14.1 Part I Counting Principle Worksheet

I. State whether the events are independent or dependent.

1. Choosing a president, vice president, secretary, and treasurer for Student Council, assuming that a person can hold only one office.

   Dependent

2. Selecting a fiction book and a nonfiction book at the library.

   Independent

3. Each of six people guess the total number of points scored in a basketball game. Each person writes down his or her guess without telling what it is.

   Independent

4. The letters A through Z are written on pieces of paper and placed in a jar. Four of them are selected one after the other without replacing any of them.

   Depending

II. Solve each problem.

5. Tim wants to buy one of three different albums he sees in a music store. Each is available on tape and CD. How many combinations of album and format does he have to choose?

   $3 \cdot 2 = 6$

6. A video store has 8 new releases this week. Each is available on DVD and Blue Ray. How many ways can a customer choose a new release and a format to rent?

   $8 \cdot 2 = 16$

7. Carlos has homework to do in Math, Chemistry, and English. How many ways can he choose the order in which to do his homework?

   $3 \cdot 2 \cdot 1 = 6$

8. The menu for a banquet has a choice of 2 types of salad, 5 main courses, and 3 desserts. How many ways can a salad, main course, and dessert be selected to form a meal?

   $2 \cdot 5 \cdot 3 = 30$

9. A golf club manufacturer makes drivers with 4 different shafts lengths, 3 different lofts, 2 different grips, and 2 different club head materials. How many different combinations are possible?

   $4 \cdot 3 \cdot 2 \cdot 2 = 48$

10. Each question on a five-question multiple-choice quiz has answer choices labeled A, B, C, and D. How many different ways can a student answer the five questions?

$\underline{4}\ \underline{4}\ \underline{4}\ \underline{4}\ \underline{4} \quad 4^5 = 1024.$

11. How many different ways can six different books be arranged on a shelf if one of the books is a dictionary and it must be on an end?

$\underline{D}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} \; +$

$\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{D}$

or $2 \cdot 5! = 240$

12. In how many orders can eight actors be listed in the opening credits of a movie if the leading actor must be listed first or last?

$2 \cdot 7! = 10{,}080$

13. Abby is registering at a Web Site. She must select a password containing 6 numerals to be able to use the site. How many passcodes are allowed if no digits may be used more than once?

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 6! = 720$

14. How many different 5 digit codes are possible if the first digit cannot be 0 and no digit may be used more than once?

$\underset{\neq 0}{\underline{9}}\ \underline{9}\ \underline{8}\ \underline{7}\ \underline{6} = 27216$

pick up the 0

15. How many different 3 letter, 4 digit license plates are there if letters can be repeated but the numbers can't?

$\underline{26}\ \underline{26}\ \underline{26}\ \underline{10}\ \underline{9}\ \underline{8}$

$= 12{,}654{,}720$

16. How many numbers between 100 and 999, inclusive, have 7 in the tens place?

$\underline{1}\ \underline{7}\ \underline{10} = 10$

one hundred

$\underline{2}\ \underline{7}\ \underline{10} = 10$

two hundred

$\longrightarrow \underline{9} \cdot \underline{1} \cdot \underline{10}$

90 ways

17. A coin is tossed four times. How many possible sequences of heads or tails are possible?

$\underline{2}\ \underline{2}\ \underline{2}\ \underline{2} = 2^4 = 16$

18. A coin is tossed 6 times and a die is rolled 3 times. How many possible outcomes are possible?

$2\ 2\ 2\ 2\ 2\ 2 \cdot 6 \cdot 6 \cdot 6 = 1{,}728$

14.1 Part II Permutations and 14.2 Permutations with Repetitions & Circular Permutations Notes

1. **Permutation:** A permutation of n differenct elements is an ordering of the elements such that one element is first, one is second, one is third, and so on. **ORDER MATTERS!!**

2. Permutation is an ordered arrangement of items that occurs when
   a. No item is used more than once.
   b. The order of arrangement makes a difference

Ex: There are 10 finalists in a figure skating competition. How many ways can gold, silver, and bronze medals be awarded?

$$10 \cdot 9 \cdot 8 = 720$$

Ex: You and 19 friends have decided to form an Internet marketing consulting firm. The group needs to choose three officers-a CEO, an operating manager, and a treasurer. In how many ways can those offices be filled?

$$20 \cdot 19 \cdot 18 = 6,840$$

3. **Permutation Formula:** ${}_nP_r = \dfrac{n!}{(n-r)!}$ or $P(n,r) = \dfrac{n!}{(n-r)!}$

$${}_5P_3 = P(5,3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = 60$$

4. Eight people enter the Best Pie contest. How many ways can blue, red, and green ribbons be awarded?

$${}_8P_3 = \frac{8!}{5!} = 8 \cdot 7 \cdot 6 = 336$$

5. Suppose you want to rearrange the letters of the word ALGEBRA to see if you can make a different arrangement. If the two A's were not identical, the seven letters in the word could be arranged in P(7,7) or 7! ways. But since the A's are identical, what are we going to do?

${}_7P_7$ same thing.

$$\frac{7!}{2!} = 2520$$

$2!$ ← repeating A's

6. **Permutations with Repetitions**:

Find the number arrangements for the word:

A. BANANA

$$\frac{6!}{3!\,2!} = 60$$

A's, N's

B. BASEBALL

$$\frac{8!}{2!\,2!\,2!} = 5040$$

C. MISSISSIPPI

$$\frac{11!}{4!\,4!\,2!} = 34,650$$

I's, S's, P's

7. How many ways can 4 Math books and 5 English books be put on a shelf if all the math books have to put together?

Math 4!

imagine math book like a block, so 6 places for the block.

English → $5! \, 4! \cdot 6 = 17,280$

8. How many ways can 4 Math books and 5 English books be put on a shelf if all the math books and the English books have to be put together?

$4! \cdot 5! \cdot 2 = 5760$

4! math

5! English

can be Math then Eng or Eng then Math (2 positions)

9. **Circular Permutations:** If n objects are arranged in a circle, then there are $\frac{n!}{n}$ or $(n-1)!$ permutations of the n objects around the circle; they do not have a beginning or an end.

P A J B C   A B P C J   B C A J P ← All the same arrangements.

10. On the buffet there are 7 different appetizers from which to choose. The appetizers are arranged on a revolving tray. How many ways can the appetizers be organized?

$$\frac{7!}{7} = 6! = 720$$

When you rotate the order doesn't change.

11. How many ways can 8 people be seated at a square table?

$$\frac{8!}{8} = 7! = 5040$$

12. How many ways can 5 men and 5 women be seated at a round table if they have to alternate the men and women?

Sit men   Women

$$\frac{5!}{5} = 4! \cdot 5! = 2880$$

If n objects on a circle are arranged in relation to a fixed point, then there are $n!$ Permutations.

If the arrangement can be physically turned over or flipped over, the reflection of the arrangement is possible, divide by 2. (For Example, a key ring can be flipped over but a football team in a huddle cannot)

13. How many ways can 7 beads be placed on a bracelet with no clasp?

$$\frac{7!}{7 \cdot 2} = \frac{6!}{2} = 360$$

can be flipped

14. How many ways can 7 beads be placed on a bracelet that has a clasp?

↑ reference — no longer circular

$$\frac{7!}{2} = 2520$$

Homework: Finish Permutation Worksheet from class, pg. 755 #5 – 34, 37 – 40, 46

14.3 Combinations

1. **Combination:** An arrangement of objects in which order is **NOT** important

2. **Combination Formula:** The number of combinations of n objects taken r at a time, is written C(n, r).

$$_nC_r = \frac{n!}{(n-r)! \bullet r!} \text{ or } C(n,r) = \frac{n!}{(n-r)! \bullet r!}$$

$$\boxed{\text{Notice } {}_7C_3 = {}_7C_4 \qquad \frac{7!}{4!\,3!} = \frac{7!}{3!\,4!}}$$

Ex: There are 4 students on student council that want to be on the recycling committee. How many different groups can be chosen when the recycling committee has 3 openings?

$$C(4,3) = {}_4C_3 = \frac{4!}{1!\,3!} = 4$$

Ex: A group of seven students working on a project needs to choose two from their group to present the group's report to the class. How many ways can they chose the two students?

$$_7C_2 = \frac{7!}{5!\,2!} = \frac{7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!} \cdot 2} = 21$$

Ex: Five cousins at a family reunion decide that three of them will go to pick up a pizza. How many ways can they choose the three people to go?

$$_5C_3 = \frac{5!}{2!\,3!} = \frac{5 \cdot \cancel{4}^{2} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot 1 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 10$$

Ex: From a row of 8 different varieties of soup on a shelf, how many groups of 5 cans can be selected?

$$_8C_5 = \frac{8!}{3!\,5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2} = 56$$

Ex: An ice cream shop has eight different toppings from which to choose. This week, if you buy three toppings for your sundae, you get another topping free. How many different ways can the sundae be made (with 4 toppings)?

$$_8C_4 = \frac{8!}{4!\,4!} = 70$$

For each of the following, explain whether the problem is one involving permutations or combinations.

a. Six candidates are running for president, chief technology officer, and director of marketing of an Internet company. The candidate with the greatest number of votes becomes the president, the second biggest vote-getter becomes chief technology officer, and the candidate who gets the third largest number of votes will be the director of marketing. How many different outcomes are possible for these three positions? Titles mean order

Permutation

b. From the six candidates who desire to hold office in an Internet company, a three-person committee is formed to study ways of finding new investors. How many different committees could be formed?

Combination

c. How many ways can you select 6 free videos from a list of 200 videos?

Combination

d. In a race in which there are 50 runners and no ties, in how many ways can the first three finishers come in?

Permutation