Practice with Transcendental Functions 2-17-2017 ICMAB

1. A particle moves along the x-axis so that at any time t > 0 its velocity is given by v(t) = t ln t - t.

   a. Write an expression for the acceleration of the particle.

   $a(t) = t \cdot \frac{1}{t} + \ln t - 1 \quad a(t) = \ln t$ $- \mid +$ at 1

   b. For what values of t is the particle moving to the right?

   $t(\ln t - 1) \quad \ln t = 1$

   $+ \mid - \mid +$ at $0, e$ (e, ∞)

   c. What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

   $+ \mid - \mid +$ at $0, 1$

   $v(1) = 1 \ln 1 - 1 = -1$

2. A particle moves along the x-axis so that its velocity v at any time t, for 0 ≤ t ≤16, is given by v(t) = $e^{2\sin t} - 1$. At time t = 0, the particle is at the origin.

   a. During what intervals of time is the particle moving to the left? Give a reason for your answer.

   $e^{2\sin t} = 1 \quad 2\sin t = 0 \quad t = 0, \pi, 2\pi, \ldots$

   $+ \mid - \mid + \mid - \mid +$ at $0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

   $(\pi, 2\pi) \cup (3\pi, 4\pi)$

   b. Find the total distance traveled by the particle from t = 0 to t = 4.

   $\int_0^4 \left| e^{2\sin t} - 1 \right| dt$ use calculator

   9.670657

   c. Is there any time t, 0 ≤ t ≤16, at which the particle returns to the origin? Justify your answer.

   Look at area. The graph shows the area never equals zero again.

A particle moves along the x-axis with velocity at time t ≥ 0 given by

3. $v(t) = -1 + e^{1-t}$.

a. Find the acceleration of the particle at time t = 3.

$a(t) = e^{1-t} \cdot (-1)$ $\quad a(3) = -e^{-2} = \frac{-1}{e^2}$

b. Is the speed of the particle increasing at time t = 3? Give a reason for your answer.

$v(t) < 0$ $\quad a(t) < 0$

speed $|v(t)| =$ .8646 increasing

c. Find all values of t at which the particle changes directions. Justify your answer.

$0 = -1 + e^{1-t}$

$1 = e^{1-t}$

$t = 1$

$\overset{+}{\rule{1cm}{0.4pt}}\big|_{1}\overset{-}{\rule{1cm}{0.4pt}}$

change direction at 1

d. Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

$$\int_0^3 |-1 + e^{1-t}|\, dt$$

$= 1.8536$

4. Let f and g and their inverses f^{-1} and g^{-1} be differentiable functions and let the values of f, g, and the derivatives f′ and g′ at x = 1 and x = 2 be given by the table below:

x	1	2
$f(x)$	2	3
$g(x)$	2	π
$f'(x)$	5	6
$g'(x)$	4	7

Determine the value of each of the following:

a. The derivative of f + g at x = 2. $\quad 13$

b. The derivative of fg at x = 2. $\quad \underset{3}{f}\underset{7}{g'} + gf' = 21 + 6\pi$

c. The derivative of f/g at x = 2. $\quad \frac{gf' - fg'}{g^2} = \frac{\pi 6 - 3(7)}{\pi^2} = \frac{6\pi - 21}{\pi^2}$

d. h'(1) where h(x) = f(g(x)). $h'(x) = f'(g(x)) \cdot g'(x) = 24$

6 2 4

e. The derivative of g^{-1} at x = 2. $(g^{-1})'(2) =$

not enough information

5. A particle moves along the x-axis with acceleration given by $a(t) = 2t - 10 + \frac{12}{t}$ for $t \geq 1$.

a. Write an expression for the velocity v(t), given that v(1) = 9.

b. For what values of t, $1 \leq t \leq 3$, is the velocity a maximum?

a) $\int 2t - 10 + \frac{12}{t}$

$v(t) = t^2 - 10t + 12\ln|t| + C$

$v(1) = 1 - 10 + C$

$9 = 1 - 10 + C$

$C = 18$

$v(t) = t^2 - 10t + 12\ln|t| + 18$

b) $a(t) = 2t - 10 + 12/t$ set = 0

$v(1) = 22$ max

$v(3) = 18$