Key

AFM Unit 7 Probability Review

1. How many subcommittees of 2 Democrats and 3 Republicans can be formed from a committee whose membership is 6 Democrats and 8 Republicans?

Combination

$_6C_2 \cdot {}_8C_3$

$(15)(56) = 840$

2. In how many ways can 5 people be seated in a room containing 2 chairs?

Permutation

$\frac{5 \cdot 4}{{}_5P_2} = 20$

3. A quality control engineer must inspect a sample of 3 fuses from a box of 100. How many different samples can he choose?

Combination

$_{100}C_3 = 161{,}700$

4. How many numbers of three or fewer digits can be formed from the digits 2, 3, 4, 5, and 6? Assume there is no repetition of digits.

$5 \cdot 4 \cdot 3$ OR $5 \cdot 4$ OR $5 = 85$

5. How many three-digit numbers can be formed from the digits 2, 3, 4, 5, & 6 if repetitions are allowed?

$5 \cdot 5 \cdot 5 = 125$

6. A witness to a holdup reports that the license of the getaway car consisted of 6 different digits. He remembers the first three but has forgotten the rest. How many licenses do the police have to check?

$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$

Knows | ↑ Last 3

$7 \cdot 6 \cdot 5 = 210$

7. In how many ways can the letters from the word ***television*** be arranged?

10 letters
2e's, 2i's

$\frac{10!}{2!2!} = 907{,}200$

8. How many ways can 8 people be seated at a round table?

$(8-1)!$ $7! = 5040$

9. A clown has 8 balloons, each a different color. There are 6 children. How many ways can the clown give each child a balloon?

${}_8P_6$ order matters $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 20{,}160$

10. How many 9-member baseball teams can be formed from 15 players if only 3 pitch while the others play the remaining 8 positions?

12 Reg. Players 3 pitchers

${}_{12}C_8 \cdot {}_3C_1$ $(495)(3) =$ **1485**

11. A photographer is taking a picture of a bride and a groom together with 6 attendants. How many ways can he arranged the 8 people in a line if the bride and groom stand in the middle?

$6! 2! =$ **1440**

$6 \cdot 5 \cdot 4 \cdot \underbrace{2 \cdot 1}_{2!} \cdot 3 \cdot 2 \cdot 1$

12. Two dice are rolled. What is the probability that their sum is 6 or 8?

$P(\text{sum } 6) = \frac{5}{36}$

$P(\text{sum } 8) = \frac{5}{36}$

$P(\text{sum 6 or 8}) = \frac{5}{36} + \frac{5}{36} = \frac{10}{36} =$ **$\frac{5}{18}$**

13. A bag contains 4 red balls and 2 white balls. If two different balls are selected are random, what is the probability of getting: (a) both red; (b) one of each color?

order does not matter

$\frac{{}_4C_2}{{}_6C_2} = \frac{6}{15} = \boxed{\frac{2}{5}}$

(1R 1W) $\frac{{}_4C_1 \cdot {}_2C_1}{{}_6C_2} = \boxed{\frac{8}{15}}$

14. A committee of 5 is selected from a group of 9 people (6 women and 3 men). What is the probability that it will have exactly 3 women and 2 men?

3w 2m $\frac{{}_6C_3 \cdot {}_3C_2}{{}_9C_5} = \frac{60}{126} = \frac{10}{21}$

15. Find the probability of drawing two aces from a deck of cards if the first card is not replaced before the second is drawn.

P(2 Aces) $\frac{{}_4C_2}{{}_{52}C_2} = \frac{6}{1326} = \boxed{\frac{1}{221}}$

16. A box contains 10 red, 8 green, and 12 blue tickets. Two successive tickets are drawn without replacement. Find the probability of drawing (without regard to order):

30 TOTAL

a) one blue and one green ticket

a) 1 Blue 1 Green $\frac{{}_{12}C_1 \cdot {}_8C_1}{{}_{30}C_2} = \frac{32}{145}$

b) two red tickets

b) $\frac{{}_{10}C_2}{{}_{30}C_2} = \frac{3}{29}$

c) no blue ticket

1 − (both Blue OR 1 Blue)

$1 - \left(\frac{{}_{12}C_2 + {}_{12}C_1 \cdot {}_{18}C_1}{{}_{30}C_2}\right) = 1 - \left(\frac{66 + 216}{435}\right) = 1 - \frac{282}{435}$

$\frac{435}{435} - \frac{282}{435} = \frac{153}{435} = \boxed{\frac{51}{145}}$

17. Solve using the binomial probability theorem: What is the probability of getting exactly 2 "fives" in 4 rolls of a die?

$n=4 \quad r=2 \quad P(5)=1/6 \qquad {}_4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = \boxed{25/216}$

18. A coin is tossed 4 times. Find the probability of getting the same number of heads and tails.

$n=4 \quad$ 2 Heads $\quad r=2 \quad P(H)=.5 \qquad {}_4C_2 (.5)^2 (.5)^2 = \boxed{3/8}$

19. A coin is flipped eight times. Find the probability of getting exactly six heads.

$n=8 \quad r=6 \quad P(H)=.5 \qquad {}_8C_6 (.5)^6 (.5)^2 = \boxed{7/64}$

20. Assume that ¾ of all drivers use seat belts on long trips. If 4 cars are checked on the highway, find the probability that seat belts are used by **at most** two drivers.

$n=4 \quad P(\text{seatbelt}) = 3/4$

$r = 0, 1, 2$ calc $(4, 3/4, \{0,1,2\})$ ← sum $= \boxed{67/256}$

21. There are 6 women and 7 men on the committee for city park enhancement. A subcommittee of 5 members is being selected at random to study the feasibility of redoing the landscaping in one of the parks. What is the probability that the committee will have at least 3 women?

3W 2M or 4W 1M or 5W

$$\frac{{}_6C_3 \cdot {}_7C_2 + {}_6C_4 \cdot {}_7C_1 + {}_6C_5}{{}_{13}C_5} = \frac{531}{1287} = \boxed{\frac{59}{143}}$$

22. In his pocket, Ben has 5 dimes, 6 nickels, and 4 pennies. He selects 3 coins. What is the probability that he selects exactly dimes and 1 penny?

2 Dimes 1 Penny

$$\frac{{}_5C_2 \cdot {}_4C_1}{{}_{15}C_3} = \frac{40}{455} = \boxed{\frac{8}{91}}$$

23. How many ways can 8 charms be arranged on a bracelet **with no clasp** if 3 of the charms are identical?

$$\frac{(8-1)!}{3!\,2} = 420$$

Reflective

24. A college library has three math books, 4 social science books and 3 biology books displayed on a shelf. In how many ways can the 10 books be arranged on the shelf if books on the same subject matter are together?

$$3! \cdot 4! \cdot 3! \cdot 3! \cdot 3! = \boxed{5184}$$

3! (math) · 4! (Soc. Science) · 3! (Bio) · 3!

25. One card is drawn at random from a standard deck. What is the probability of drawing an ace or a red card?

$$\frac{4 + 26 - 2}{52} = \frac{28}{52} = \frac{7}{13}$$

26. Determine if the following is a fair game: Two dice are rolled. If the sum is less than 7, then player A wins $5 from player B; otherwise, B wins $4 from A.

	Sum < 7	Sum ≥ 7
A	5	-4
Prob	$\frac{15}{36}$	$\frac{21}{36}$

Expected value for A: $5\left(\frac{15}{36}\right) - 4\left(\frac{21}{36}\right) = -\frac{9}{36} = -\frac{1}{4} = -.25$

Not Fair (should be zero)

27. Kay Paso, who is 3 years old, tears the labels off all 10 of the soup cans on her mother's shelf. Her mother knows that there were 2 cans of tomato soup and 8 cans of vegetable soup. She selects 4 cans at random. What is the probability that exactly one of the cans is tomato?

3 V 1 T

$\frac{{}_8C_3 \cdot {}_2C_1}{{}_{10}C_4} = \frac{112}{210} = \boxed{\frac{8}{15}}$

28. A coin is tossed three times. What is the probability that not all three tosses are the same?

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

$1 - P(\text{same})$

$1 - (HHH \text{ OR } TTT)$

$P(HHH) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$ $P(TTT) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

$1 - \left(\frac{1}{8} + \frac{1}{8}\right) = 1 - \frac{2}{8} = \frac{6}{8} = \boxed{\frac{3}{4}}$

29. Five cards are dealt from a deck of 52 cards. Which of the following shows the probability that 4 aces will be dealt?

(a) $\frac{{}_4C_4}{{}_{12}C_5}$ (b) $\frac{{}_{52}C_4}{{}_{52}C_5}$ (c) $\frac{{}_4C_4 \cdot {}_{48}C_1}{{}_{52}C_5}$ (d) $\frac{{}_4C_4 \cdot {}_{52}C_1}{{}_{52}C_5}$

30. Two cards are drawn from a deck of 52 cards with the first card replaced before the second card is drawn. What is the probability that neither card is a spade?

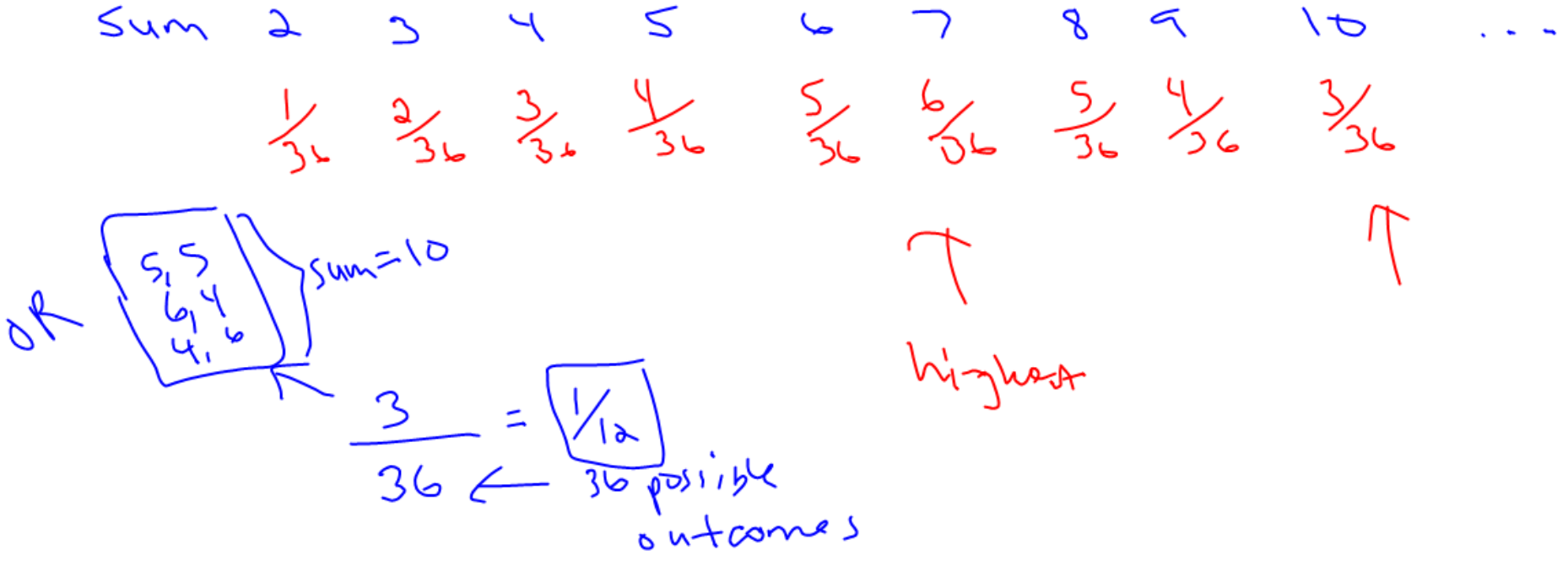
(a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{19}{34}$

$P(\text{NOT SPADE}) \cdot P(\text{NOT SPADE})$

$\frac{39}{52} \cdot \frac{39}{52} = \frac{3}{4} \cdot \frac{3}{4} = \boxed{\frac{9}{16}}$

31. A pair of dice is tossed. What is the probability that the sum of the faces showing on top is 10?

(a) $\frac{2}{9}$ (b) $\frac{1}{12}$ (c) $\frac{1}{9}$ (d) $\frac{1}{6}$



32. Suppose you play a game in which you make a bet and then draw a card from a standard deck of 52 cards as well as 2 jokers. If you draw a joker, you keep your bet and win $5; if you draw a face card, you keep your bet and win $2; and if you draw any other card, you lose your bet. What is your expected value on this game if you bet $1?

	Joker	Face	any other
You	5	2	-1
Prob.	$\frac{2}{54}$	$\frac{12}{54}$	$\frac{40}{54}$

← always adds to 1

expected value: $5\left(\frac{2}{54}\right) + 2\left(\frac{12}{54}\right) - 1\left(\frac{40}{54}\right) = \frac{-6}{54} = -\frac{1}{9}$

$\approx -.11$

you will lose an average of 11¢ per game