Rolle's Theorem:

If f is continuous on the closed interval $[a, b]$, and differentiable on the open interval (a, b)

If then $f(a) = f(b)$ then there is at least one number c in (a, b) such that $f'(c) = 0$.

(The graph has to turn around somewhere in between (a, b))

I. Find the intercepts of the given function and show that $f'(x) = 0$
at some point between the intercepts.

Ex: $f(x) = x^2 - x - 20$

$(x-5)(x+4)$

$x=5 \quad x=-4$

$f'(x) = 2x - 1 \qquad x = 1/2$

$x = 1/2 \quad [-4, 5]$

Ex: $f(x) = 2x^3 - x^2 - 13x - 6$

$(x+2)(2x+1)(x-3)$

$f'(x) = 6x^2 - 2x - 13$

$x = \dfrac{2 \pm \sqrt{4 - 4(6)(-13)}}{12} = \dfrac{2 \pm 2\sqrt{79}}{12}$

$x = \dfrac{2+2\sqrt{79}}{12} \quad [-1/2, 3] \qquad x = \dfrac{2-2\sqrt{79}}{12} \quad [-2, -1/2]$

II. Given an interval, find the c that satisfies Rolle's Theorem:

Ex: $f(x) = \dfrac{x^2 - 2x - 15}{-x+6}; [-3, 5]$

$f(-3) = 0 \quad f(5) = 0$

$f'(x) = (-x+6)\cdot(2x-2) - (x^2+2x+15)\cdot -1$

$= \cancel{-2x^2} + \cancel{2x} + \cancel{12x} - 12 + \cancel{x^2} - \cancel{2x} - 15$

$= -x^2 + 12x - 27 \qquad x^2 - 12x + 27$

$(x-3)(x-9)$

$x=3 \quad [-3, 5]$

Ex: $f(x) = -2\sin(x); [-\pi, \pi]$

$f(\pi) = 0 \quad f(-\pi) = 0$

$f'(x) = -2\cos(x)$

$x = -\pi/2 \quad x = \pi/2 \quad [-\pi, \pi]$

III. For each problem, determine if Roll's Theorem can be applied. If so, find all c values.

Ex: $f(x) = \dfrac{4 - x^2}{4x}; [-2, 2]$

discontinuous at

$x=0$

Ex: $f(x) = \tan(x); [0, \pi]$

discontinuous at

$x = \pi/2$

Ex: $f(x) = 2\cos(x); [-\pi, \pi]$

$f(-\pi/2) = f(\pi/2)$

$f'(x) = -2\sin x$

$0 = -2\sin x$

$x = 0$

$x=0 \quad [-\pi, \pi]$

Mean Value Theorem:

If f is continuous on the closed interval $[a, b]$, and differentiable on the open interval (a, b),

then there exists a number c in (a, b) such that

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$

The average rate of change will equal the instantaneous rate at some point on the interval.

In other words, the slope of the secant line equals the slope of the tangent line.

Ex: Cooking a hamburger: A hamburger is pulled out of the refrigerator at 50 degrees. The burger is cooked on a grill set at 500 degrees for three minutes. At some point, the temperature of the hamburger must be 150 degrees.

For the following, find the c value that satisfies the Mean Value Theorem.

Show slope of secant = tangent

Ex: $f(x) = x^3 - 9x^2 + 24x - 18; [2, 4]$

$$\frac{f(4) - f(2)}{4 - 2} = -2$$

$$-2 = 3x^2 - 18x + 24$$

$$x = \frac{18 \pm \sqrt{12}}{6} = \frac{9 \pm \sqrt{3}}{3}$$

Ex: $f(x) = \frac{x^2 - 9}{3x}; [1, 4]$

$$\frac{\frac{7}{12} - -\frac{8}{3}}{3} = \frac{13}{12}$$

$$\frac{3x(2x) - (x^2 - 9)3}{9x^2} = \frac{13}{12}$$

$$x = \pm 2$$

$$x = 2$$

Ex: $f(x) = \frac{5x - 4}{x}; [1, 4]$

$$\frac{4 - 1}{4 - 1} = 1$$

$$\frac{x \cdot 5 - (5x - 4)}{x^2} = \frac{4}{x^2}$$

$$x = \pm 2$$

Ex: $f(x) = \cos(x); [0, \pi]$

$$\frac{-1 - 1}{\pi - 0} = \frac{-2}{\pi}$$

$$-\sin x = -2/\pi$$

$$\sin x = 2/\pi$$

$$x = \arcsin(2/\pi)$$