Rates of Change 11/17/2014

When you integrate a rate of change you get an amount of change! This is important and has many applications. If you integrate a velocity curve (positive in the interval) you get the distance traveled.

Ex1. The tide moves sand from Sandy Point Beach at a rate modeled by the function R, given by $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$. A pumping station adds sand to the beach at a rate modeled by the function S, given by $S(t) = \frac{15t}{1+3t}$. Both R(t) and S(t) have units of cubic yards per hour and t is measured in hours for $0 \leq t \leq 6$. At time t = 0, the beach contains 2500 cubic yards of sand.

a. How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

b. Write an expression for Y(t), the total number of cubic yards of sand on the beach at time t.

c. Find the rate at which the total amount of sand on the beach is changing at time t = 4.

a. For $0 \leq t \leq 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

Ex2. At an intersection in Thomasville, Oregon, cars turn left at the rate

$L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours.

a. To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

b. Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

c. Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

3. A particle starts at the point (5,0) at t=0 and moves along the x-axis in such a way that at time t > 0 its velocity v(t) is given by $v(t) = \frac{t}{1+t^2}$.

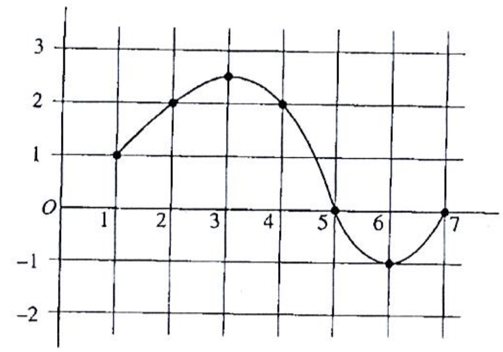
a. Determine the maximum velocity attained by the particle. Justify your answer.

b. Determine the position of the particle at t = 6.

c. Find the limiting value of the velocity as t increases without bound.

4. The graph of a differentiable function f on the closed interval [1,7] is shown.

Let h(x) = ∫ f(t) dt for $1 \le x \le 7$.



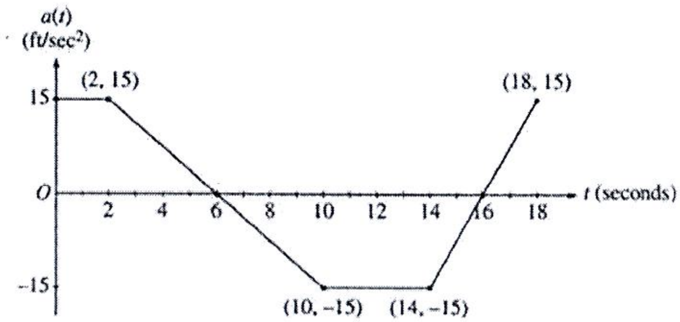
a. Find h(1).

b. Find h′ (4).

c. On what interval or intervals is the graph of h concave upward? Justify your answer.

d. Find the value of x at which h has its minimum on the closed interval [1,7]. Justify your answer

5.



A car traveling on a straight road with velocity 55 ft/sec at time t = 0. For $0 \leq t \leq 18$ seconds, the car's acceleration a(t), in ft/sec², is the piecewise linear function defined by the graph above.

a. Is the velocity of the car increasing at t = 2 seconds? Why or why not?

b. At what time in the interval $0 \leq t \leq 18$, other than t = 0, is the velocity of the car 55 ft/sec? Why?

c. On the time interval $0 \leq t \leq 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

d. At what times in the interval $0 \leq t \leq 18$, if any, is the car's velocity equal to zero? Justify your answer.