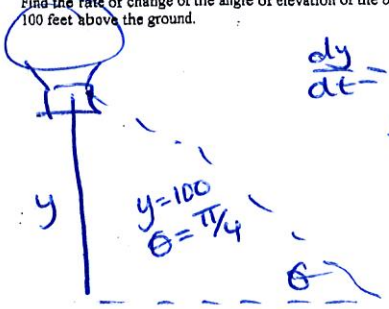
AP Calculus AB Name Key
Section 2.6 Related Rates Period
Date

3. A balloon rises at the rate of 10 feet per second from a point on the ground 100 feet from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 100 feet above the ground.



$\frac{dy}{dt} = 10$ ft/sec $\quad y = 100$

$\tan\theta = \frac{y}{100}$

$100\tan\theta = y$

$100\sec^2\theta \frac{d\theta}{dt} = \frac{dy}{dt}$

$100\sec^2(\pi/4)\frac{d\theta}{dt} = 10$

$\frac{d\theta}{dt} = \frac{1}{20}$ rads/sec.

4. Coffee is poured at a uniform rate of 20 cm³/sec into a cup whose inside is shaped like a truncated cone (see the figure below). If the upper and lower radii of the cup are 4 cm and 2 cm and the height of the cup is 6 cm, how fast will the coffee level be rising when the coffee is halfway up? (*Hint:* Extend the cup downward to form a cone.)

$\frac{dV}{dt} = 20\text{cm}^3/\text{sec}$.

$\frac{4}{6+h} = \frac{2}{h} \Rightarrow 4h = 12 + 2h \quad h = 6$

cone has height of 6.

radius = r of coffee h of coffee.

$\frac{4}{12} = \frac{r}{h}$

$r = \frac{1}{3}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(\frac{1}{3}h)^2 h$

$V = \frac{1}{27}\pi h^3$

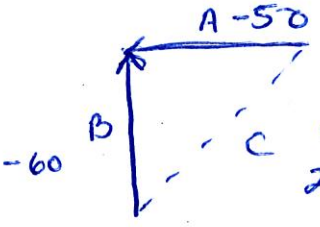
$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \cdot \frac{dh}{dt}$

halfway is 6+3 ← cup

$20 = \frac{1}{9}\pi(81)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{20}{9\pi}$ cm/sec

5. Car A is traveling west at 50 mi/h and car B is traveling north at 60 mi/h. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?



$A^2 + B^2 = C^2$

$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2C\frac{dC}{dt}$

distance shrinking (−)

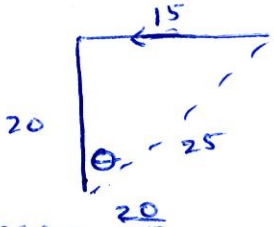
$2(.3)(50) + 2(.4)(60) = 2(.5)\frac{dC}{dt}$

$C = .5$

$a = .3 \quad b = .4$

$\frac{dC}{dt} = -78$ mi/hr

7. A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?



$\tan\theta = \frac{x}{20}$

$20\tan\theta = x$

$20\sec^2\theta\frac{d\theta}{dt} = \frac{dx}{dt}$

$20(5/4)^2\frac{d\theta}{dt} = 4$

$25/16\frac{d\theta}{dt} = 1/5$

$\cos\theta = \frac{20}{25}$

$\sec\theta = \frac{25}{20} = \frac{5}{4}$

$\frac{d\theta}{dt} = \frac{16}{125}$ rad/sec

1. The length L of a rectangle is decreasing at the rate of 2 cm/sec while the width w is increasing at the rate of 2 cm/sec. When $L = 12$ and $w = 5$,

a. find the rate of change of the area.

$A = l \cdot w$

$\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$

$\frac{dA}{dt} = 12(2) + 5(-2)$

$\frac{dA}{dt} = 14\text{cm}^2/\text{sec}$

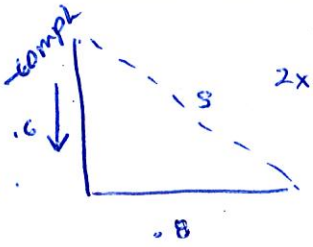
b. find the rate of change of the perimeter.

$P = 2l + 2w$

$\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$

$= 2(-2) + 2(2) = 0$

2. A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?



$x^2 + y^2 = s^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}$

$2(.8)\frac{dx}{dt} + 2(.6)(-60) = 2(1)(20)$

$1.6\frac{dx}{dt} = 72$

$\frac{dx}{dt} = 70$ mph

6. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

$x^2 + y^2 = 10^2$

$\frac{dx}{dt} = 1$ ft/s $\quad \frac{dy}{dt} = ?$

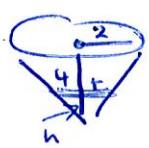
$x = 6 \quad y = 8$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(6)(1) + 2(8)\left(\frac{dy}{dt}\right) = 0$

$\frac{dy}{dt} = \frac{-3}{4}$ ft/s

8. A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 m³/min, find the rate at which the water level is rising when the water is 3 m deep.



$\frac{2}{4} = \frac{r}{h} \quad 2h = 4r \quad \frac{1}{2}h = r$

$\frac{dV}{dt} = 2\text{m}^3/\text{min}$

$h = 3$ m

$\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$2 = \frac{\pi}{4}\cdot 9\cdot\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{8}{9\pi}$ m/min