Unit 1: Polynomial Review Problems

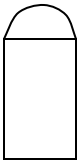
1. An open-top box is to be made from a 16-in by 30-in piece of cardboard by cutting out squares of equal size from the 4 corners and bending up the sides. What size should the squares be to obtain a box with the largest possible volume?

2. A rectangular field is to be bounded by a fence on three sides and by a straight stream on the fourth side. Find the dimensions of the field with maximum area that can be enclosed with 1000 feet of fence.

3. A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $y = 16 - x^2$. For all such rectangles, what are the dimensions of the one with largest area?

4. A closed rectangular container with a square base is to have a volume of 2250 in^3. The material for the top and bottom of the container will cost \$2 per sq. in., and the material for the sides will cost \$3 per sq. in. Find the dimensions of the container that can be made for the least cost.

5. A church window consisting of a rectangle topped by a semicircle is to have a perimeter of 16 ft. Find the radius of the semicircle if the area of the window is to be a maximum.



6. A container with a square base, vertical sides, and an open top is to be made from 1000 ft^2 of materials. Find the dimensions of the container with greatest volume.

7. Given: $2x^4 - 7x^3 + 4x^2 + 7x - 6 = 0$

   a) List all possible rational zeros.
   b) Find the least positive integral upper bound and the greatest negative integral lower bound.
   c) Use Descartes' Rule of Signs to determine the possible combinations of positive, negative, and imaginary roots.
   d) Find all the roots.
   e) Sketch the graph.

8. Find all roots of the equation: $x^3 - x^2 - 5x + 2 = 0$.

9. Using a calculator, estimate the real roots of the following (nearest hundredth):

   a) $2x^4 - 3x^3 + 6x^2 + x - 15 = 0$
   b) $x^3 - 2x = -7$
   c) $x^4 - 3x^3 - 7x^2 - x + 2 = 0$

10. Write a polynomial equation of least degree with roots -3 and $2i$.

Key:

1) 3.3 in 2) 250' x 500' 3) 4.6 x 10.7 4) 15 x 15 x 10 5) r = 2.24' 6) 18.3 x 18.3 x 9

7) a) $\pm1, \pm2, \pm3, \pm6, \pm1/2, \pm3/2$ b) UB = 4, LB = -1 c) 3, 1, 0 / 1, 1, 2 d) 2, 3/2, ±1

8) $-2, \frac{3 \pm \sqrt{5}}{2}$ 9) a) $-1.15, 1.5$ b) -2.3 c) $-1, 0.438, 4{,}562$ 10) $x^3 + 3x^2 + 4x + 12 = 0$