PS AB 2017
due 4-24

CALCULUS AB

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $f(x) = 5x^{\frac{4}{3}}$, then $f'(8) =$

(A) 10 (B) $\frac{40}{3}$ (C) 40 (D) 80 (E) $\frac{160}{3}$

---

2. $\lim_{x \to \infty} \frac{5x^3 - 3x + 1}{4x^2 + 2x + 5}$ is

(A) 0 (B) $\frac{4}{5}$ (C) $\frac{3}{11}$ (D) $\frac{5}{4}$ (E) ∞

3. If $f(x) = \frac{3x^2 + x}{3x^2 - x}$ then $f'(x)$ is

(A) 1

(B) $\frac{6x^2 + 1}{6x^2 - 1}$

(C) $\frac{-6}{(3x-1)^2}$

(D) $\frac{-2x^2}{(x^2 - x)^2}$

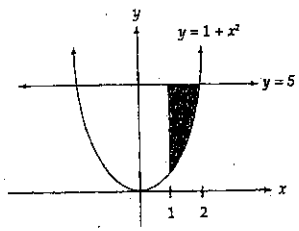
(E) $\frac{36x^3 - 2x}{(x^2 - x)^2}$

---

4. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$, then $f(4) =$

(A) 1 (B) $\frac{8}{7}$ (C) –1 (D) 0 (E) undefined

NO 5 ☺



6. Which of the following integrals correctly corresponds to the area of the shaded region in the figure above ?

(A) $\int_1^2 (x^2 - 4)dx$

(B) $\int_1^2 (4 - x^2)dx$

(C) $\int_1^5 (x^2 - 4)dx$

(D) $\int_1^5 (x^2 + 4)dx$

(E) $\int_1^5 (4 - x^2)dx$

---

7. If $f(x) = \sec x + \csc x$, then $f'(x) =$

(A) 0
(B) $\sec^2 x + \csc^2 x$
(C) $\csc x - \sec x$
(D) $\sec x \tan x + \csc x \cot x$
(E) $\sec x \tan x - \csc x \cot x$

8. An equation of the line normal to the graph of $y = \sqrt{(3x^2 + 2x)}$ at (2, 4) is

(A) $-4x + y = 20$ (B) $4x + 7y = 20$ (C) $-7x + 4y = 2$ (D) $7x + 4y = 30$ (E) $4x + 7y = 36$

---

9. $\int_{-1}^{1} \frac{4}{1 + x^2} dx =$

(A) 0 (B) π (C) 1 (D) 2π (E) 2

---

10. If $f(x) = \cos^2 x$, then $f''(\pi) =$

(A) –2 (B) 0 (C) 1 (D) 2 (E) 2π

11. If $f(x)=\frac{5}{x^2+1}$ and $g(x)=3x$, then $g(f(2))=$

(A) -3 (B) $\frac{5}{37}$ (C) 3 (D) 5 (E) $\frac{37}{5}$

12. $\int x\sqrt{5x^2-4}\,dx=$

(A) $\frac{1}{10}(5x^2-4)^{\frac{3}{2}}+C$

(B) $\frac{1}{15}(5x^2-4)^{\frac{3}{2}}+C$

(C) $-\frac{1}{5}(5x^2-4)^{-\frac{1}{2}}+C$

(D) $\frac{20}{3}(5x^2-4)^{\frac{3}{2}}+C$

(E) $\frac{3}{20}(5x^2-4)^{\frac{3}{2}}+C$

---

13. The slope of the line tangent to the graph of $3x^2+5\ln y=12$ at (2, 1) is

(A) $-\frac{12}{5}$ (B) $\frac{12}{5}$ (C) $\frac{5}{12}$ (D) 12 (E) -7

---

14. The equation $y=2-3\sin\frac{\pi}{4}(x-1)$ has a fundamental period of

(A) $\frac{1}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{4}{\pi}$ (D) 8 (E) 2π

15. If $f(x)=\begin{cases}x^2+5 \text{ if } x<2\\ 7x-5 \text{ if } x\geq 2\end{cases}$, for all real numbers x, which of the following must be true?

I. $f(x)$ is continuous everywhere.
II. $f(x)$ is differentiable everywhere.
III. $f(x)$ has a local minimum at $x=2$.

(A) I only (B) I and II only (C) II and III only (D) I and III only (E) I, II, and III

---

16. For what value of x does the function $f(x)=x^3-9x^2-120x+6$ have a local minimum?

(A) 10 (B) 4 (C) 3 (D) -4 (E) -10

17. The acceleration of a particle moving along the x-axis at time t is given by $a(t)=4t-12$. If the velocity is 10 when $t=0$ and the position is 4 when $t=0$, then the particle is changing direction at

(A) $t=1$
(B) $t=3$
(C) $t=5$
(D) $t=1$ and $t=5$
(E) $t=1$ and $t=3$ and $t=5$

18. The average value of the function $f(x)=(x-1)^2$ on the interval from $x=1$ to $x=5$ is

(A) $-\frac{16}{3}$ (B) $\frac{16}{3}$ (C) $\frac{64}{3}$ (D) $\frac{66}{3}$ (E) $\frac{256}{3}$

---

19. $\int\left(e^{3\ln x}+e^{3x}\right)dx=$

(A) $3+\frac{e^{3x}}{3}+C$

(B) $\frac{x^4}{4}+3e^{3x}+C$

(C) $\frac{e^{x^4}}{4}+3e^{3x}+C$

(D) $\frac{e^{x^4}}{4}+\frac{e^{3x}}{3}+C$

(E) $\frac{x^4}{4}+\frac{e^{3x}}{3}+C$

20. If $f(x)=\sqrt{(x^3+5x+121)}\,(x^2+x+11)$ then $f'(0)=$

(A) $\frac{5}{2}$ (B) $\frac{27}{2}$ (C) 22 (D) $22+\frac{2}{\sqrt{5}}$ (E) $\frac{247}{2}$

---

21. If $f(x)=5^{3x}$ then $f'(x)=$

(A) $5^{3x}(\ln 125)$ (B) $\frac{5^{3x}}{3\ln 5}$ (C) $3(5^{3x})$ (D) $3(5^{3x})$ (E) $3x(5^{3x-1})$

22. A solid is generated when the region in the first quadrant enclosed by the graph of $y = (x^2+1)^3$, the line $x = 1$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals?

(A) $\pi\int_1^8 (x^2+1)^3\,dx$

(B) $\pi\int_1^8 (x^2+1)^6\,dx$

(C) $\pi\int_0^1 (x^2+1)^3\,dx$

(D) $\pi\int_0^1 (x^2+1)^6\,dx$

(E) $2\pi\int_0^1 (x^2+1)^6\,dx$

23. $\lim_{x\to 0} 4\,\dfrac{\sin x\cos x - \sin x}{x^2}$

a) 2 b) $\frac{40}{3}$ c) ∞ d) 0 e) undefined

24. If $\frac{dy}{dx} = \frac{(3x^2+2)}{y}$ and $y = 4$ when $x = 2$, then when $x = 3$, $y =$

(A) 18 (B) $\pm\sqrt{66}$ (C) 58 (D) $\pm\sqrt{74}$ (E) $\pm\sqrt{58}$

---

25. $\int \frac{dx}{9+x^2} =$

(A) $3\tan^{-1}\left(\frac{x}{3}\right)+C$

(B) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

(C) $\frac{1}{9}\tan^{-1}\left(\frac{x}{3}\right)+C$

(D) $\frac{1}{3}\tan^{-1}(x)+C$

(E) $\frac{1}{9}\tan^{-1}(x)+C$

26. If $f(x) = \cos^3(x+1)$ then $f'(\pi) =$

(A) $-3\cos^2(\pi+1)\sin(\pi+1)$

(B) $3\cos^2(\pi+1)$

(C) $3\cos^2(\pi+1)\sin(\pi+1)$

(D) $3\pi\cos^2(\pi+1)$

(E) 0

27. $\int x\sqrt{x+3}\,dx =$

(A) $\frac{2}{3}(x)^{\frac{3}{2}} + 6(x)^{\frac{1}{2}} + C$

(B) $\frac{2(x+3)^{\frac{3}{2}}}{3} + C$

(C) $\frac{2}{5}(x+3)^{\frac{5}{2}} - 2(x+3)^{\frac{3}{2}} + C$

(D) $\frac{3(x+3)^{\frac{3}{2}}}{2} + C$

(E) $\frac{4x^2(x+3)^{\frac{3}{2}}}{3} + C$

28. $f(x) = \ln(\ln(1-x))$

$f'(x) =$

CALCULUS AB

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. $\int_0^{\frac{\pi}{4}} \sin x\,dx + \int_{-\frac{\pi}{4}}^{0} \cos x\,dx =$

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

30. Boats A and B leave the same place at the same time. Boat A heads due north at 12 km/hr. Boat B heads due east at 18 km/hr. After 2.5 hours, how fast is the distance between the boats increasing (in km/hr)?

(A) 21.63 (B) 31.20 (C) 75.00 (D) 9.84 (E) 54.08

---

31. $\lim_{h\to 0} \dfrac{\tan\left(\frac{\pi}{6}+h\right) - \tan\left(\frac{\pi}{6}\right)}{h} =$

(A) $\frac{\sqrt{3}}{3}$ (B) $\frac{4}{3}$ (C) $\sqrt{3}$ (D) 0 (E) $\frac{3}{4}$

32. If $\int_{30}^{100} f(x)dx = A$ and $\int_{50}^{100} f(x)dx = B$, then $\int_{30}^{50} f(x)dx =$

(A) $A + B$ (B) $A - B$ (C) 0 (D) $B - A$ (E) 20

---

33. If $f(x) = 3x^2 - x$, and $g(x) = f^{-1}(x)$, then $g'(10)$ could be

(A) 59 (B) $\frac{1}{59}$ (C) $\frac{1}{10}$ (D) 11 (E) $\frac{1}{11}$

34. $y = x^3 - 5x^2 + 4x + 2$ has a local min at (___, ___)

35. Find the volume of solid generated by revolving the region bounded by $y = 9 - x^2$ and $y = 9 - 3x$ for $0 \le x \le 2$ is

36. The average value of $f(x) = \ln x$ on $[2,4]$ is

37. $\frac{d}{dx}\int_0^{3x} \cos(t)dt =$

a) $\sin(3x)$ b) $-3\sin(3x)$ c) $\cos(2x)$ d) $3\sin(3x)$ e) $3\cos(3x)$

38. If the definite integral $\int_1^3 (x^2+1)dx$ is approximated by using the Trapezoid Rule with $n = 4$, the error is

(A) 0 (B) $\frac{7}{3}$ (C) $\frac{1}{12}$ (D) $\frac{65}{6}$ (E) $\frac{97}{3}$

39. The radius of a sphere is increasing at a rate proportional to its radius. If the radius is 4 initially, and the radius is 10 after two seconds, what will the radius be after three seconds?

(A) 62.50 (B) 13.00 (C) 15.81 (D) 16.00 (E) 25.00

---

40. Use differentials to approximate the change in the volume of a sphere when the radius is increased from 10 to 10.02 cm.

(A) 4213.973 (B) 1261.669 (C) 1256.637 (D) 25.233 (E) 25.133

41. $\int \ln 2x \, dx =$

(A) $\frac{\ln 2x}{x} + C$

(B) $\frac{\ln 2x}{2x} + C$

(C) $x \ln x - x + C$

(D) $x \ln 2x - x + C$

(E) $2x \ln 2x - 2x + C$

45. $\int \tan^6 x \sec^2 x \, dx =$

(A) $\frac{\tan^7 x}{7} + C$

(B) $\frac{\tan^7 x}{7} + \frac{\sec^3 x}{3} + C$

(C) $\frac{\tan^7 x \sec^3 x}{21} + C$

(D) $7\tan^7 x + C$

(E) $\frac{2}{7}\tan^7 x \sec x + C$

---

42. If the function $f(x)$ is differentiable and $f(x) = \begin{cases} ax^3 - 6x; & \text{if } x \le 1 \\ bx^2 + 4; & x > 1 \end{cases}$, then $a =$

(A) 0 (B) 1 (C) −14 (D) −24 (E) 26

---

43. Two particles leave the origin at the same time and move along the y-axis with their respective positions determined by the functions $y_1 = \cos 2t$ and $y_2 = 4\sin t$ for $0 < t < 6$. For how many values of t do the particl have the same acceleration?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

44. Find the distance traveled (to three decimal places) in the first four seconds, for a particle whose velocity is given by $v(t) = 7e^{-t^2}$; where t stands for time.

(A) 0.976 (B) 6.204 (C) 6.359 (D) 12.720 (E) 7.000

Calc may be used:

onsider the equation $x^2 - 2xy + 4y^2 = 64$.

) Write an expression for the slope of the curve at any point (x,y).

) Find the equation of the tangent lines to the curve at the point $x = 2$.

) Find $\frac{d^2y}{dx^2}$ at $(0,4)$

---

particle moves along the x-axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position is $x(1) = 9$.

) Write an expression for the velocity of the particle $v(t)$.

) At what values of t does the particle change direction?

) Write an expression for the position $x(t)$ of the particle.

) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 6$.

---

et R be the region enclosed by the graphs of $y = 2 \ln x$ and $y = \frac{x}{2}$, and the lines $x = 2$ and $x = 8$.

) Find the area of R.

) Set up, but do not integrate, an integral expression, in terms of a single variable, for the volume of the solid generated when R is revolved about the x-axis.

) Set up, but do not integrate, an integral expression, in terms of a single variable, for the volume of th solid generated when R is revolved about the line $x = -1$.

PART B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

---

4. Water is draining at the rate of 48π ft³/second from the vertex at the bottom of a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.
   (a) Find an expression for the volume of water in the tank in terms of its radius at the surface of the water.
   (b) At what rate is the radius of the water in the tank shrinking when the radius is 16 feet?
   (c) How fast is the height of the water in the tank dropping at the instant that the radius is 16 feet?

---

5. Let f be the function given by $f(x) = 2x^4 - 4x^2 + 1$.
   (a) Find an equation of the line tangent to the graph at $(-2,17)$.
   (b) Find the x- and y-coordinates of the relative maxima and relative minima. Verify your answer.
   (c) Find the x- and y-coordinates of the points of inflection. Verify your answer.

---

6. Let $F(x) = \int_0^x \left[\cos\left(\frac{t}{2}\right) + \left(\frac{3}{2}\right)\right] dt$ on the closed interval $[0,4\pi]$.
   (a) Approximate $F(2\pi)$ using four inscribed rectangles.
   (b) Find $F'(2\pi)$.
   (c) Find the average value of $F'(x)$ on the interval $[0,4\pi]$.