PS 2 Q4
AB 2017
NO CALC
1-28

1. $\ln(x+3) \geq 0$ if and only if

A. $-3 < x < -2$
B. $x > -2$
C. $x \geq -2$
D. $x > 4$
E. $x \geq 4$

2. Which of the following is NOT symmetric with respect to the y-axis?

I. $y = 2\cos x$
II. $y = (x+2)^2$
III. $y = \ln|x|$

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

3. $\sin 2\theta =$

A. $\cos^2\theta - \sin^2\theta$
B. $2\sin\theta$
C. $\sin^2\theta$
D. $2\sin\theta\cos\theta$
E. $\frac{1}{2}(1-\cos 2\theta)$

4. What is $\lim_{b \to -\infty}\left(\frac{\sqrt{b^2+5}}{4-3b}\right)$?

A. $\frac{-5}{3}$
B. $\frac{-1}{3}$
C. $\frac{1}{3}$
D. 1
E. $\frac{5}{4}$

5. What is $\lim_{x \to 2^+}\left(\frac{3}{x-2}+x\right)$?

A. $+\infty$
B. 2
C. 0
D. $-\infty$
E. none of these

6. What is $\lim_{t \to 1}\left(\frac{\cos(t-1)-1}{t-1}\right)$?

A. 0
B. 1
C. 2
D. 3
E. The limit does not exist.

7. What is $\lim_{x \to 3^+}\left(\frac{5x}{5-x}\right)$?

A. $+\infty$
B. 15
C. $\frac{15}{2}$
D. 0
E. $-\infty$

8. $\lim_{x \to 0}\left(\frac{x}{\sin 2x}\right) =$

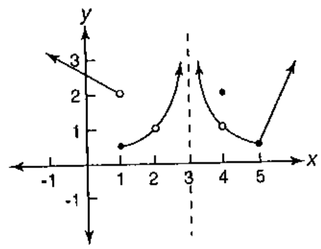
A. -1
B. 0
C. $\frac{1}{2}$
D. 1
E. 2

9. Find a value for b such that $f(x)$ will be continuous, given that

$$f(x)\begin{cases} \frac{x^2-x}{x-1} & \text{for } x \neq 1 \\ b & \text{for } x = 1 \end{cases}$$

A. $b = 0$
B. $b = 1$
C. $b = 2$
D. $f(x)$ is continuous for any value of b.
E. $f(x)$ is not continuous for any value of b.

Use the following graph of f(x) for problems 10–12.



10. $f(x)$ is discontinuous for

A. $x = 1, 3$ only

B. $x = 1, 2, 4$ only

C. $x = 2, 3, 4$ only

D. $x = 1, 2, 3, 4$ only

E. $x = 1, 2, 3, 4, 5$

11. $\lim_{x \to a} f(x)$ does not exist for which of the following values of a?

A. $a = 1, 3$ only

B. $a = 1, 2, 4$ only

C. $a = 2, 3, 4$ only

D. $a = 1, 2, 3, 4$ only

E. $a = 1, 2, 3, 4, 5$

12. $f(x)$ is NOT differentiable at

A. $x = 1, 3$ only

B. $x = 1, 2, 4$ only

C. $x = 2, 3, 4$ only

D. $x = 1, 2, 3, 4$ only

E. $x = 1, 2, 3, 4, 5$

13. If $y = \frac{3}{4+x^2}$ then $\frac{dy}{dx} =$

A. $\frac{-6x}{(4+x^2)^2}$

B. $\frac{3x}{(4+x^2)^2}$

C. $\frac{6x}{(4+x^2)^2}$

D. $\frac{-3}{(4+x^2)^2}$

E. $\frac{3}{2x}$

14. Given that $y = x^{2x}$, find $\frac{dy}{dx}$.

A. $x^{2x}[2 + 2\ln x]$

B. $(2x)(x^{2x-1})$

C. $(\ln x)(x^{2x})$

D. $2 + 2\ln x$

E. $2x^{2x-1}$

15. A particle moves along a horizontal path so its velocity at any time t $(t > 0)$ is given by $v(t) = t\ln t$ *ft/s*. Its acceleration is given by

A. $a(t) = \frac{1}{t}$ ft/s^2

B. $a(t) = 1 + \ln t$ ft/s^2

C. $a(t) = t + \ln t$ ft/s^2

D. $a(t) = \frac{\ln t}{t}$ ft/s^2

E. $a(t) = \frac{t^2}{2}$ ft/s^2

16. If $V = \frac{4}{3}\pi r^3$, what is $\left.\frac{dV}{dr}\right|_{r=3}$?

A. 4π

B. 12π

C. 24π

D. 36π

E. 42π

17. The graph of $y = 3xe^{2x}$ has a relative extremum at

A. $x = 0$ only

B. $x = 0$ and $x = \frac{-1}{2}$

C. $x = \frac{-1}{2}$ only

D. $x = -2$ only

E. The graph has no relative extrema.

18. Find the equation of the line that is normal to the curve $y = 3\tan\frac{x}{2}$ at the point where $x = \frac{\pi}{2}$.

A. $2x + 12y = \pi + 36$

B. $2x + 6y = 18 + \pi$

C. $-6x + 2y = 3\pi - 6$

D. $-2x + 6y = 18 + \pi$

E. $6x - 2y = 3\pi - 6$

19. Find the area of the largest rectangle that has two vertices on the x-axis and two vertices on the curve $y = 9 - x^2$.

A. $\sqrt{3}$

B. $4\sqrt{3}$

C. $12\sqrt{3}$

D. $16\sqrt{3}$

E. $24\sqrt{3}$

20. Sand is falling into a conical pile at the rate of $10\text{ m}^3/\text{s}$ such that the height of the pile is always half the diameter of the base of the pile. Find the rate at which the height of the pile is changing when the pile is 5 m high. (Volume of a cone: $V = \frac{1}{3}\pi r^2 h$)

A. $\frac{1}{25\pi}$ m/s

B. $\frac{2}{5\pi}$ m/s

C. $\frac{4}{5\pi}$ m/s

D. $\frac{8}{5\pi}$ m/s

E. 250π m/s

21. The antiderivative of $\frac{3}{x^2}$ is

A. $\frac{3}{x} + C$

B. $\frac{-6}{x^3} + C$

C. $\frac{-3}{x} + C$

D. $\frac{1}{x^3} + C$

E. $\frac{-3}{x^2} + C$

22. $\int_0^{\pi} \cos\frac{x}{2} = dx$

A. -2

B. -1

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

E. 2

23. $\int 3^{2x}\, dx =$

A. $\frac{\ln 3}{2} 3^{2x} + C$

B. $\frac{1}{2\ln 3} 3^{2x} + C$

C. $(2\ln 3)3^{2x} + C$

D. $\frac{2}{\ln 3} 3^{2x} + C$

E. $\frac{1}{\ln 3} 3^{2x} + C$

24. $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\, dx =$

A. $1 - \frac{\sqrt{3}}{2}$

B. $\frac{1}{2}\ln\frac{3}{4}$

C. $\sqrt{3} - 2$

D. $\frac{\pi}{6} - 1$

E. $2 - \sqrt{3}$

25. $\int \frac{5}{\sqrt{9-4x^2}}\, dx =$

A. $\frac{-5}{2}\ln\left|9 - 4x^2\right| + C$

B. $\frac{-5}{8}\ln\left|9 - 4x^2\right| + C$

C. $\frac{-5}{4}\sqrt{9 - 4x^2} + C$

D. $\frac{-5}{2}\sqrt{9 - 4x^2} + C$

E. $\frac{5}{2}\arcsin\frac{2x}{3} + C$

26. $\int_1^{e^3} \frac{\ln x}{x}\, dx =$

A. 1

B. 4

C. $\frac{9}{2}$

D. $2e^3 - 1$

E. $e^3 - 2$

27. The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is

A. $\frac{e^4}{2} - e$ square units

B. $\frac{e^4}{1} - 1$ square units

C. $\frac{e^4}{2} - \frac{1}{2}$ square units

D. $2e^4 - e$ square units

E. $2e^4 - 2$ square units

28. The average value of $y = e^{3x}$ over the interval from $x = 0$ to $x = 4$ is

A. $\frac{e^{12} - 1}{12}$

B. $\frac{e^{12} - 1}{4}$

C. $\frac{e^{12}}{12}$

D. $\frac{e^{12}}{4}$

E. $e^{12} - 1$

You may use calc 1 – 17

1. If $f(x) = |x|$, then $f'(2)$ is

A. –2

B. –1

C. 1

D. 2

E. nonexistent

2. If $y = 2e^6$, then $y' =$

A. $\frac{2e^7}{7}$

B. $12e^5$

C. $2e^6$

D. 2

E. 0

3. The absolute maximum of $f(x) = 2x - \sin^{-1} x$ on its domain is approximately

A. 0.523

B. 0.571

C. 0.685

D. 0.866

E. 0.923

4. Which of the following is equivalent to $\left.\frac{d(\sin\theta)}{d\theta}\right|_{\theta = \pi/3}$?

A. $\lim_{\theta \to \pi/3} \frac{\sin\theta - \frac{1}{2}}{\theta - \frac{\pi}{3}}$

B. $\lim_{\theta \to \pi/3} \frac{\sin\theta - \frac{\sqrt{3}}{2}}{\theta - \frac{\pi}{3}}$

C. $\lim_{\theta \to 0} \frac{\sin\theta - \frac{\sqrt{3}}{2}}{\theta - \frac{\pi}{3}}$

D. $\lim_{h \to 0} \frac{\sin(\theta + h) - \sin\theta}{h}$

E. $\lim_{h \to 0} \frac{\cos(\theta + h) - \cos\theta}{h}$

5. The function $g(x) = (\cos x)(e^x) - \frac{3}{2}$ has two real zeros between 0 and 2. If $(a, 0)$ and $(b, 0)$ represent these two zeros, then $a + b$ is approximately

A. 2.17

B. 2.00

C. 1.55

D. 0.99

E. 0.45

6. Find the number guaranteed by the mean value theorem for the function $f(x) = e^{(1/2)x}$ on the interval [0, 2].

A. 1.083

B. 0.709

C. 0.614

D. –0.304

E. The mean value theorem cannot be applied on [0, 2].

7. Let A be the region completely bounded by $y = \ln x + 2$ and $y = 2x$. Correct to three decimal places, the area of A is approximately

A. 0.053

B. 0.162

C. 0.203

D. 1.216

E. 2.358

8. Which of the following is equivalent to $\lim_{n \to \infty} \sum_{i=1}^{n} \left[\left(1 + \frac{2i}{n}\right)^2 + 1 \right] \left(\frac{2}{n}\right)$?

A. $\int_2^4 (x^2 + 1)\, dx$

B. $\int_1^3 x^2\, dx$

C. $\int_1^2 (x^2 + 1)\, dx$

D. $\int_1^3 (x^2 - 1)\, dx$

E. $\int_1^3 (x^2 + 1)\, dx$

9. $\frac{d}{dz}\left[\int_0^z \left(e^{4x^2}\right) dx \right] =$

A. $e^{4x^2} + C$

B. $\frac{e^{4x^2}}{8x} + C$

C. $e^{4z^2} - 1$

D. e^{4z^2}

E. $\frac{e^{4z^2}}{8z} + C$

10. Approximate the slope of the line tangent to the graph of $f(x) = 4x \ln x$ at the point where $x = 2.1$.

A. 4.93

B. 5.07

C. 6.23

D. 6.97

E. 7.27

11. If $\frac{dy}{dt} = \pi y$, which of the following could represent y?

A. $\frac{1}{\pi} e^t$

B. $\pi e^{x/t}$

C. $e^{\pi t} + \pi$

D. πe^t

E. $\pi e^{\pi t}$

12. If $f(x) = \frac{x^3}{\sqrt[3]{x}}$, then $f'(x) =$

A. $\frac{8}{3} x \sqrt[3]{x^2}$

B. $\frac{3}{11} x \sqrt[3]{x^2}$

C. $\frac{8}{3} x^2 \sqrt[3]{x^2}$

D. $3x^3$

E. $\frac{10}{3} x \sqrt[3]{x^2}$

13. Let A be the area bounded by one arch of the sine curve. Which of the following represents the volume of the solid generated when A is revolved around the x-axis?

A. $2\pi \int_0^{\pi} x \sin x\, dx$

B. $\pi \int_0^{\pi} \sin^2 x\, dx$

C. $\pi \int_0^{\pi} x \sin x\, dx$

D. $\pi \int_0^{2\pi} \sin^2 x\, dx$

E. $2\pi \int_0^1 \arcsin y\, dy$

14. $\lim_{h \to 0} \frac{\sin(1+h) - \sin 1}{h}$ is approximately

A. 0

B. 0.54

C. 0.63

D. 0.89

E. none of these

15. The fundamental period of $y = \sin 3x + \cos 2x$ is

A. π

B. $\frac{2\pi}{3}$

C. $\frac{4\pi}{3}$

D. $\frac{5\pi}{3}$

E. 2π

16. Approximate the value of $\int_1^3 \ln x \, dx$ using 4 circumscribed rectangles.

A. 1.007

B. 1.296

C. 1.557

D. 2.015

E. 3.114

17. $\int \sin x \cos^2 x \, dx =$

A. $-\frac{2}{3}\cos^3 x + C$

B. $-\frac{1}{2}\sin^2 x + C$

C. $\frac{1}{2}\sin^2 x + C$

D. $\frac{1}{3}\cos^3 x + C$

E. $-\frac{1}{3}\cos^3 x + C$

Free Response 1-3 with calculator

1. A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

(a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?

(b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?

(c) At what time t, for $0 \le t \le 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

(d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

2. A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$. The particle is at position $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

(c) Find the position of the particle at time $t = 2$.

(d) Find the average speed of the particle over the interval $0 \le t \le 2$.

3. A particle moves along the y-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

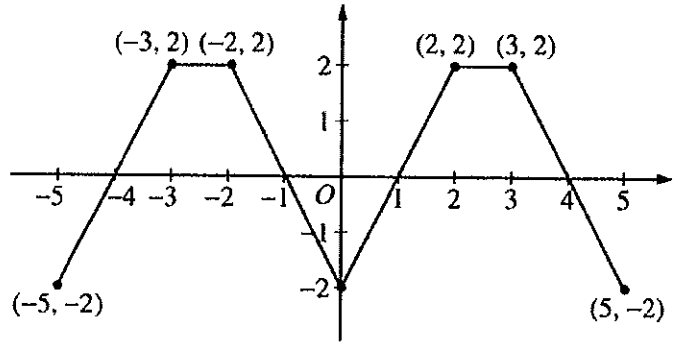
(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \ge 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

FR 4-6 No calc

4\. The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.



Graph of f

(a) Find $g(4)$, $g'(4)$, and $g''(4)$.

(b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.

(c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

5\. Consider the curve given by $y^2 = 2 + xy$.

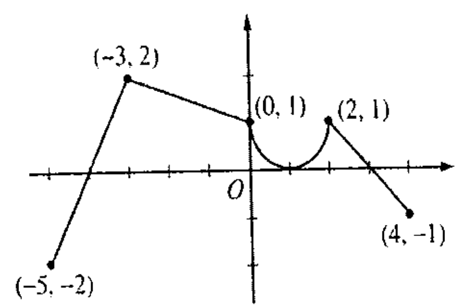
(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

6\. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^x f(t)\,dt$.



Graph of f

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.