Inverses Cont:

Let $f(x) = \sqrt{x+2}$ find the inverse. Label the inverse as a function named g.

What is the slope of the tangent line to f at x = 7?

$(7, 3)$

$f(x) = \sqrt{x+2}$

$f'(x) = \frac{1}{2}(x+2)^{-1/2}$

$= 1/6$

$g(x) = x^2 - 2 \quad [0, \infty)$

$g'(x) = 2x \quad$ at 3

$= 6$

We know that $f(x) = \ln x$ and $g(x) = e^x$ are inverses. Namely that $\ln\left(e^x\right) = x$ and $\cancel{e}^{\ln x} = x$.

Ex: A girl invests \$500 in a bank with an interest of 3 % compounding continuously. How long before her investment doubles?

$1000 = 500e^{.03t}$

$2 = e^{.03t}$

$\ln 2 = .03t \cancel{\ln e}$

$t = \dfrac{\ln 2}{.03}$

Ex: Solve $9 = e^{2x+1}$

$\ln 9 = 2x + 1$

$\ln 9 - 1 = 2x$

$\dfrac{\ln 9 - 1}{2} = x$

Ex: Solve $\ln(x - 5) = 4$

$e^4 = x - 5$

$e^4 + 5 = x$

Ex: Solve $\ln(2x + 3) = \dfrac{3}{2}$

$e^{3/2} = 2x + 3$

$e^{3/2} - 3 = 2x$

$\dfrac{e^{3/2} - 3}{2} = x$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}\left[e^x\right] = e^x \qquad\qquad \frac{d}{dx}\left[e^u\right] = e^u\frac{du}{dx}$$

Find the derivative of each of the following:

Ex: $y = e^{3x+1}$

$u = 3x + 1$

$du = 3$

$y' = 3e^{3x+1}$

Ex: $y = e^{\frac{1}{2}x - 5}$

$u = \frac{1}{2}x - 5$

$du = 1/2$

$y' = \frac{1}{2}e^{1/2x - 5}$

Ex: $y = e^{\frac{-3}{x}}$

$e^{-\frac{3}{x}}$

$y' = \dfrac{3}{x^2} \cdot e^{-\frac{3}{x}}$

Ex: $y = e^{\cos(x)}$

$y' = -\sin x \cdot e^{\cos x}$

Ex: $y = x^2 e^{2x}$

$y' = x^2 \cdot 2e^{2x} + 2xe^{2x}$

$= 2xe^{2x}(x + 1)$

Integration – The Natural Exponential Function

$$\int e^x\,dx = e^x + C \qquad\qquad \int e^u\,dx = e^u + C$$

Ex: $\int e^{2x+5}\,dx$

$u = 2x+5$

$du = 2\,dx$

$\frac{1}{2}\int e^u\,du$

$\frac{1}{2}e^{2x+5} + C$

Ex: $\int e^{\frac{1}{3}x-1}\,dx$

$u = 1/3x - 1$

$du = 1/3\,dx$

$3\int e^u\,du$

$3e^{1/3x-1} + C$

Ex: $\int 7x^2 e^{x^3}\,dx$

$u = x^3\,dx$

$du = 3x^2\,dx$

$\frac{7}{3}\int e^u\,du$

$7/3\,e^{x^3} + C$

Ex: $\int \sec^2(x)\,e^{\tan x}\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$e^{\tan x} + C$

Ex: $\int \frac{e^x}{2-e^x}\,dx$

$u = 2 - e^x$

$du = -e^x$

$-\int \frac{1}{u}\,du$

$-\ln|2-e^x| + C$

Ex: $\int e^{\sec(2x)}\sec(2x)\tan(2x)\,dx$

$u = \sec(2x)$

$du = \sec(2x)\tan(2x)\cdot 2$

$\frac{1}{2}\int e^u$

$\frac{1}{2}e^{\sec(2x)} + C$

Discuss $f(x) = x^2$ and the $g(x) = \sqrt{x}$.

$\sqrt{x^3+1} = 3$

$x = 2$

$(7,3)$

Ex: $f(x) = \sqrt{x^3+1}$ Find $(f^{-1})'(3) =$

Ex: $f(x) = \ln x$ Find $(f^{-1})'(1) =$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$x = \sqrt{y^3+1}$

$f(x) = \sqrt{x^3+1}$ $\qquad \sqrt[3]{x^2-1} = g(x)$

$f'(x) = \frac{1}{2}(x^3+1)^{-1/2} \cdot 3x^2$ $\qquad \frac{1}{3}(x^2-1)^{-2/3} \cdot 2x$ $\quad g'(x)$

$\boxed{g'(3) = \frac{1}{2}}$

$(2,3)$