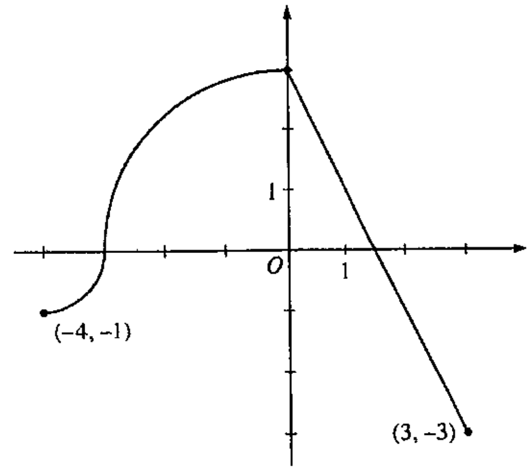
The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.



Graph of *f*

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

Multiple-Choice Questions

No calculator is allowed for these questions.

1. If $F(x) = \int_0^x \sin^2(2t)\,dt$, then $F'(x) =$

(A) $-\cos^2(2x)$
(B) $\cos^2(2x)$
(C) $\sin^2(2x)$
(D) $\frac{1}{2}\sin^2(2x)$
(E) $4\sin(2x)\cos(2x)$

2. $\frac{d}{dx}\left(\int_2^5 (1 + t^{2/3})\,dt\right) =$

(A) 0
(B) $1 + 5^{2/3}$
(C) $t + \frac{3}{5}t^{5/3}$
(D) $5^{2/3} - 2^{2/3}$
(E) $\frac{(1 + t^{2/3})^2}{2}$

3. $\frac{d}{dt}\left(\int_{6t}^1 \left(1 + \sqrt{x}\right)^2 dx\right) =$

(A) $\sqrt{1 + 6t}$
(B) $-\left(1 + \sqrt{6t}\right)^2$
(C) $\left(1 + \sqrt{6t}\right)^2$
(D) $-6\left(1 + \sqrt{6t}\right)^2$
(E) $-\frac{1}{6}\left(1 + \sqrt{6t}\right)^2$

For Questions 4 and 5, if $F'(x) = \int_{-x}^{x} e^{-t^2}\,dt$, *then*

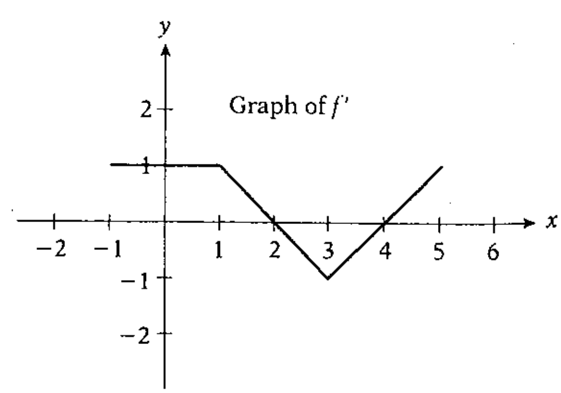
4. $F'(0) =$

(A) 0
(B) $2e^{-1}$
(C) 2
(D) $2e$
(E) $-2e^{-1}$

5. $F''(x) =$

(A) 0
(B) $-4xe^{-x^2}$
(C) $-2xe^{-x^2}$
(D) $4xe^{-x^2}$
(E) $2e^{x^2}$

For Questions 6–9, $f(x) = \int_0^x f'(t)\,dt$ *and the graph of* f′ *is shown.*



6. Which of the following are true?

I $f(-1) = -1$
II $f(1) < f(3)$
III $f'(1) < f'(3)$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I, II, and III

7. Which of the following are true about the graph of f?

I f is increasing on $(-1, 2)$ only
II f is increasing on $(-1, 2)$ or $(4, 5)$
III f is decreasing on $(1, 3)$

(A) I only
(B) II only
(C) III only
(D) I and III
(E) none

8. Which of the following are true about the graph of f?

I f is concave up on $(-1, 1)$
II f is concave up on $(1, 3)$
III f is concave down on $(3, 5)$

(A) I only
(B) II only
(C) III only
(D) II and III
(E) none

9. Which of the following are true about the graph of f?

I f has a relative minimum at $x = 2$
II f has a relative minimum at $x = 4$
III f has a relative maximum at $x = 2$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

graphing calculator is required for Question 10.

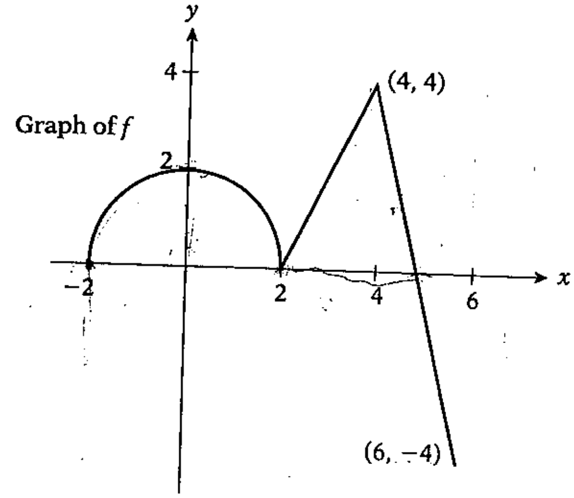
10. $F(x) = \int \cos(x^2)\,dx$ and $F(2) = 10$. Find $F(3)$.

(A) 0.140
(B) 0.241
(C) 0.703
(D) 10.241
(E) 2.414

Free-Response Questions

No calculator is allowed for these questions.

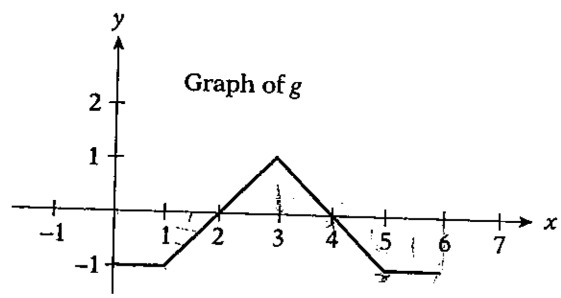
1. The graph of f shown consists of a semicircle and two line segments on the interval $[-2, 6]$.



If $g(x) = \int_2^x f(t)\,dt$, use the graph of f to do the following:

(a) Find $g(-2)$, $g(0)$, $g(2)$, $g(4)$ and $g(6)$.
(b) Find the intervals where g is decreasing.
(c) Find the intervals where g is concave up.
(d) Find the absolute extrema of g on the interval $[-2, 6]$.
(e) Find the point(s) of inflection of g.
(f) Sketch g on the interval $[-2, 6]$.

2. The graph of $g(t)$ is shown on the interval $[0, 6]$ and $f(x) = \int_0^x g(t)\,dt$.



(a) Find the critical points of $f(x)$, and identify each as a relative maximum, a relative minimum, or neither. Justify your conclusion.
(b) Find the absolute extrema of $f(x)$. Justify your conclusion.
(c) If $f(0) = k$, is there a number c in the interval $[0, 6]$ such that $f(c) = k$? Justify your conclusion.
(d) On what interval(s) is $f(x)$ concave up? On what interval(s) is $f(x)$ concave down?
(e) Sketch a graph of $f(x)$.