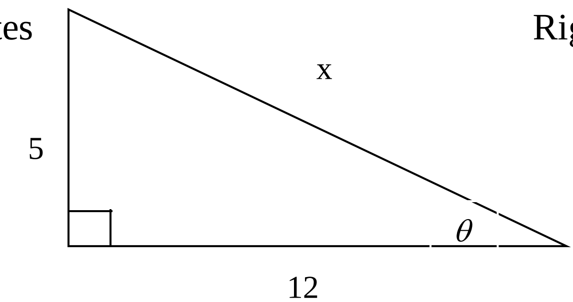
Precalculus Notes
Trigonometry

Right Triangle

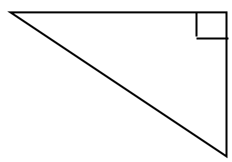


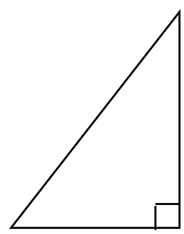
How do we find the hypotenuse?

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

Reciprocals:

Hint: Every function pair has a "co" in it.





$\sin\theta =$ $\csc\theta =$
$\cos\theta =$ $\sec\theta =$
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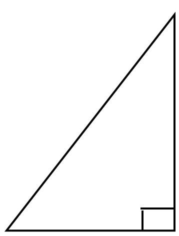
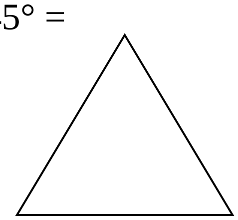


$\sin 45° =$ $\cos 45° =$ $\tan 45° =$

Equilateral Triangle:



$\sin 30° =$
$\cos 30° =$
$\tan 30° =$

$\sin 60° =$
$\cos 60° =$
$\tan 60° =$

sine and cosine are co-functions of complementary angles.
All co-functions of complementary angles are congruent.

Examples:

Identities: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Ex: Know $\sin\theta = \dfrac{\sqrt{2}}{2}$, $\tan\theta = 1$. Ex: Know $\csc\theta = \dfrac{\sqrt{15}}{2}$, $\cot\theta = \dfrac{\sqrt{11}}{2}$

Find the other values.

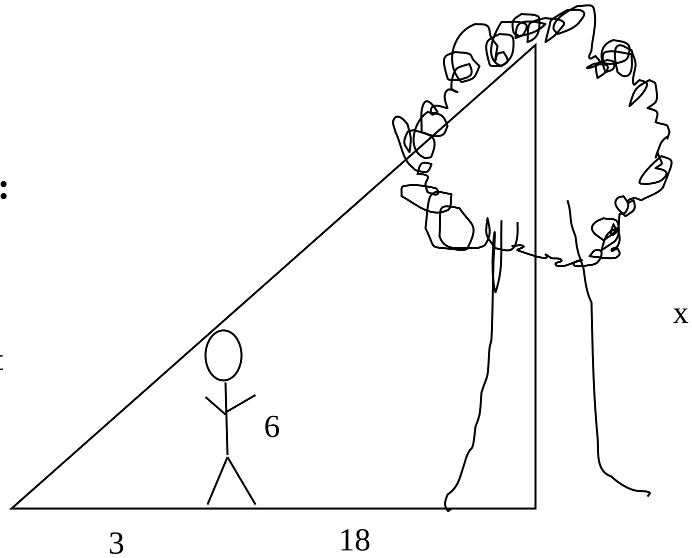
Ex: Know $\tan\theta = \dfrac{6}{5}$ Ex: Know $\csc\theta = \dfrac{16}{7}$

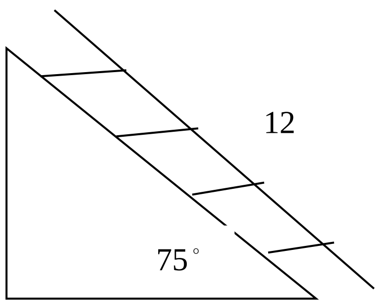
Ex: Know $\sin\theta = \dfrac{3}{4}$, find $\cos\theta$, $\tan\theta$.
Using identities:

Ex: $\tan\theta = 5$ find $\cot\theta$, $\sec\theta$, $\csc\theta$

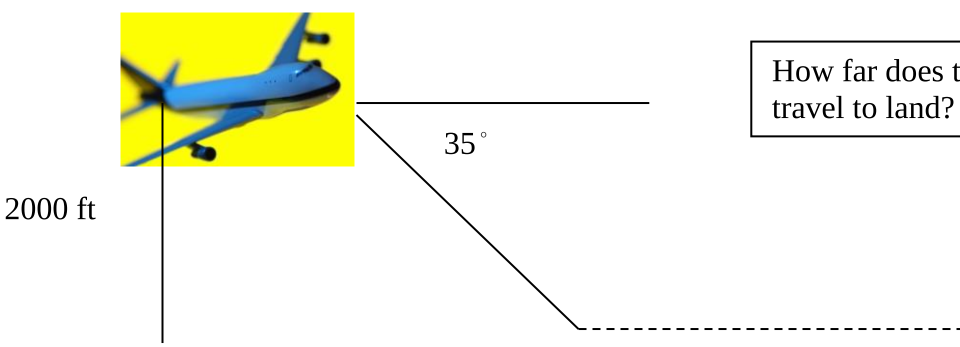
Trig Word Problems:

Given the following information, find the height of the tree.





Find how high the ladder is on the wall.



How far does the plane need to travel to land?

Finding values using a calculator:

a) sin 65° 30’ b) tan 34° 20’ 15” c) csc 49° 36’ d) sec 71°

Finding Angle Measures using a calculator:

a) $\cos\theta = .3947$ b) $\tan\theta = .8932$ c) $\sec\theta = 1.2207$

An angle is determined by rotating array at its endpoint.

Starting side is **initial** – ending side is **terminal**
Endpoint of ray is vertex of angle.
Origin = vertex
Standard Position: When an angle is at the origin and its initial side lies along the positive x-axis.

Positive angles: counter-clockwise **Negative angles: clockwise**

Quadrantal Angle: An angle whose terminal side lies on the x-axis or the y-axis

Measurement of angle is amount of rotation from initial side to terminal side.

Draw each angle in standard position:

a) 45 º b) 240º c) -150º d) 405º

Radians: One radian is the measure of a central angle θ that intercepts an arc equal in length to the radius of the circle.

Just over 6 radians in a full circle hence **2** π
Make sure to make clear π in radians is 180 degrees and π as a distance is 3.14.

Because the radian measure of an angle of one full revolution is 2 π you obtain.

$\frac{1}{2}$ revolution $\frac{2\pi}{2} = \pi$ radians = 180 degrees

$\frac{1}{6}$ revolution $\frac{2\pi}{6} = \frac{\pi}{3}$ radians = 60 degrees

Degrees: – 1 degree is equivalent to a rotation of $\frac{1}{360}$ a revolution about the vertex.

360 degrees = 2 π **radians**

How to convert from Degrees to Radians: radians = 1 degree = $\dfrac{\pi}{180}$

How to convert from Radians to Degrees: degrees = 1 radian = $\dfrac{180}{\pi}$ degrees

Ex:	$45°$	Ex:	$\dfrac{\pi}{2}$
	$150°$		$\dfrac{3\pi}{4}$
	$72°$		$\dfrac{2\pi}{5}$
	$270°$		$\dfrac{5\pi}{6}$
	$99°$		$\dfrac{3\pi}{14}$

Coterminal: An angle of $x°$ is coterminal with angles of $x° + k \cdot 360°$, where k is an integer. Coterminal angles have the same initial sides and terminal sides.

0 and 2π are coterminal

$\dfrac{\pi}{6}$ and $\dfrac{13\pi}{6}$ are coterminal.

Determine two coterminal angles for each:

a. $165°$ b. $420°$ c. $-120°$

a) positive angles you will subtract 2π
b) negative angles you will add 2π

$$\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}$$

Radian measure : Consider an arc of length s on a circle of radius r. The measure of the central angle, θ, that intercepts the arc is $\theta = \frac{s}{r}\, radians$. ($\theta$ is measured in radians)

Ex: Find the length of the arc is a circle has diameter of 12 in. and a central angle of 120 degrees.

Ex: A circle has a radius of 6 inches. Find the length of the arc intercepted by a central angle of $45°$.

The formula for the length of a circular arc can be used to analyze the motion of a particle moving at a constant speed along a circular path.

$$\textbf{Speed} = \frac{distance}{time} = \frac{s}{t} = \frac{r\theta}{t}$$

θ is the angle (in radians) corresponding to the arc length s, the angular speed.

$$\textbf{Angular speed} = \frac{\theta}{t}$$

Ex: The minute hand of a clock is 10.2 cm long. Find the speed of the tip of the hand.

Ex: A 16 in. wheel rotates one revolution every $\frac{3}{4}$ sec.

a) Find angular speed
b) how fast is the wheel moving?

a) angular speed = $\frac{2\pi}{\frac{3}{4}} = \frac{8\pi}{3}$ radians per sec.

b) speed = $\frac{16\pi}{\frac{3}{4}} = \frac{64\pi}{3}$ in. per sec

1. Linear Speed in Terms of Angular Speed

2. If the propeller of a plane makes ½ revolution around its axis, find the angular displacement in radians of a point on the end of the propeller.

3. A record player turntable rotates at 33 1/3 revolutions per minute (rpm). Find the angular velocity of the turntable in radians per second.
   The linear speed, v, of a point a distance r from the center of the rotation is given by $v = r\omega$, where ω is the angular speed in radians per unit of time.

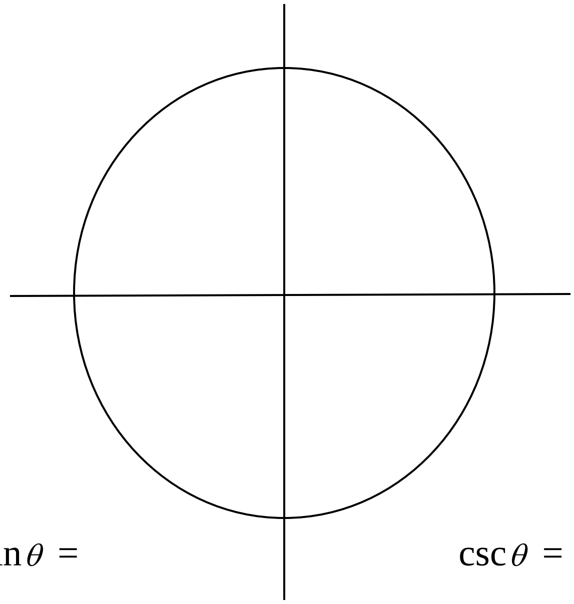
4. A wind machine used to generate electricity has blades that are 10 feet in length. The propeller is rotating at four revolutions per second. Find the linear speed, in feet per second, of the tips of the blades.

5. A compact disc player uses a laser to read music from a disc. The player varies the rotational speed of the disc depending on the position of the laser. When the laser is at the outer edge of the disc, the player spins the disc at the slowest speed, 200rpm.
   a. At the slowest speed, through how many degrees does the disc turn in a minute?

   b. If the diameter of the disc is 11.9 cm, find the approximate distance that a point on the outer edge travels at the slowest speed in 1 min.

   c. Use part (b) to give the speed in cm/s.

Unit Circle:



$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

When you reflect over the y-axis (x, y) = (-x, y)
So x’s become negative
Reflect over the x-axis (x, y) = (x, -y)
So y’s become negative
Reflect through origin (x, y) = (-x, -y)

Notice since sin θ = y and cos θ = x

$x^2 + y^2 = 1$ so $\sin^2\theta + \cos^2\theta = 1$ is an identity

Notice since $\sin\theta$ = y that sine is positive in quadrant one and two.

Also $\cos\theta$ is = x and that is positive in quadrant one and four.

Since tangent is = $\frac{\sin\theta}{\cos\theta}$ it is positive in one and three.

$\theta = \frac{\pi}{6}$ = 30 degrees Find all six trig functions.

$\sin\frac{\pi}{6} = y = \frac{1}{2}$ $\csc\frac{\pi}{6} = 2$

$\cos\frac{\pi}{6} = x = \frac{\sqrt{3}}{2}$ $\sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}$

$\tan\frac{\pi}{6} = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{2}$ $\cot\frac{\pi}{6} = \frac{\sqrt{3}}{1}$

Find all six trig values for?

1. $\theta = \frac{5\pi}{4}$ = 225 degrees

2. $\theta = \pi$

3. $\theta = \frac{\pi}{2}$

Notice $\cot\theta$, $\csc\theta$ are not defined

Notice the domain of sine & cosine is all real numbers
The range is from -1 to 1
And the period is 2π
Notice tan & sec are not defined when x = 0 so tan $\frac{\pi}{2}$, sec $\frac{\pi}{2}$ are undefined.

Ex: cot $\theta = \frac{3}{5}$ and θ is in quad III, find 5 remaining functions

Ex: Let (-3, -4) be a point on terminal side of θ**,** find six functions.

Ex: $\tan\theta$ = -1 and $\sin\theta < 0$, find 5 remaining functions.

cosine and secant are even functions

The other four are odd functions

Reference angle: is the acute angle θ formed by the terminal side of θ and the horizontal axis.

What are the reference angles?

1) -45 degrees

2) 200 degrees

3) 123 degrees

4) -400 degrees

5) $\frac{2\pi}{5}$

6) $\frac{25\pi}{12}$

7) $\frac{-3\pi}{4}$

8) $\frac{-12\pi}{5}$

9) 2.3 radians

Ex: Determine which quadrant $\sin\theta < 0$ and $\sec\theta > 0$

Ex: Find the reference angle of -400 degrees & $\frac{-5\pi}{3}$

Find the trig values if:

Ex: $\tan\theta = \frac{5}{3}$ and θ is in quad III

Ex: $\csc\theta = \frac{-12}{5}$ and $\cot\theta > 0$

True or False: the cosine of an angle can be $-\frac{3}{2}$

Table:

θ	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$
θ	0						
$\sin\theta$							
$\cos\theta$							
$\tan\theta$							

Graph y = sin x:

Graph y = cos x:

To graph with a calculator set mode to radians

Each function has a period of 2 π

To graph by hand you should find five key points. Intercepts, max, min points.

Ex: Graph y = 2 sin x

Ex: y = $\frac{1}{2}$ cos x

Amplitude = |a|

Period: – how long it takes one complete cycle.

The period of y = a sinbx and y = a cosbx is $\frac{2\pi}{|b|}$.

Ex: y = cos $\frac{1}{2}$ x = cos $\frac{x}{2}$

Ex: y = -2 sin (3x)

Amplitude = 1

Period $\frac{2\pi}{\frac{1}{2}}$ = 4 π

Ex: y = 5 sin $\frac{1}{3}$ x

Ex: y = -cos $\frac{2}{3}$ x

Amplitude = 5

Period = $\frac{2\pi}{\frac{1}{3}}$ = 6 π

y = a sin (bx – c) and y = a cos (bx – c)

shifts curve horizontally

This implies c/b is the phase shift!

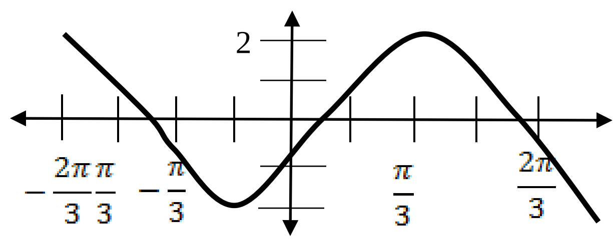
Ex: y = -2sin $\left(x - \frac{\pi}{2}\right)$

Ex: y = 3cos $(2\pi x + 4\pi)$

Ex: y = 3cos (2x) + 2

Ex: y = 3sin $(2x - \pi) + 1$

Write the equation of the graph:

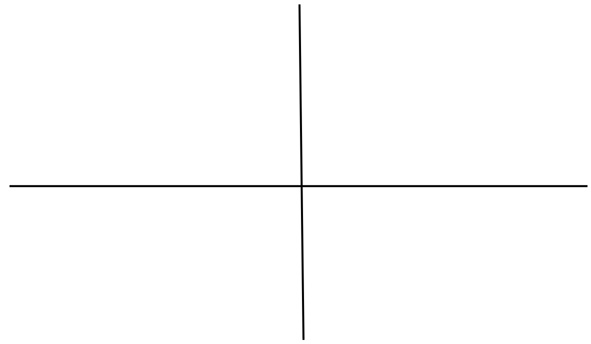


sin x. y = ______________________________ cos x. y = ____________________________

Ex: y = -2sin $(4x + \pi)$

Ex: y = -4sin $\left(\frac{2}{3}x - \frac{\pi}{3}\right)$

Graphs of other Trig Functions:



y = tan x ---------- $y = \frac{\sin\theta}{\cos\theta}$

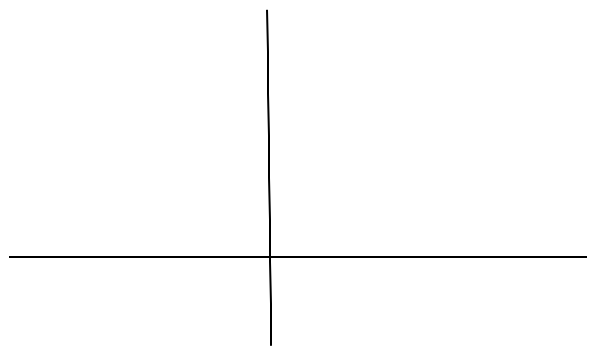
Look at where cos x = 0 (vertical asymptotes)

Domain:
Range:
Period:

Sketching y = a tan(bx – c) + d

Graph $y = \tan \frac{1}{2} x$

Graph $y = -2\tan\left(2x - \frac{\pi}{4}\right) + 3$



Graph y = cot x

How are we going to graph y = csc x and y = sec x relationship with sin x and cos x are reciprocals

y = csc x

y = sec x

Graph $y = 2\csc\left(x + \frac{\pi}{4}\right)$

Inverse Trig Functions:

To have an inverse the function must be one to one (horizontal line test.)

Find the following values:

arc sin 1

arc cos 1

arc sin $-\frac{1}{2}$

arc cos $\frac{\sqrt{2}}{2}$

arc cos $-\frac{\sqrt{3}}{2}$

Inverse tangent function:

Ex: tan $[\arctan -5]$ = -5

Ex: arc sin (sin $\frac{5\pi}{3}$)

y = arc sin x means y is an angle whose sin is x

Ex: find sin (arc cos $\frac{2}{3}$)

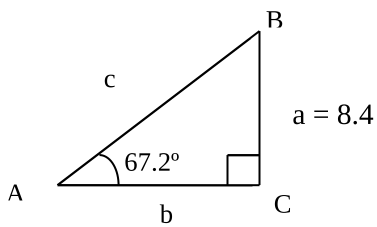
Ex: tan (arc cos $\frac{2}{3}$)

Ex: tan (arc cos $\frac{2}{3}$)

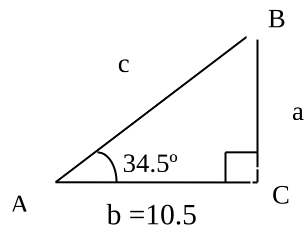
Ex: cos $\left[\arcsin \frac{-3}{5}\right]$

Ex: sin (arc cos 3x)

1. Solving triangles.



2. Solve the following triangle.



Ex: . From a point on level ground 125 feet from the base of a tower, the angle of elevation is 57.2 º . Approximate the height of the tower to the nearest foot.

Ex: A kite flies at a height of 30 ft. when 65 ft. of string is out. If the string is in a straight line, find the angle that it makes with the ground. Round to the nearest tenth of a degree.

Ex: You are taking your first hot-air balloon ride. Your friend is standing on level ground, 100 feet away from your point of launch, making a video of the terrified look on your rapidly ascending face. How rapidly? At one instant, the angle of elevation form the video camera to your face is 31.7 º One minute later, the angle of elevation is 76.2 º How far did you travel during that minute?

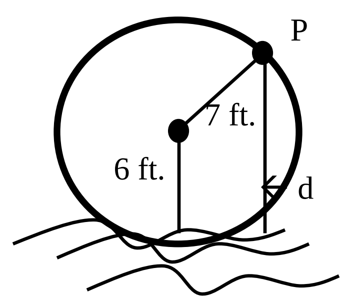
Ex: You are standing on level ground 800 feet from Mt. Rushmore, looking at the sculpture of Abraham Lincoln's face. Then angle of elevation to the bottom of the sculpture is 32º and the angle of elevation to the top is 35º. Find the height of the sculpture of Lincoln's face to the nearest tenth of a foot.

Sinusoidal Examples:

1. A person on a Ferris wheel of radius 100 ft. that makes one rotation every 30s. The center of the wheel is 105 ft. above ground. Find and graph a function to represent the person's height above the ground at any time t for a two minute ride.

2. Tarzan is swinging back and forth on his vine. As he swings, he goes back and forth across the river bank, going alternately over land and water. Let y be the number of meters Tarzan is from the river bank. Assume y varies sinusoidally with t, and that y is positive when Tarzan is over water and negative when he is over land. Jane decides to mathematically model his motion and finds that when t = 2, Tarzan is at one end of his swing, where y = -23. She finds that when t = 5 he reaches the other end of his swing and y = 17. Write the equation expressing Tarzan's distance from the river bank in terms of t.

3. Suppose that a waterwheel has a radius of 7 feet and rotates at 6 revolutions per minute (rpm). You start your stopwatch and two seconds later point P on the rim of the wheel is at its greatest point. You are to model the distance d of point P from the surface of the water in terms of the number of seconds t the stopwatch reads.



A) Sketch a graph of the curve

B) Write the equation of the curve.

C) Predict the height of P when t = 5.5