A **sequence** is an ordered list of numbers. Each number in the list is called a **term** of the sequence. The first term of a sequence is denoted as a_1. The second term is denoted as a_2. The term in the nth position is called the nth term and is denoted as a_n. The term before a_n is a_{n-1}.

A sequence is a function whose <u>range</u> is the <u>terms</u> of the sequence and the <u>domain</u> is the <u>position</u> of each term.

There are many types of sequences:

1) An **infinite sequence** is a sequence with an infinite number of terms.
   Can we come up with some examples?

   Ex: 2, 4, 6, 8, 10, 12,....

   Ex: 1, -3, 5, -7, 9, -11, 13 .....

2) A **finite sequence** is a sequence with a finite number of terms. The sequence ends at a certain point.

Three sequences we will learn about:

3) An **Arithmetic sequence** is a sequence in which the difference between each term and the proceeding term is always constant. This difference is known as *d* and is constant term.

4) A **Geometric sequence** is a sequence in which terms are found by multiplying a preceding term by a non-zero constant. This term is known as *r* and is called the common ratio of a geometric sequence.

5) A **Recursive Sequence** is a sequence in which each term is defined using the previous terms.

I. Write a rule for the n^{th} term for the following examples:

6) Ex 1. 1, 2, 3, 4, 5, ... Ex. 2 2,4,6,8,10,... Ex.3 2,3,4,5,6,...

$a_n = 1n + 0$ $a_n = 2n + 0$ $a_n = n + 1$

7) Ex. 4 5,8,11,14,... Ex. 5 1,3,5,7,9,... Ex. 6 $\frac{2}{5}, \frac{2}{25}, \frac{2}{125}, \frac{2}{625}...$

$a_n = 3n + 2$ $a_n = 2n - 1$ Geometric

$a_n = 2/5 \cdot (1/5)^{n-1}$

II. List the first four terms of the sequence given by

Ex: $a_n = 4n - 3$

$a_1 = 1$
$a_2 = 5$
$a_3 = 9$
$a_4 = 13$

Ex: $a_n = \frac{(-1)^n}{2n-1}$

$a_1 = -1$
$a_2 = 1/3$
$a_3 = -1/5$
$a_4 = 1/7$

Ex: $a_n = 7(3^{-n})$

$a_1 = 7/3$
$a_2 = 7/9$
$a_3 = 7/27$
$a_4 = 7/81$

Arithmetic sequences:

The sequence $a_1, a_2, a_3, a_4, \ldots a_n$ is arithmetic if there is a number d such that:

$a_2 - a_1 = d$
$a_3 - a_2 = d$ Where d is the common difference.

Ex: 6, 9, 12, 15, ...3n + 3 The common difference is 3 because 9 – 6 = 3. d =3

Ex: 2, -3, -8, -13, …, 5n + 7 The common difference is –5 because -3–2 = -5. d = -5

Explicit Formula: a formula that defines the n^{th} term.

The n^{th} term of an arithmetic sequence: $\mathbf{a_n = dn + c}$

Is the form for the n^{th} term of an arithmetic sequence. Where d is the common difference and $c = a_1 - d$ (The first term minus the common difference).

$a_n = nd + a_0$

Your book represents this differently ($a_n = a_1 + (n-1)d$ or $a_n = a_{n-1} + d$)

Ex: Find a formula of an arithmetic sequence whose common difference is 4 and whose first term is 3.

$a_n = dn + c$ We know d = 4. $a_1 = 3$. So c = 3 – 4. c = -1

$a_n = 4n - 1$. The terms of this sequence are: 3, 7, 11, 15, …, 4n – 1.

Ex: Find the formula of the arithmetic sequence whose first term is 3 and whose second term is –1.

$a_n = dn + c$ We know $a_1 = 3$ and $a_2 = -1$. So d = -4. c must be 3–(-4) = 7

$a_n = -4n + 7$

I. Write a rule for the n^{th} term for the following examples:

Ex: 2,4,6,8,10,... Ex: $\frac{2}{5}, \frac{9}{10}, \frac{7}{5}, \frac{19}{10} \ldots$ Ex: 4,8,12,16,...

$a_n = 2n + 0$ $a_n = \frac{1}{2}n - \frac{1}{10}$ $a_n = 4n + 0$

Ex 1: The fifth term of an arithmetic sequence is 25 and the 12^{th} term is 60. Write the first several terms of this sequence.

$a_5 = 25$ $a_{12} = 60$
$a_{12} = a_5 + 7d$ Where 7 is the difference in the term numbers.
$60 = 25 + 7d$
$35 = 7d$
$5 = d$

Since $a_5 = 25$ we can subtract 5 to get each term in the sequence down to the first.
5, 10, 15, 20, 25

Ex 2: Find the eighth term of an arithmetic sequence that begins with 1 and 7.

$d = 7 - 1 = 6$

Method 1: Write out the first 8 terms. 1, 7, 13, 19, 25, 31, 37, 43

Method 2: Find the nth term by finding c.

$a_n = dn + c$ $\qquad c = a_1 - d \qquad c = 1 - 6$ or -5

$a_n = 6n - 5$

So $a_8 - 6(8) - 5 = 43$

Ex 3: Find a rule for the nth term and fill in the missing terms.

$3d = 18$

____, __4__, ____, ____, __22__, ____

$d = 6$ $\qquad a_n = 6n - 8$

Ex 4: $a_7 = 34$ $a_{18} = 122$ Write a rule for the nth term. $\quad 11d = 88 \quad d = 8$

$a_n = 8n - 22$

Arithmetic means: the terms between any two nonconsecutive terms of an arithmetic sequence.

The terms between 2 given terms of an arithmetic sequence are called *arithmetic means*.

$10, \underbrace{13, 16, 19}_{\text{3 arithmetic means}}, 22$ $\qquad\qquad 10, \underbrace{14, 18}_{\text{2 arithmetic means}}, 22$

Ex 5. Insert 4 arithmetic means between 15 and 50.

15, __22__, __29__, __36__, __43__, 50 $\qquad 5d = 35$, $d = 7$

Ex 6: Form an arithmetic sequence that has six arithmetic means between -12 & 23.

-12 __ __ __ __ __ __ 23 $\qquad 7d = 35$, $d = 5$

Summation Notation: the sum of a sequence is also known as an **Arithmetic Series**.

$$\sum_{k=1}^{m} c_k = c_1 + c_2 + c_3 + \ldots + c_m$$

Sigma Notation: the sum of the first n terms of a sequence (called a series)

Ex: $\displaystyle\sum_{i=2}^{5} 4i$ $\quad 8 + 12 + 16 + 20 =$

$= 56$

Ex: $\displaystyle\sum_{k=3}^{8} (4 - k)$ $\quad 1 + 0 + -1 + -2 +$

$-3 + -4$

$= -9$

Partial Sums of an **Arithmetic Sequence**:

$S_n = \frac{n}{2}(a_1 + a_n)$ This means that we add the first and last terms, then multiply by the number of terms divided by 2.

Ex: Find the sum of 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20.

We know that this is an arithmetic sequence because d = 2 and there are 10 terms.

$n = 10$, $a_1 = 2$ and $a_n = 20$

$$S_n = \frac{10}{2}(2 + 20) = 5(22) = 110$$

Gauss: asked to add number from 1 to 100 in 3rd grade.

1 + 2 + 3 + 4 + 5 + … + 98 + 99 + 100

101

101

101

(50 pairs)(101) = 5050 or _____________

Ex: Find the sum of the integers from 1 to 500.

$S_n = \frac{n}{2}(a_1 + a_n)$ $n = 500$, $a_1 = 1$ and $a_n = 500$

$$S_n = \frac{500}{2}(1 + 500) = 250(501) = 125,250$$

Ex: Find the sum of the first 200 terms of the arithmetic sequence: 3, 7, 11, 15…..

$S_n = \frac{n}{2}(a_1 + a_n)$ $n = 200$, $a_1 = 3$, $a_n = ?$, and $d = 4$

1) Find a_n

$a_n = dn + c$ Where $c = 3 - 4 = -1$

$a_n = 4n + -1$

2) Find the 200th term

$a_{200} = 4(200) - 1 = 800 - 1 = 799$

3) Find S_n

$$S_n = \frac{200}{2}(3 + 799) = 100\,(802) = 80,200$$

Ex: Find the sum of the arithmetic series where $a_1 = -111$, $d = 3$, and $a_n = 9$

$S_n = \frac{41}{2}(-111 + 9)$

$= -2091$

$-111 \quad -108 \quad -105 \quad -102 \quad \_ \quad \_ \quad 9$

$a_n = 3n - 114$

$9 = 3n - 114$

$n = 41$

Ex: Find the first three terms of the arithmetic series where $a_1 = 10$, $a_n = -46$, and $S_n = -522$.

10, 8, 6

$-522 = \frac{n}{2}(10 + -46)$

$-522 = -18n \quad n = 29$

$a_1 = 10 \quad a_{29} = -46$

$d = \frac{-46 - 10}{29 - 1} = \frac{-56}{28} = -2$

Recursive Sequences: A formula for a sequence that gives the value of a term t_n in terms of the preceding term t_{n-1}. The first term is represented by t_1, the second term in represented by t_2, the third term in represented by t_3, and so forth.

Ex: Find the eighth term of an arithmetic sequence that begins with 1 and 7.

Find the next three terms in each sequence.

a. 4, 9, 14, 19, …

24, 29, 34

b. 6, -3, 1.5, -.75,…

3/8, -3/16, 3/32

c. 0, 3, 7, 12, 18, …

25, 33, 42

d. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

$\frac{1}{64}, \frac{1}{128}, \frac{1}{256}$

Ex: If $t_1 = 22$ & $t_n = t_{n-1} - 3$, find the next three terms.

$t_2 = 19$ $t_3 = 16$ $t_4 = 13$

Ex: If $t_1 = 3$ and $t_n = t_{n-1} + 4$, find the next five terms in the sequence.

$t_2 = 7$ $t_3 = 11$ $t_4 = 15$

Ex: If $t_1 = 64$ and $t_n = \frac{1}{2} t_{n-1}$, find the next four terms.

$t_2 = 32$ $t_3 = 16$ $t_4 = 8$

Ex: The winner of a contest received \$200 the first year, with a 25% increase over the preceding year's payment for each subsequent year. How much did the contest winner receive during the first 10 years of payments?

$a_1 = 200$ $a_4 = 390.625$ $a_7 = 762.94$ $a_{10} = 1490.12$

$a_2 = 250$ $a_5 = 488.28$ $a_8 = 953.67$

$a_3 = 312.5$ $a_6 = 610.35$ $a_9 = 1192.09$

Ex: The owners of a certain store reduce the price of their items at the end of each week. If the original price of a blouse is \$250 and its price each week is 4/5 of the previous week, what will be the price of the blouse at the end of the 10th week?

4

end week 1 = 200

2 = 160

3 = 128

4 = 102.40

Geometric Sequence: the ratio of any term to the previous term is constant.

r = common ratio

Ex: $a_n = 2^n = 2, 4, 8, 16\ldots$ Ex: $a_n = \left(\frac{-1}{4}\right)^4 = \frac{-1}{4}, \frac{1}{16}, \frac{-1}{64}, \frac{1}{256}\ldots$

Finding the nth term of a geometric sequence:

$$\mathbf{a_n = a_1 r^{n-1}} \Rightarrow \cancel{a_n = a_0 r^n}$$

Ex: Find the first 5 terms of the geometric sequence whose first term is $a_1 = 4$ and whose ratio is r = 3.

4, 12, 36, 108, 324

Ex: Find the 18[th] term of the geometric sequence whose first term is 20 and whose common ratio is 1.4.

$a_n = 20(1.4)^{n-1}$

$a_{18} = 20(1.4)^{17} = 6098.27$

Ex: Write a rule for the n[th] term. –8, -12, -18, -27, …

$a_n = -8(3/2)^{n-1}$

Ex: $a_4 = 3$, r = 3. Write a rule for the n[th] term.

$\underline{1/9}$ $\underline{1/3}$ $\underline{1}$ $\underline{3}$ $a_n = 1/9\,(3)^{n-1}$

Geometric Means: the terms between any two nonconsecutive terms of a geometric sequence.

Geometric Mean: $x = \sqrt{ab}$ Recall from Geometry: $\frac{x}{a} = \frac{b}{x}$

Ex: Insert 2 geometric means between 8 and 512.

$8, \underbrace{\;}_{r}\underline{32}, \underbrace{\;}_{r}\underline{128}, \underbrace{\;}_{r}512$

$8r^3 = 512$

$r^3 = 64$ $r = 4$

Geometric series

$$s_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

r = common ratio n = number of terms

works for a finite sequence

Ex: Find the sum of the first 10 terms of the geometric series 1 + 5 + 25 + 125 + 625 …

Ex: Find the sum of the 1[st] 8 terms of the geometric series where $a_1 = 8$ and $a_4 = 512$.