Supplemental Problems for UNIT 1
Fundamental Thm

3. Let $G(x) = \int_1^x (t^2 - 2)\,dt$. Calculate $G(1)$, $G'(1)$ and $G'(2)$. Then find a formula for $G(x)$.

4. Find $F(0)$, $F'(0)$, and $F'(3)$, where $F(x) = \int_0^x \sqrt{t^2+t}\,dt$.

5. Find $G(1)$, $G'(0)$, and $G'(\pi/4)$, where $G(x) = \int_1^x \tan t\,dt$.

6. Find $H(-2)$ and $H'(-2)$, where $H(x) = \int_{-2}^x \frac{du}{u^2+1}$.

In Exercises 7–16, find formulas for the functions represented by the integrals.

7. $\int_2^x u^4\,du$

8. $\int_2^x (12t^2 - 8t)\,dt$

9. $\int_0^x \sin u\,du$

10. $\int_{-\pi/4}^x \sec^2\theta\,d\theta$

21. $\frac{d}{dx}\int_0^x (t^5 - 9t^3)\,dt$

22. $\frac{d}{d\theta}\int_1^\theta \cot u\,du$

23. $\frac{d}{dt}\int_{100}^t \sec(5x-9)\,dx$

24. $\frac{d}{ds}\int_{-2}^s \tan\left(\frac{1}{1+u^2}\right)du$

25. Let $A(x) = \int_0^x f(t)\,dt$ for $f(x)$ in Figure 8.

(a) Calculate $A(2)$, $A(3)$, $A'(2)$, and $A'(3)$.

(b) Find formulas for $A(x)$ on $[0, 2]$ and $[2, 4]$ and sketch the graph of $A(x)$.

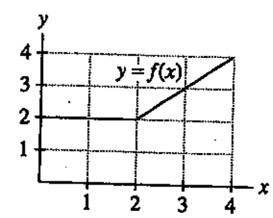

FIGURE 8

26. Make a rough sketch of the graph of $A(x) = \int_0^x g(t)\,dt$ for $g(x)$ in Figure 9.

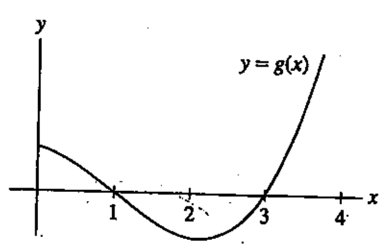

FIGURE 9

27. Verify: $\int_0^x |t|\,dt = \frac{1}{2}x|x|$. *Hint:* Consider $x \ge 0$ and $x \le 0$ separately.

28. Find $G'(1)$, where $G(x) = \int_0^{x^2} \sqrt{t^3+3}\,dt$.

In Exercises 29–34, calculate the derivative.

29. $\frac{d}{dx}\int_0^{x^2} \frac{t\,dt}{t+1}$

30. $\frac{d}{dx}\int_1^{1/x} \cos^3 t\,dt$

31. $\frac{d}{ds}\int_{-6}^{\cos s} u^4\,du$

32. $\frac{d}{dx}\int_{x^2}^{x^4} \sqrt{t}\,dt$

Hint for Exercise 32: $F(x) = A(x^4) - A(x^2)$.

33. $\frac{d}{dx}\int_{\sqrt{x}}^{x^2} \tan t\,dt$

34. $\frac{d}{du}\int_{-u}^{3u} \sqrt{x^2+1}\,dx$

In Exercises 35–38, with $f(x)$ as in Figure 10 let

$$A(x) = \int_0^x f(t)\,dt \quad \text{and} \quad B(x) = \int_2^x f(t)\,dt.$$

35. Find the min and max of $A(x)$ on $[0, 6]$.

36. Find the min and max of $B(x)$ on $[0, 6]$.

37. Find formulas for $A(x)$ and $B(x)$ valid on $[2, 4]$.

38. Find formulas for $A(x)$ and $B(x)$ valid on $[4, 5]$.

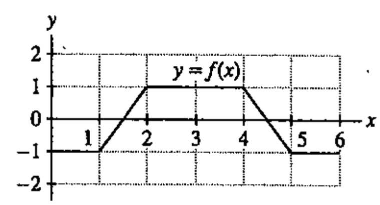

FIGURE 10

39. Let $A(x) = \int_0^x f(t)\,dt$, with $f(x)$ as in Figure 11.

(a) Does $A(x)$ have a local maximum at P?

(b) Where does $A(x)$ have a local minimum?

(c) Where does $A(x)$ have a local maximum?

(d) True or false? $A(x) < 0$ for all x in the interval shown.

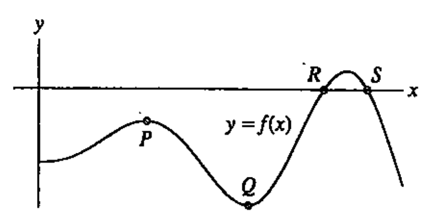

FIGURE 11 Graph of $f(x)$.

40. Determine $f(x)$, assuming that $\int_0^x f(t)\,dt = x^2 + x$.

41. Determine the function $g(x)$ and all values of c such that

$$\int_c^x g(t)\,dt = x^2 + x - 6$$

42. Find $a \le b$ such that $\int_a^b (x^2 - 9)\,dx$ has minimal value.

In Exercises 43–44, let $A(x) = \int_a^x f(t)\,dt$.

43. **Area Functions and Concavity** Explain why the following statements are true. Assume $f(x)$ is differentiable.

a) If c is an inflection point of $A(x)$, then $f'(c) = 0$.

b) $A(x)$ is concave up if $f(x)$ is increasing.

c) $A(x)$ is concave down if $f(x)$ is decreasing.

44. Match the property of $A(x)$ with the corresponding property of the graph of $f(x)$. Assume $f(x)$ is differentiable.

Area function $A(x)$

a) $A(x)$ is decreasing.

b) $A(x)$ has a local maximum.

c) $A(x)$ is concave up.

d) $A(x)$ goes from concave up to concave down.

Graph of $f(x)$

i) Lies below the x-axis.

ii) Crosses the x-axis from positive to negative.

iii) Has a local maximum.

iv) $f(x)$ is increasing.

45. Let $A(x) = \int_0^x f(t)\,dt$, with $f(x)$ as in Figure 12. Determine:

(a) The intervals on which $A(x)$ is increasing and decreasing

(b) The values x where $A(x)$ has a local min or max

(c) The inflection points of $A(x)$

(d) The intervals where $A(x)$ is concave up or concave down

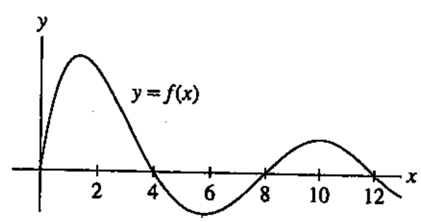

FIGURE 12

46. Let $f(x) = x^2 - 5x - 6$ and $F(x) = \int_0^x f(t)\,dt$.

(a) Find the critical points of $F(x)$ and determine whether they a local minima or local maxima.

(b) Find the points of inflection of $F(x)$ and determine whether tl concavity changes from up to down or from down to up.

(c) GU Plot $f(x)$ and $F(x)$ on the same set of axes and confir your answers to (a) and (b).

47. Sketch the graph of an increasing function $f(x)$ such that bo $f'(x)$ and $A(x) = \int_0^x f(t)\,dt$ are decreasing.

48. Figure 13 shows the graph of $f(x) = x\sin x$. Let $F(x) = \int_0^x t \sin t\,dt$.

(a) Locate the local max and absolute max of $F(x)$ on $[0, 3\pi]$.

(b) Justify graphically: $F(x)$ has precisely one zero in $[\pi, 2\pi]$.

(c) How many zeros does $F(x)$ have in $[0, 3\pi]$?

(d) Find the inflection points of $F(x)$ on $[0, 3\pi]$. For each one, sta whether the concavity changes from up to down or from down to up

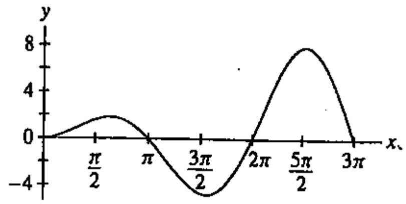

FIGURE 13 Graph of $f(x) = x \sin x$.

49. GU Find the smallest positive critical point of

$$F(x) = \int_0^x \cos(t^{3/2})\,dt$$

and determine whether it is a local min or max. Then find the sma est positive inflection point of $F(x)$ and use a graph of $y = \cos(x^{3/}$ to determine whether the concavity changes from up to down or fr down to up.

Integration Practice

16. $\int \sin(4\theta - 7)\,d\theta$

17. $\int \sin^2\theta \cos\theta\,d\theta$

18. $\int \sec^2 x \tan x\,dx$

19. $\int xe^{-x^2}\,dx$

20. $\int (\sec^2 t)e^{\tan t}\,dt$

23. $\int x^3 \cos(x^4)\,dx$

24. $\int x^2 \cos(x^3 + 1)\,dx$

25. $\int x^{1/2} \cos(x^{3/2})\,dx$

26. $\int \cos x \cos(\sin x)\,dx$

Rates of Change

Preliminary Questions

1. A hot metal object is submerged in cold water. The rate at which the object cools (in degrees per minute) is a function $f(t)$ of time. Which quantity is represented by the integral $\int_0^T f(t)\,dt$?

2. A plane travels 560 km from Los Angeles to San Francisco in 1 hour. If the plane's velocity at time t is $v(t)$ km/h, what is the value of $\int_0^1 v(t)\,dt$?

3. Which of the following quantities would be naturally represented as derivatives and which as integrals?

(a) Velocity of a train

(b) Rainfall during a 6-month period

(c) Mileage per gallon of an automobile

(d) Increase in the U.S. population from 1990 to 2010

Exercises

1. Water flows into an empty reservoir at a rate of $3000 + 20t$ liters per hour. What is the quantity of water in the reservoir after 5 hours?

2. A population of insects increases at a rate of $200 + 10t + 0.25t^2$ insects per day. Find the insect population after 3 days, assuming that there are 35 insects at $t = 0$.

3. A survey shows that a mayoral candidate is gaining votes at a rate of $2000t + 1000$ votes per day, where t is the number of days since she announced her candidacy. How many supporters will the candidate have after 60 days, assuming that she had no supporters at $t = 0$?

4. A factory produces bicycles at a rate of $95 + 3t^2 - t$ bicycles per week. How many bicycles were produced from the beginning of week 2 to the end of week 3?

5. Find the displacement of a particle moving in a straight line with velocity $v(t) = 4t - 3$ m/s over the time interval $[2, 5]$.

6. Find the displacement over the time interval $[1, 6]$ of a helicopter whose (vertical) velocity at time t is $v(t) = 0.02t^2 + t$ m/s.

7. A cat falls from a tree (with zero initial velocity) at time $t = 0$. How far does the cat fall between $t = 0.5$ and $t = 1$ s? Use Galileo's formula $v(t) = -9.8t$ m/s.

8. A projectile is released with an initial (vertical) velocity of 100 m/s. Use the formula $v(t) = 100 - 9.8t$ for velocity to determine the distance traveled during the first 15 seconds.

In Exercises 9–12, a particle moves in a straight line with the given velocity (in m/s). Find the displacement and distance traveled over the time interval, and draw a motion diagram like Figure 3 (with distance and time labels).

9. $v(t) = 12 - 4t$, $[0, 5]$

10. $v(t) = 36 - 24t + 3t^2$, $[0, 10]$

11. $v(t) = t^{-2} - 1$, $[0.5, 2]$

12. $v(t) = \cos t$, $[0, 3\pi]$

13. Find the net change in velocity over $[1, 4]$ of an object with $a(t) = 8t - t^2$ m/s^2.

14. Show that if acceleration is constant, then the change in velocity is proportional to the length of the time interval.

15. The traffic flow rate past a certain point on a highway is $q(t) = 3000 + 2000t - 300t^2$ (t in hours), where $t = 0$ is 8 AM. How many cars pass by in the time interval from 8 to 10 AM?

16. The marginal cost of producing x tablet computers is $C'(x) = 120 - 0.06x + 0.00001x^2$ What is the cost of producing 3000 units if the setup cost is \$90,000? If production is set at 3000 units, what is the cost of producing 200 additional units?

17. A small boutique produces wool sweaters at a marginal cost of $40 - 5[[x/5]]$ for $0 \le x \le 20$, where $[[x]]$ is the greatest integer function. Find the cost of producing 20 sweaters. Then compute the average cost of the first 10 sweaters and the last 10 sweaters.

18. The rate (in liters per minute) at which water drains from a tank is recorded at half-minute intervals. Compute the average of the left- and right-endpoint approximations to estimate the total amount of water drained during the first 3 minutes.

t (min)	0	0.5	1	1.5	2	2.5	3
r (l/min)	50	48	46	44	42	40	38

19. The velocity of a car is recorded at half-second intervals (in feet per second). Use the average of the left- and right-endpoint approximations to estimate the total distance traveled during the first 4 seconds.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
$v(t)$	0	12	20	29	38	44	32	35	30

21. A megawatt of power is 10^6 W, or 3.6×10^9 J/hour. Which quantity is represented by the area under the graph in Figure 5? Estimate the energy (in joules) consumed during the period 4 PM to 8 PM.

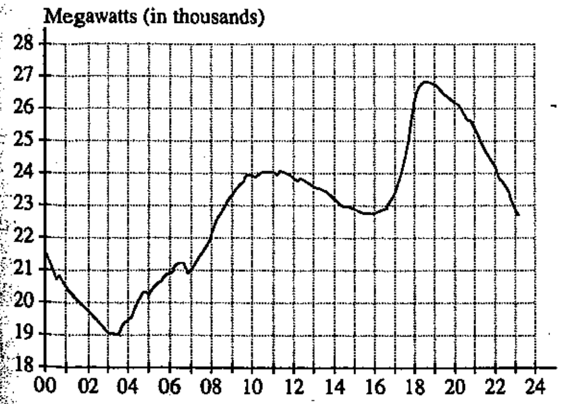

FIGURE 5 Power consumption over 1-day period in California (February 2010).

22. Figure 6 shows the migration rate $M(t)$ of Ireland in the period 1988–1998. This is the rate at which people (in thousands per year) move into or out of the country.

(a) Is the following integral positive or negative? What does this quantity represent?

$$\int_{1988}^{1998} M(t)\,dt$$

(b) Did migration in the period 1988–1998 result in a net influx of people into Ireland or a net outflow of people from Ireland?

(c) During which two years could the Irish prime minister announce, "We've hit an inflection point. We are still losing population, but the trend is now improving."

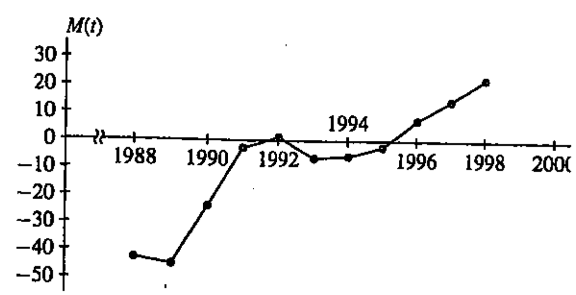

FIGURE 6 Irish migration rate (in thousands per year).

25. Figure 7 shows the rate $R(t)$ of natural gas consumption (in billions of cubic feet per day) in the mid-Atlantic states (New York, New Jersey, Pennsylvania). Express the total quantity of natural gas consumed in 2009 as an integral (with respect to time t in days). Then estimate this quantity, given the following monthly values of $R(t)$:

3.18, 2.86, 2.39, 1.49, 1.08, 0.80,
1.01, 0.89, 0.89, 1.20, 1.64, 2.52

Keep in mind that the number of days in a month varies with the month.

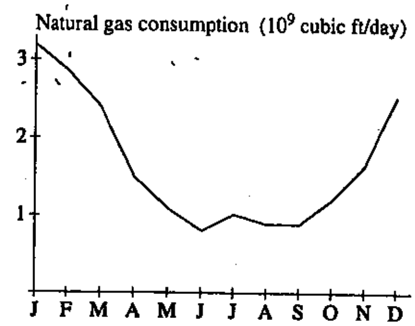

FIGURE 7 Natural gas consumption in 2009 in the mid-Atlantic states

Integration Practice

27. $\int (4x+5)^9\,dx$

28. $\int \frac{dx}{(x-9)^5}$

29. $\int \frac{dt}{\sqrt{t+12}}$

30. $\int (9t+2)^{2/3}\,dt$

31. $\int \frac{x+1}{(x^2+2x)^3}\,dx$

32. $\int (x+1)(x^2+2x)^{3/4}\,dx$

33. $\int \frac{x}{\sqrt{x^2+9}}\,dx$

34. $\int \frac{2x^2+x}{(4x^3+3x^2)^2}\,dx$

35. $\int (3x^2+1)(x^3+x)^2\,dx$

36. $\int \frac{5x^4+2x}{(x^5+x^2)^3}\,dx$

37. $\int (3x+8)^{11}\,dx$

38. $\int x(3x+8)^{11}\,dx$

39. $\int x^2\sqrt{x^3+1}\,dx$

40. $\int x^5\sqrt{x^3+1}\,dx$

41. $\int \frac{dx}{(x+5)^3}$

42. $\int \frac{x^2\,dx}{(x+5)^3}$

43. $\int z^2(z^3+1)^{12}\,dz$

44. $\int (z^5+4z^2)(z^3+1)^{12}\,dz$

45. $\int (x+2)(x+1)^{1/4}\,dx$

46. $\int x^3(x^2-1)^{3/2}\,dx$

47. $\int \sin(8-3\theta)\,d\theta$

55. $\int \sec^2(4x+9)\,dx$

49. $\int \frac{\cos\sqrt{t}}{\sqrt{t}}\,dt$

57. $\int \frac{\sec^2(\sqrt{x})\,dx}{\sqrt{x}}$

51. $\int \tan(4\theta+9)\,d\theta$

59. $\int \sin 4x\sqrt{\cos 4x+1}\,dx$

53. $\int \cot x\,dx$

61. $\int \sec\theta\tan\theta(\sec\theta-1)\,d\theta$