Graphing Quadratics & Inequality Notes

1. **Completing the Square:**

You can change an expression like $x^2 + bx$ into a perfect square trinomial by adding $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$.

2. Solve by completing the square. Completing the square helps find vertex!! Q.F problem

a. $x^2 - 8x - 36 = 0$

$(x^2 - 8x + 16) - 36 - 16 = 0$

$\frac{1}{2} \cdot -8 = (-4)^2$

$(x-4)^2 - 52 = 0$

$\sqrt{(x-4)^2} = \sqrt{52}$

$x - 4 = \pm\sqrt{52}$ $\quad x = 4 \pm 2\sqrt{13}$

$x - 4 = \pm 2\sqrt{13}$ $\quad$ 2 real irrat.

b. $x^2 + 6x - 41 = 0$

$(x^2 + 6x + 9) - 41 - 9 = 0$

$(x+3)^2 - 50 = 0$

vertex $(-3, -50)$

$\sqrt{(x+3)^2} = \sqrt{50}$

$x = -3 \pm 5\sqrt{2}$

c. $2x^2 = 6x + 5$

$2x^2 - 6x - 5 = 0$

$2(x^2 - 3x \quad) - 5 \quad = 0$

$\frac{1}{2} \cdot -3 = \left(-\frac{3}{2}\right)^2$

$2(x^2 - 3x + 9/4) - 5 - 9/2$

$2(x - 3/2)^2 - 19/2$

vertex $(3/2, -19/2)$

3. To graph a quadratic, the best form to have the equation is in vertex form: $y = a(x-h)^2 + k$

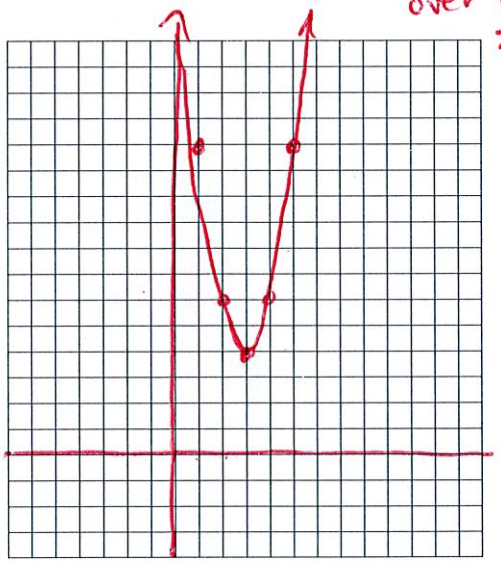
vertex (h, k)

4. If the equation is not in vertex form then either complete the square or use $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

5. Once you have the vertex form, then you can plot the vertex and use the patterns for parabolas to complete your sketch. If you are asked to also find the roots (x-intercepts), you can include those on the sketch as well. It is also helpful to find the y-intercept and use that to help sketch as well.

6. Sketch each quadratic.

a. $y = 2(x-3)^2 + 4$ $\quad (3, 4)$ over 1 up 1(2) 2 up 4(2)

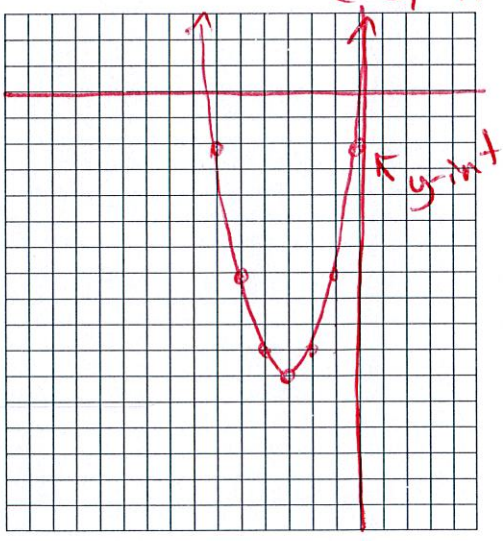


b. $y = x^2 + 6x - 2$

$x^2 + 6x + \underline{+9} - 2 \underline{-9}$

$(x+3)^2 - 11$

$(-3, -11)$



b. $y = x^2 - 3x - 1$

$\frac{-b}{2a} = \frac{3}{2}$

$(3/2, -13/4)$ or $(1.5, -3.25)$

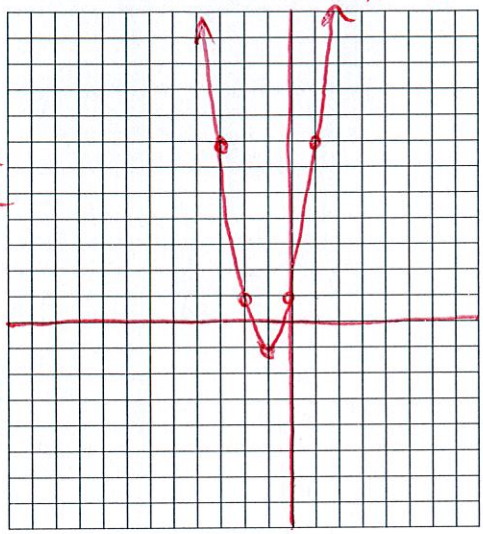
still is over 1 up 1a 2up 4a

ATLEAST 5 POINTS

d. $y = 2x^2 + 4x + 1$

$\frac{-b}{2a} = \frac{-4}{4} = -1$

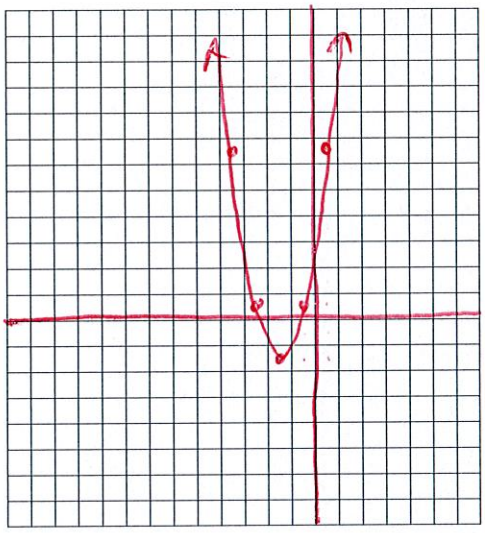
$(-1, -1)$



e. $y = 2x^2 + 6x + 3$

$\frac{-b}{2a} = \frac{-6}{4} = -1.5$

$(-1.5, -1.5)$



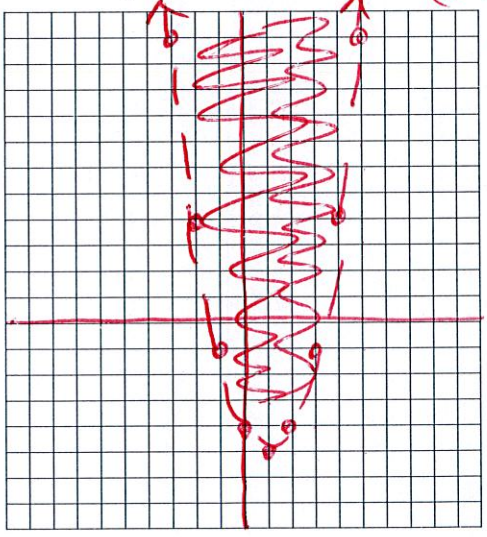
7. Sketch each quadratic inequality.

a. $y > x^2 - 2x - 4$ (above)

$x^2 - 2x + 1 \quad -4 - 1$

$(x-1)^2 - 5$

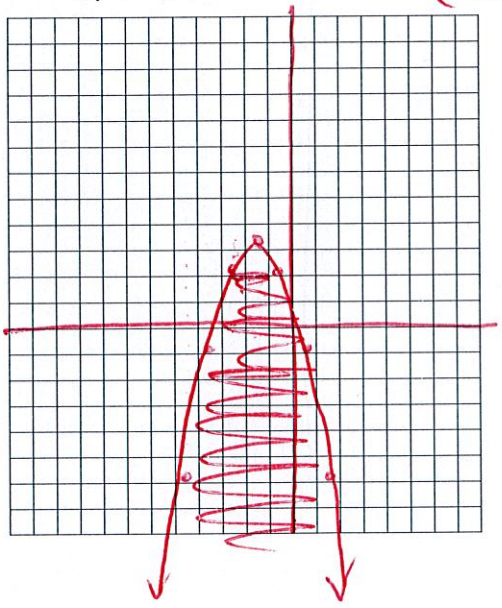
$(1, -5)$



b. $y \leq -x^2 - 3x + 1$ (below or equal)

$\frac{-b}{2a} = \frac{3}{-2} = -1.5$

$(-1.5, 3.25)$



Solving Quadratic Inequalities (These have only one variable - NO Y)

1. Interval Notation: All answers to quadratic inequalities will be in interval notation.

2. Solve each quadratic inequality. Put answers in interval notation

a. $x^2 - 4x - 12 > 0$ (above x-axis)

$(x-6)(x+2) > 0$

$x = 6 \quad x = -2$

Line

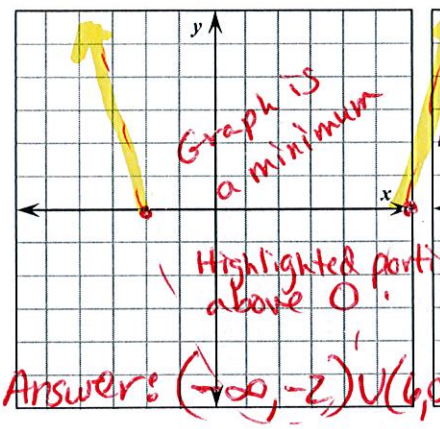
b. $3x^2 + 14x - 5 \le 0$ (below or = to x-axis)

$(3x-1)(x+5) \le 0$

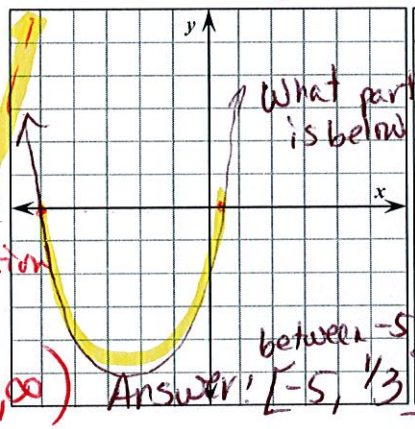
c. $2x^2 + x \ge 15$ (above or =)

$(2x-5)(x+3)$

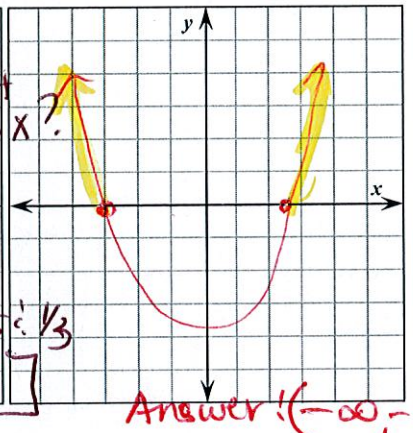
$x = 5/2 \quad x = -3$



Answer: $(-\infty, -2) \cup (6, \infty)$



Answer: $[-5, 1/3]$



Has to open up. Want portion that is above or = to x-axis. What x's make graph go above or = to x axis

Answer: $(-\infty, -3] \cup [5/2, \infty)$

d. $6x^2 + x > 1$

$6x^2 + x - 1 > 0$

$(3x-1)(2x+1) > 0$

Answer $(-\infty, -1/2) \cup (1/3, \infty)$

e. $5x < 2 - 3x^2$

$3x^2 + 5x - 2 < 0$

$(3x-1)(x+2) < 0$

$x = 1/3 \quad x = -2$

Answer $(-2, 1/3)$

f. $x^4 - 9 > 0$

Factor first

$(x^2+3)(x^2-3) = 0$

$x = \sqrt{3}, -\sqrt{3}$

$(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

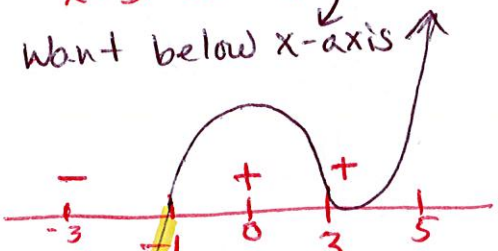
g. $x^3 + 27 < 0$

h. $12x^3 - 10x \le 12$

i. $(x-3)^2(x+1) < 0$

$x = 3 \quad x = -1$

Want below x-axis



Answer $(-\infty, -1)$

sketch of graph

① Factor & find critical points

② Put critical points on # line

③ Test Intervals

④ Look at what intervals match the inequality.

⑤ * Graph can help

⑥ Put into Interval Notation.

Homework: 4.1 Worksheet