Notes on Related Rates AB 2016

Derivatives tell us how fast the amount of something is changing. In other words, its rate of change.

When you are working these problems take notice of which quantities are changing and which are constant. **(Be sure to treat changing quantities like variable and constant like constants.)**

Also take the derivatives BEFORE you plug in temporary constants.

Strategy:
1. Read the problem and write down knowns and unknowns. Draw pic if appropriate.
2. Write the equation that relates the variables representing the quantity that you know, the rate of change of, to the quantity that you are looking for the rate of change of.
3. Eliminate any variable that you don't know the rate of change of and aren't looking for the rate of change of. (Not by erasing it ☺)
4. Differentiate both sides with respect to time.
5. Plug in knowns and solve for unknowns, answer the question with correct units.

EXAMPLES:

1. If you know that $y = x^2 + 3$ and we know $\frac{dx}{dt} = 2$ when x = 1, find $\frac{dy}{dt}$.

   What if you knew that $\frac{dy}{dt} = 8$, what would $\frac{dx}{dt} = ?$ when $x = 5$

   $\frac{dy}{dt} = 2x\frac{dx}{dt}$ $\quad 8 = 2(5)\frac{dx}{dt}$

   $\frac{dy}{dt} = 2(1)(2)$ $\quad \frac{dx}{dt} = \frac{8}{10}$

   $= 4$

2. A stone is dropped into a pond, forming concentric circles. We know that $\frac{dr}{dt} = 1\,ft/\sec$. Find the change in the area $\frac{dA}{dt}$ when the radius is 4 feet.

   $A = \pi r^2$

   $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

   $= 2\pi(4)\cdot(1)$

   $\frac{dA}{dt} = 8\pi\ in^2/sec$

3. Air is pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius of the balloon when the radius is 6 feet.

$V = \frac{4}{3}\pi r^3$

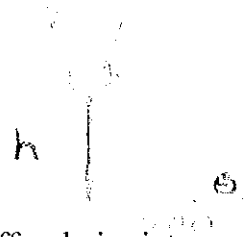
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4.5 = 144\pi \cdot \frac{dr}{dt}$

$\frac{4.5}{144\pi} = \frac{dr}{dt}$

$\frac{dr}{dt} = .0099$ ft/min

4. A hot air balloon, rising straight up from a level field, is tracked by a range finder 500 feet from its lift off spot. At the moment that the angle of elevation of the range finder is $\frac{\pi}{4}$, the angle is increasing at the rate of 0.14 radians/minute. How fast is the balloon rising then?



$\frac{d\theta}{dt} = .14$ rads/min $\quad \theta = \frac{\pi}{4}$

$\tan\theta = \frac{h}{500}$

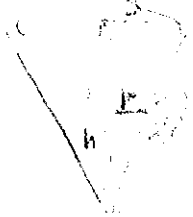
$500\tan\theta = h$

$500\sec^2\theta \frac{d\theta}{dt} = \frac{dh}{dt}$

$1000 \cdot .14 = 140$

$\frac{dh}{dt} = 140$ ft/min

5. Coffee drains into a very large conical coffee cup at a rate of 9 cubic feet per minute. (The cone is inverted or vertex down ☺) The cup has a height of 10 feet and a base radius of 5 feet. How fast is the coffee level changing when it is 6 feet deep, again ☺.



$V = \frac{1}{3}\pi r^2 h$

$\frac{r}{h} = \frac{5}{10}$

$10r = 5h$

$r = \frac{1}{2}h$

$V = \frac{1}{3}\pi(\frac{1}{2}h)^2 h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

when $h = 6$

$9 = \frac{1}{4}\pi(6)^2 \frac{dh}{dt}$

$\frac{1}{\pi} = \frac{dh}{dt}$

ft/min

6. How fast does the water level drop when a cylindrical tank is drained at 3 liters per second?

(1 ml = 1 cubic cm)

3 liters = 3000 cm^3

3000 cm^3

$\frac{dV}{dt} = -3000$ cm^3/sec $\quad \frac{dh}{dt} = ?$

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$

$-3000 = \pi r^2 \frac{dh}{dt}$

$\frac{-3000}{\pi r^2} = \frac{dh}{dt}$

cm/sec

radius of a cylinder does not change.

7. Two trucks convoys leave a truck stop, at the same time, with convoy A traveling east at 40 mph and convoy B traveling north at 30 mph. How fast are the convoys separating 6 minutes later?

$\frac{1}{10} \cdot 30 = 3$ mph

$\frac{6}{60} = \frac{1}{10}$

$a^2 + b^2 = c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

$2(4)(30) + 2(3)(40) = 2(5)\frac{dc}{dt}$

$\frac{dc}{dt} = 50$ mph