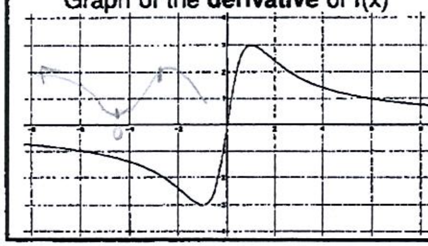
1) x-coordinate of local minimum: V
2) x-coordinate of local maximum: I
3) x-coordinates of inflection points: O
4) Intervals where f(x) is increasing: T
5) Intervals where f(x) is decreasing: S
6) Intervals where f(x) is concave up: G
7) Intervals where f(x) is concave down: C

Graph of **f(x)**



8) x-coordinate of local minimum: C
9) x-coordinate of local maximum: E
10) x-coordinates of inflection points:
11) Intervals where f(x) is increasing: I
12) Intervals where f(x) is decreasing: R
13) Intervals where f(x) is concave up: V Y
14) Intervals where f(x) is concave down:
15) Equation of vertical asymptote: N

Graph of the **derivative** of f(x)



Given the graph of **f′(x)** to the left, answer the following for **f(x)**.

16) x-coordinate of local minimum: N
17) x-coordinate of local maximum: I
18) x-coordinates of inflection points: H
19) Intervals where f(x) is increasing: A
20) Intervals where f(x) is decreasing: B
21) Intervals where f(x) is concave up: M
22) Intervals where f(x) is concave down: S

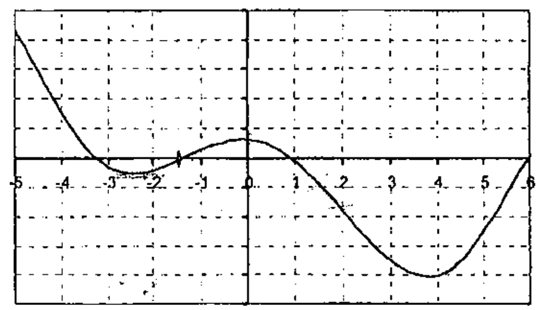
x-coordinates and intervals.

C. x = −2	E. x = 2	G. x = 1	H. x = ±1	I. none
N. x = 0	O. x = 0, x = 2	Q. x = 0, x = 1	V. x = −1	Y. x = ±3
A. (0,∞)	B. (−∞,0)	C. (0,2)	M. (−1,1)	O. (−∞,−1)
G. (−∞,0) ∪ (2,∞)	I. (−2,0) ∪ (0,2)	R. (−∞,−2) ∪ (2,∞)	S. (−∞,−1) ∪ (1,∞)	
T. (−1,2) ∪ (2,∞)	U. (−∞,0) ∪ (0,∞)	V. (−3,0) ∪ (3,∞)	Y. (−∞,−3) ∪ (0,3)	

B	Y		R	E	M	O	V	I	N	G		H	I	S		C	O	N	C	A	V	I	T	Y
20	10		12	9	21	5	1	17	16	6		18	2	22		8	3	15	7	19	13	11	4	14

READING GRAPHS

$f'(x)$



1. A graph of $f'(x)$ is given at the right.

A. On what interval(s) is $f(x)$ increasing? Decreasing? Explain.

dec $(-3,-1) \cup (1,6)$
inc $(-5,-3) \cup (-1,1)$

B. On what interval(s) is $f'(x)$ increasing? Decreasing? Explain.

− | + | − | +
-2.5 0 4 6

Inc $(-2.5, 0) \cup (4, 6)$
dec $(-5, -2.5) \cup (0, 4)$

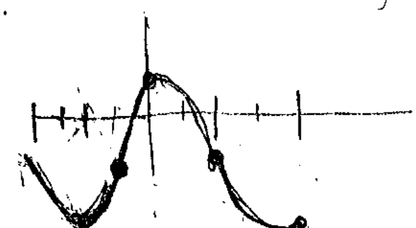
C. On what interval(s) is $f(x)$ concave up? Concave down? Explain.

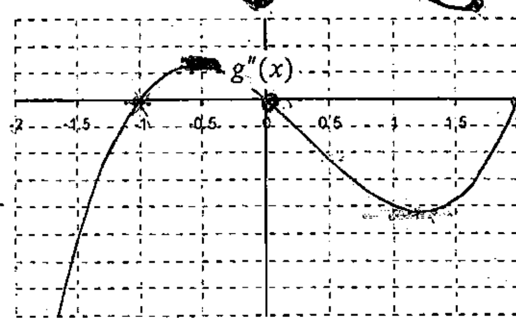
ccu $(-2.5, 0) \cup (4, 6)$
ccd $(-5, -2.5) \cup (0, 4)$

$g'(x)$

D. On what interval(s) is $f'(x)$ concave up? Concave down? Explain.

ccu $(-5, -1.5) (2, 6)$
ccd $(-1.5, 2)$





g''

2. A graph of $g''(x)$ is given at the right.

A. On what interval(s) is $g(x)$ concave up? Concave down? Explain.

+ | − | − | +
-1 0 2

ccu $(-1, 0)$
ccd $(-2, -1) \cup (0, 2)$

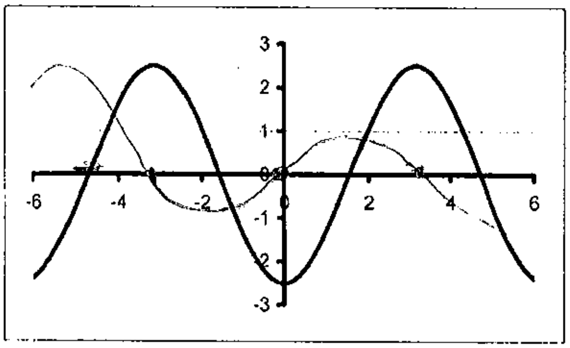
− | + | −
-2 -1 0 2

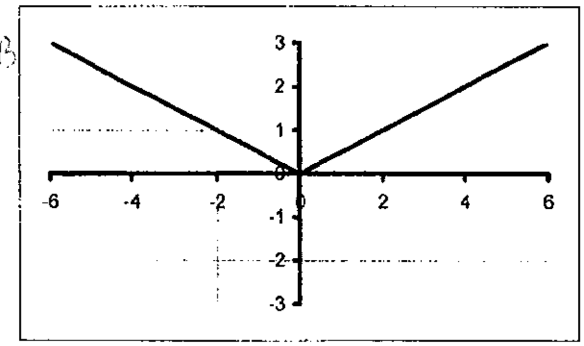
B. On what interval(s) is $g'(x)$ increasing? Decreasing? Explain.

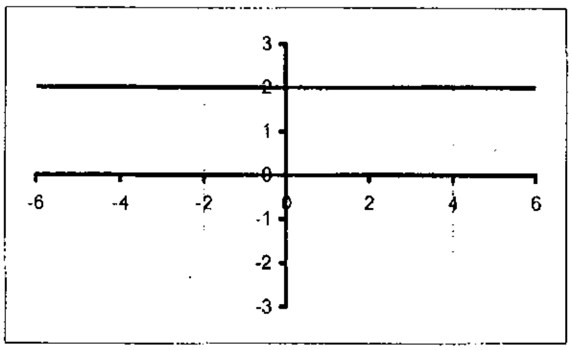
inc $(-1, 0)$
dec $(-2, -1) \cup (0, 2)$

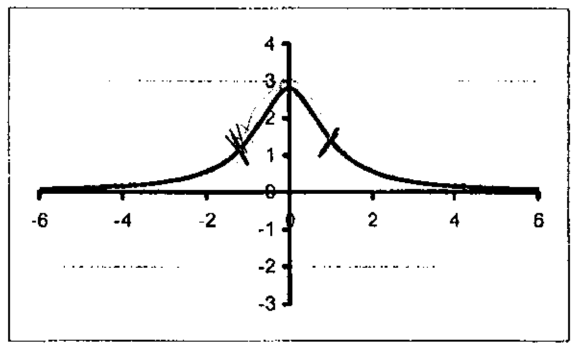
DERIVATIVE GRAPHS (2.3) NAME_______________________

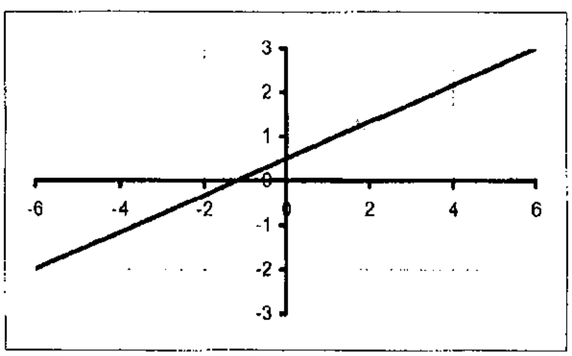
3. Sketch the graph of the derivative of each of the following functions.

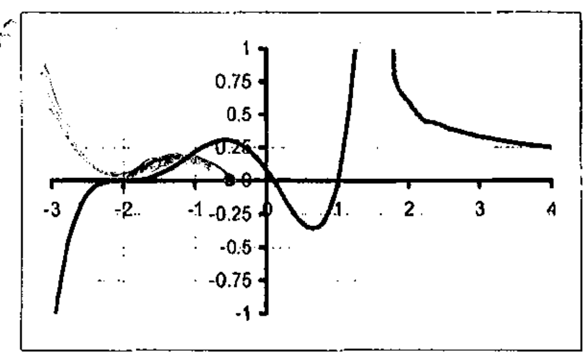
A


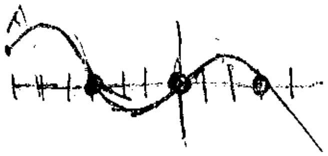
B


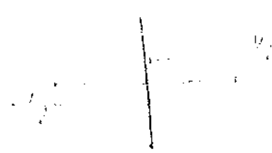
C


D


E


F


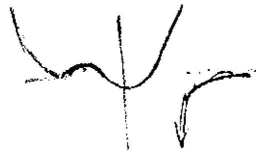
A


B


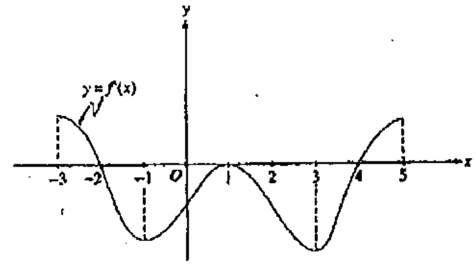
C


D


E


F


4.



f'

Note: This is the graph of the derivative of f, not the graph of f.

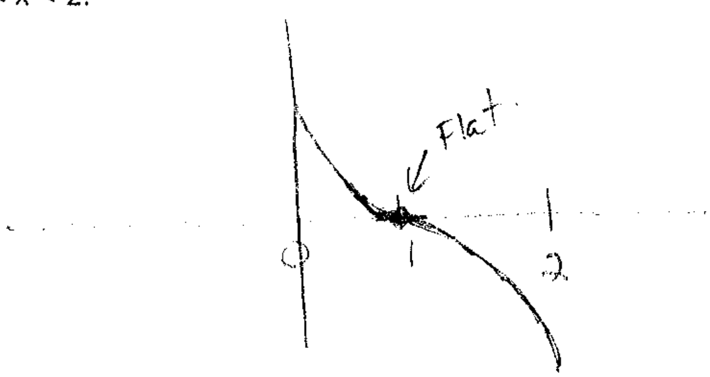
The figure above shows the graph of $f'(x)$, the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$.

a. For what values of x does f have a relative maximum? Why? $x = -2$

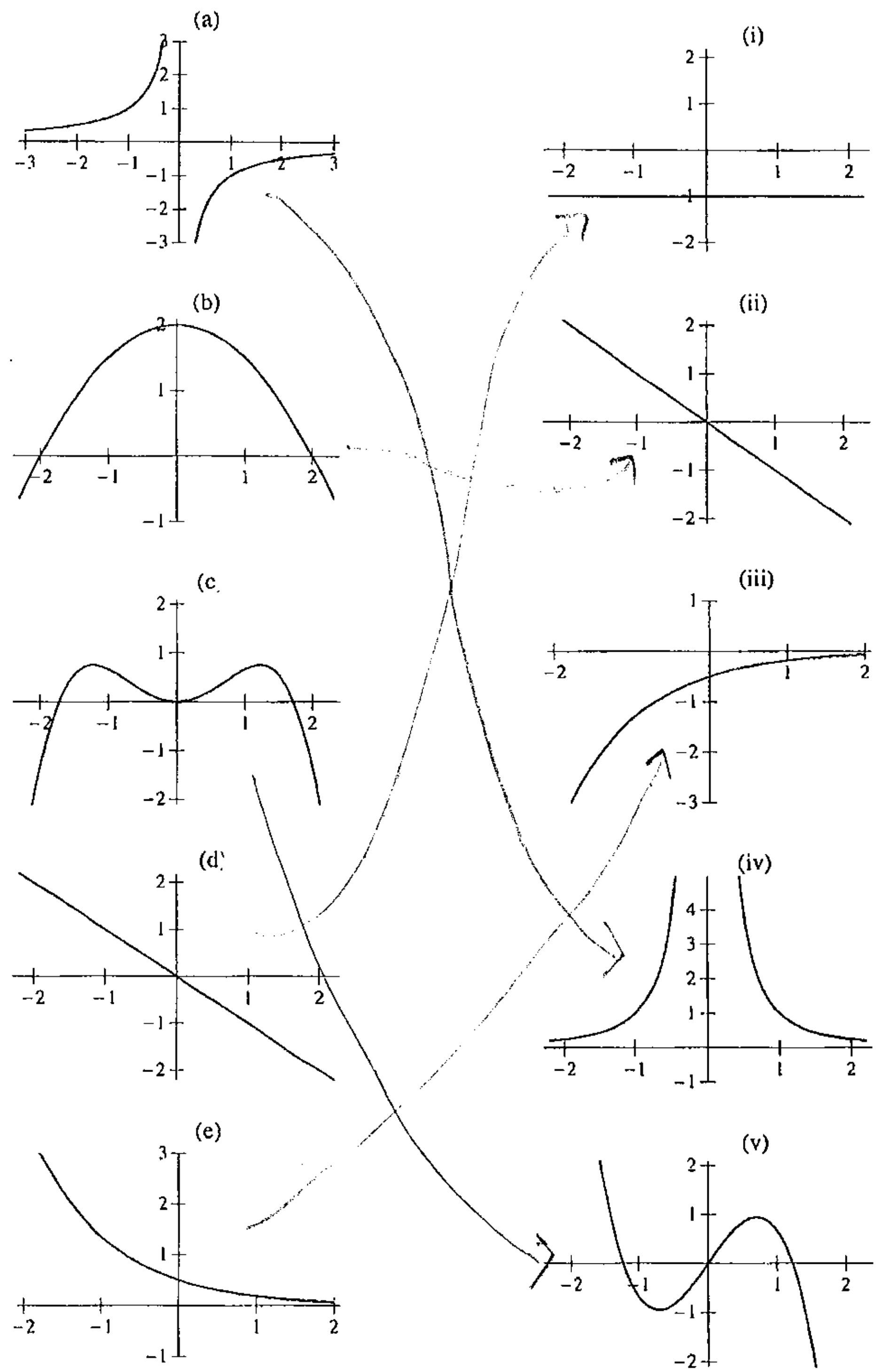
b. For what values of x does f have a relative minimum? Why? $x = 4$

c. On what intervals is the graph of f concave upward? Use f' to justify your answer. CCU $(-1, 1) \cup (3, 5)$
CCD $(-3, -1) \cup (1, 3)$

d. Suppose that $f(1) = 0$. In the xy-plane provided, draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.



5. Match the five functions a–e, given below, with their derivatives i–v. (You must be able to explain your reasoning).



CRITICAL POINTS - PART 2

6. In each case, sketch a graph of a <u>continuous</u> function with the given properties.

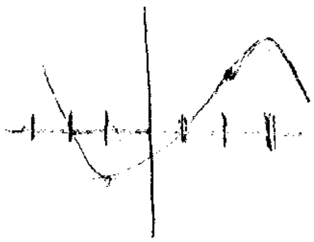
A. $f'(-1)=0$ and $f'(3)=0$

$f'(x)$

--	-1	+	3	--

$f''(x)$

+	2	--



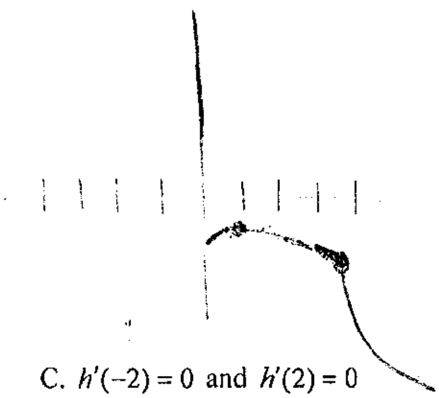
B. $g'(1)=0$ and $g'(4)$ is undefined

$g'(x)$

+	1	--	4	--

$g''(x)$

--	4	+



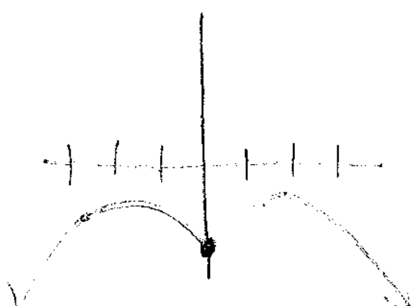
C. $h'(-2)=0$ and $h'(2)=0$
$h'(0)$ is undefined

$h'(x)$

+	-2	--	0	+	2	--

$h''(x)$

--	0	--



7. Use Calculus to determine i) critical points, ii) local extrema, iii) inflection points, and iv) intervals where $f(x)$ is concave up or down. Include an accurate graph that illustrates these features. Do this on a separate sheet of paper.

A. $f(x)=x^4+2x^3-1$

B. $f(x)=\dfrac{8x-16}{x^2}$

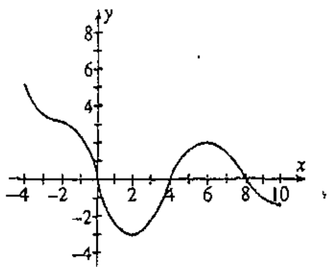
C. $f(x)=2x+3x^{2/3}$

$f'(x) = 4x^3 + 6x^2$

$f'' = 12x^2 + 12x$

$f'' = -2/3\,x^{-4/3}$

1. Use the graph below to answer true or false to each.



a) $f''(x) > 0$ for $x \in (2, 4)$ T

b) $f''(x) < 0$ for $x \in (-4, -2)$ F

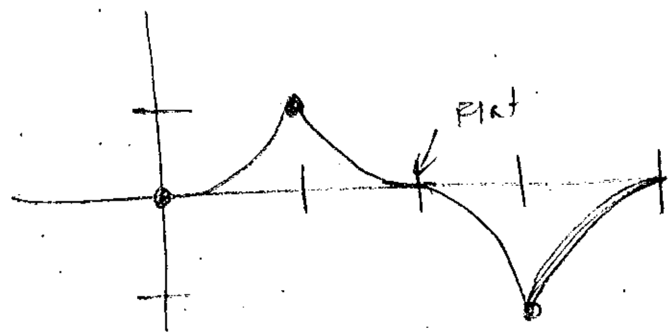
c) $f''(6) = 0$ F

d) $f''(2) > 0$ T

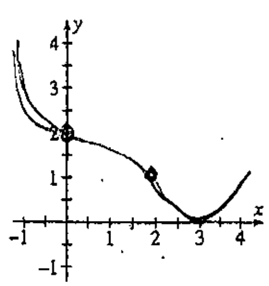
e) f is concave upward on $(0, 2)$ T

2. A function $f(x)$ and its first and second derivatives have values described in the table below. Sketch the graph of $f(x)$.

x	0	(0, 1)	1	(1, 2)	2	(2, 3)	3	(3, 4)	4
f	0		1		0		−1		0
f'		+	undef.	−	0	−	undef.	+	0
f''		+		+	0	−		−	

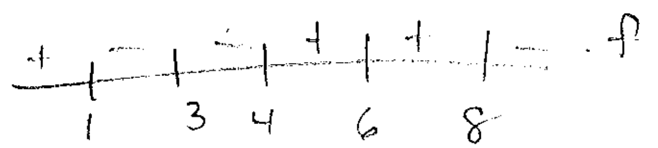


3. Sketch a graph of a function f having all these properties:
$f(-1) = 4, f(0) = 2, f(2) = 1, f(3) = 0$
$f'(x) \le 0$ for $x < 3$ and
$f'(x) \ge 0$ for $x > 3$.
$f''(x) < 0$ for $0 < x < 2$ and
$f''(x) \ge 0$ elsewhere.



4. What can you conclude about the local extrema of a differential function f from the following? Critical numbers for f are $x = 1, 3, 4, 6, 8$.

Interval	Sign of $f'(x)$
$(-\infty, 1)$	positive
$(1, 3)$	negative
$(3, 4)$	negative
$(4, 6)$	positive
$(6, 8)$	positive
$(8, \infty)$	negative



rel max $x = 1, 8$
rel min $x = 4$