

Trig Identities

Name: _____

Use basic identities and algebra to simplify each expression. Assume all denominators are NOT zero.

1) $(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)$

$$\begin{aligned} \sin^2 \theta - \cos^2 \theta &= \\ 1 - \cos^2 \theta - \cos^2 \theta &\text{ or } \sin^2 \theta - (1 - \sin^2 \theta) \\ 1 - 2\cos^2 \theta &\text{ or } 2\sin^2 \theta - 1 \end{aligned}$$

2) $(\sin x - \cos x)^2$

$$\begin{aligned} \sin^2 x - 2\sin x \cos x + \cos^2 x \\ 1 - 2\sin x \cos x \\ \text{or } 1 - \sin(2x) \end{aligned}$$

3) $\frac{\sin t}{\tan t} = \frac{\sin t}{\frac{\sin t}{\cos t}}$

$$= \cos t$$

4) $(\tan \theta + 2)(\tan \theta - 3) - (6 - \tan \theta) + 2 \tan \theta$

$$\begin{aligned} \tan^2 \theta - \tan \theta - 6 - 6 + \tan \theta + 2 \tan \theta \\ \tan^2 \theta + 2 \tan \theta - 12 \end{aligned}$$

5) $\left(\frac{4\cos^2 x}{\sin^2 x}\right) \left(\frac{\sin x}{4\cos x}\right)^2$

$$\frac{4\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{16\cos^2 x} = \frac{1}{4}$$

6) $\frac{5\cos \theta}{\sin^2 \theta} * \frac{\sin^2 \theta - \sin \theta \cos \theta}{\sin^2 \theta - \cos^2 \theta}$

$$\begin{aligned} \frac{5\cos \theta}{\sin^2 \theta} * \frac{\sin \theta (\sin \theta - \cos \theta)}{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)} \\ \frac{5\cos \theta}{\sin^2 \theta + \sin \theta \cos \theta} \end{aligned}$$

7) $\frac{\cos^2 t + 4\cos t + 4}{\cos t + 2}$

$$= \cos t + 2$$

8. $\frac{\sin^2 \theta - 2\sin \theta + 1}{\sin \theta - 1}$

$$\sin \theta - 1$$

9. $\frac{1}{\cos \theta} - \sin \theta \tan \theta$

$$\frac{1 - \sin^2 \theta}{\cos \theta} = \frac{\cos^2 \theta}{\cos \theta} = \cos \theta$$

10. $\frac{1 - \tan^2 x}{1 + \tan^2 x} + 2\sin^2 x$

$$\begin{aligned} \frac{1 - \tan^2 x}{\sec^2 x} + 2\sin^2 x \\ \cos^2 x - \sin^2 x + 2\sin^2 x = .1 \end{aligned}$$

11. $\sqrt{\sin^3 \theta \cos \theta} * \sqrt{\cos \theta} = \sin \theta \cos \theta \sqrt{\sin \theta}$

Fundamental Identities
Simplify.

Name Key

1. $\sec \theta (\cot \theta)$

$$\frac{1}{\sin \theta} = \csc \theta$$

2. $1 + \sec^2 x - \tan^2 x$

$$1 + 1 = 2$$

3. $\cos \theta (\sec \theta - \cos \theta)$

$$1 - \cos^2 \theta = \sin^2 \theta$$

4. $\sin \theta \tan \theta + \cos \theta$

$$\frac{\sin^2 \theta}{\cos \theta} + \frac{\cos^2 \theta}{\cos \theta} = \frac{1}{\cos \theta} = \sec \theta$$

5. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$

$$\sin^2 x + 2\sin x \cos x + \cos^2 x + \sin^2 x - 2\sin x \cos x + \cos^2 x = 2$$

6. $\sin \theta \sec \theta \tan \theta$

$$\frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} = \tan^2 \theta = \sec^2 \theta - 1$$

Prove the stated identity

7. $\sec \theta \sin \theta = \tan \theta$

$$\frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

8. $\sec^2 \theta \cot^2 \theta - 1 = \cot^2 \theta$

$$\frac{1}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} - 1 = \frac{1}{\sin^2 \theta} - 1 = \csc^2 \theta - 1 = \tan^2 \theta$$

9. $\frac{1 + \tan^2 \theta}{1 + \cot^2 \theta} = \tan^2 \theta$

$$\frac{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}{1 + \frac{\cos^2 \theta}{\sin^2 \theta}} = \frac{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta}} = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$$

10. $\sec \theta - 1 = \frac{\tan^2 \theta}{\sec \theta + 1}$

$$\frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{1}{\cos \theta} + \frac{\cos \theta}{\cos \theta}} = \frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{1 + \cos \theta}{\cos \theta}}$$

11. $(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x) - 2 \tan^2 x (\sec^2 x - \tan^2 x) = 1$

$$\sec^2 x + \tan^2 x - 2 \tan^2 x \sec^2 x + 2 \tan^4 x = 1$$

$$1 + \tan^2 x + \tan^2 x - 2 \tan^2 x (1 + \tan^2 x) + 2 \tan^4 x = 1$$

$$1 = 1 \quad \checkmark$$

$$\frac{\sin^2 \theta}{\cos \theta (1 + \cos \theta)} = \frac{(1 - \cos \theta)(1 + \cos \theta)}{\cos \theta (1 + \cos \theta)}$$

12. $\frac{1 + \csc \theta}{1 - \csc \theta} + \frac{1 + \sin \theta}{1 - \sin \theta} = 0$

$$\frac{1 + \frac{1}{\sin \theta}}{1 - \frac{1}{\sin \theta}} + \frac{1 + \sin \theta}{1 - \sin \theta} = 0$$

$$\frac{\frac{\sin \theta + 1}{\sin \theta}}{\frac{\sin \theta - 1}{\sin \theta}} + \frac{1 + \sin \theta}{1 - \sin \theta} = 0$$

$$\frac{\sin \theta + 1}{\sin \theta - 1} + \frac{1 + \sin \theta}{1 - \sin \theta} = 0$$

$$\frac{1}{\cos \theta} - \frac{\cos \theta}{\cos \theta} = 0 \quad \checkmark$$