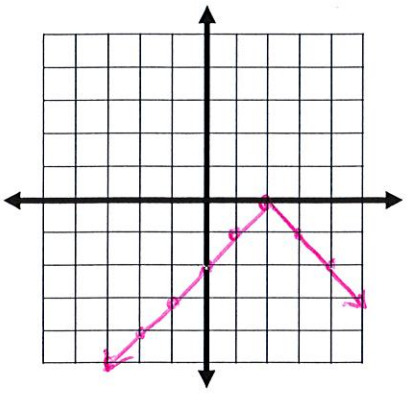
AFM Midterm Review I Fall 2014

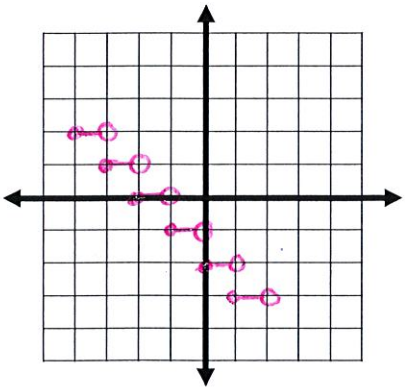
1. Determine if the relation is a function. $\{(1,6),(-3,3),(5,-1)\}$ yes

2. Determine the domain of the function $h(x)=\dfrac{7x}{x(x^2-4)}$. $x(x+2)(x-2)$ $(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$

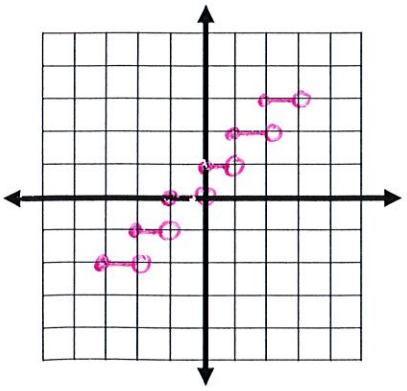
3. Sketch the graph of $f(x)=-|x-2|$



4. Sketch the graph of $f(x)=-[x+2]$



5. Sketch the graph of $f(x)=[x]+1$



6. Find the vertical, horizontal, and slant asymptotes, if any, for $f(x)=\dfrac{2x^3+2x^2-7x-12}{x^2-x-2}$. $(x-2)(x+1)$

VA: $x=2$ $x=-1$
SA: $y=2x+4$

$$x^2-x-2\overline{)\,2x^3+2x^2-7x-12}\quad 2x+4$$
$$2x^3-2x^2-4x$$

7. Given that one zero is -2, find all the zeroes of $P(x)=x^3+11x^2+38x+40$

-2 | 1 | 11 | 38 | 40
-2 | -18 | -40
1 | 9 | 20 | 0

$x^2+9x+20$ $(x+5)(x+4)$ $x=-5$, $x=-4$, $x=-2$

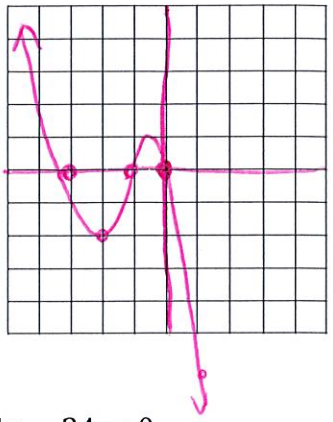
8. Find the number of complex roots of the equation $-x^3 - 4x^2 - 3x = 0$. Then find the roots and graph the related function.

$-x^3 - 4x^2 - 3x = 0$

$-x(x^2 + 4x + 3) = 0$

$-x(x+3)(x+1)$

$x = 0 \quad x = -3 \quad x = -1$



9. Find the discriminant and determine the number and kind of roots for $4x^2 - 4x - 24 = 0$.

$b^2 - 4ac$ = discriminant $\quad 16 - [4(4)(-24)] = 400$ 2 real rational unequal.

10. Divide using synthetic division: $(x^3 - 4x^2 - 16x + 64) \div (x + 4)$.

$-4 \mid 1 \;\; -4 \;\; -16 \;\; 64$

$\quad\quad -4 \;\; 32 \;\; -64$

$\quad 1 \;\; -8 \;\; 16 \;\; 0$

$x^2 - 8x + 16$

11. Use the rational-root theorem to find the roots of the equation $24x^3 + 14x^2 - x - 1 = 0$.

$P/q = \pm 1, \pm 1/2, 1/3, \pm 1/4, \pm 1/6, \pm 1/8, \pm 1/12, \pm 1/24$

$-1/2 \mid 24 \;\; 14 \;\; -1 \;\; -1$

$\quad\quad -12 \;\; -1 \;\; 1$

$\quad 24 \;\; 2 \;\; -2 \;\; 0$

$24x^2 + 2x - 2$

$(8x-2)(3x+1)$

$x = -1/2 \quad x = 1/4 \quad x = -1/3$

12. Find all the zeros of the function for $f(x) = 6x^4 + 5x^3 - 12x^2 - 5x + 6$.

$-1 \mid 6 \;\; 5 \;\; -12 \;\; -5 \;\; 6$

$\quad\quad -6 \;\; 1 \;\; 11 \;\; -6$

$1 \mid 6 \;\; -1 \;\; -11 \;\; 6 \;\; 0$

$\quad\quad 6 \;\; 5 \;\; -6$

$\quad 6 \;\; 5 \;\; -6 \;\; 0$

$(3x-2)(2x+3)$

$x = -1 \quad x = 1 \quad x = 2/3 \quad x = -3/2$

13. Convert 30° to radians. $\pi/6$

14. Convert $\frac{5}{6}\pi$ to degrees. 150°

15. Find the least positive angle measurement that is coterminal to $-\frac{4}{3}\pi$. $2\pi/3$

16. What is the reference angle for $-\frac{53\pi}{10}$ radians? $3\pi/10$

17. For a circle of radius 4 feet, find the arc length s subtended by a central angle of 30°.

$S = r \cdot \theta \quad S = 4 \cdot \pi/6 = 2\pi/3$ ft or 2.094 ft.

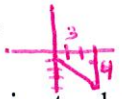
18. Find the area of a sector with a central angle of $\frac{8\pi}{9}$ and a radius of 8.2 m. Round the answer to one decimal place.

$A = 1/2 r^2 \theta \quad A = 1/2 (8.2)^2 \cdot 8\pi/9 = \frac{6724\pi}{225}$ m^2 or 93.88 m^2

19. Find $\sin\theta$ if θ is an angle is standard position and the point with coordinates (4,3) lies on the terminal side of the angle.

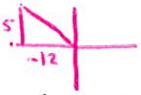
$\sin\theta = 3/5$

20. Find $\cos\theta$ if θ is an angle in standard position and the point with coordinates $(3, -4)$ lies on the terminal side of the angle.



$\cos\theta = 3/5$

21. Find $\tan\theta$ if θ is an angle in standard position and the point with coordinates $(-12,5)$ lies on the terminal side of the angle.



$\tan\theta = -5/12$

22. Find the values of the three trigonometric functions of an angle in standard position and the point with coordinates (2,4) lies on the terminal side.

$\sin\theta = \frac{4}{2\sqrt{5}} = 2\sqrt{5}/5$

$\cos\theta = \frac{2}{2\sqrt{5}} = \sqrt{5}/5$

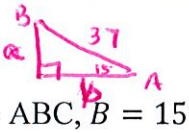
$\tan\theta = 4/2 = 2$

23. Find the values of the three trigonometric functions of an angle in standard position if the point with coordinates $(-2,0)$ lies on the terminal side. $\sin\theta = 0$, $\cos\theta = -1$, $\tan\theta = 0$

24. Find $\cos(-\frac{7}{6}\pi)$. $-\frac{\sqrt{3}}{2}$

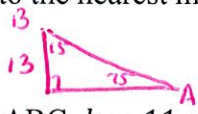
25. Use a calculator to approximate the value of $\cos(-113°)$ to four decimal places. $-.3907$

26. In right triangle ABC, $A = 15°$ and $c = 37$. Angle C is the right angle. Solve the triangle. Round angle measures to the nearest minute and side measures to the nearest tenth.



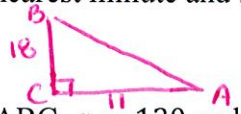
$\sin 15 = \frac{a}{37}$ $\cos 15 = \frac{b}{37}$ $A = 15$ $B = 75$ $C = 90$ $a = 9.6$ $b = 35.7$ $c = 37$

27. In right triangle ABC, $B = 15°$ and $a = 13$. Angle C is the right angle. Solve the triangle. Round angle measures to the nearest minute and side measures to the nearest tenth.



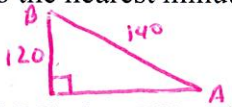
$\sin 75 = \frac{13}{c}$ $\tan 75 = \frac{13}{b}$ $A = 75$ $B = 15$ $C = 90$ $a = 13$ $b = 3.5$ $c = 13.5$

28. In right triangle ABC, $b = 11$ and $a = 18$. Angle C is the right angle. Solve the triangle. Round angle measures to the nearest minute and side measures to the nearest tenth.



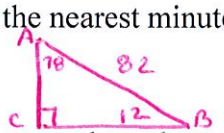
$A = \tan^{-1}(18/11)$ $A = 58°34'$ $B = 31°26'$ $C = 90°$ $a = 18$ $b = 11$ $c = 21.1$

29. In right triangle ABC, $a = 120$ and $c = 140$. Angle C is the right angle. Solve the triangle. Round angle measures to the nearest minute and side measures to the nearest tenth.



$A = \sin^{-1}(120/140)$ $A = 59°$ $B = 31°$ $C = 90°$ $a = 120$ $b = 72.1$ $c = 140$

30. In right triangle ABC, $A = 78°$ and $c = 82$. Angle C is the right angle. Solve the triangle. Round angle measures to the nearest minute and side measures to the nearest tenth.



$\sin 78 = \frac{a}{82}$ $\cos 78 = \frac{b}{82}$ $A = 78°$ $B = 12°$ $C = 90°$ $a = 80.2$ $b = 17$ $c = 82$

31. How many triangles are there that satisfy the conditions $a = 7, b = 8, A = 70°$? 0

32. Solve triangle ABC given that $A = 54°, B = 56°$ and $b = 73$.

$\frac{\sin 54}{a} = \frac{\sin 56}{73}$ $A = 54$ $B = 56$ $C = 80$ $a = 71.2$ $b = 73$ $c = 86.7$

33. Given a triangle with $b = 5, c = 9$, and $A = 110°$, what is the length of a? Round the answer to two decimal places.

$a^2 = 5^2 + 9^2 - [2(5)(9)\cos 110]$

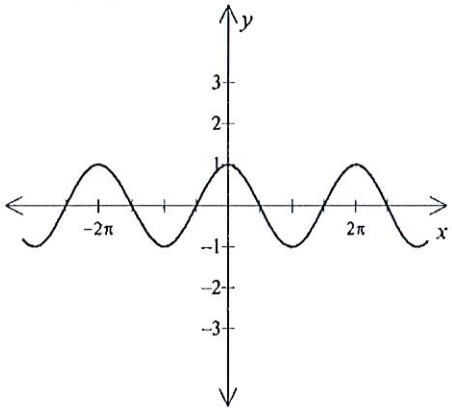
$a = 11.70$

34. Find the area of the triangle with $A = 98°, b = 2$ feet and $c = 7$ feet. Round your answer to two decimal places.

$A = \frac{1}{2}(2)(7)\sin 98$

$A = 6.93 \text{ u}^2$

35. Use the graph of the cosine function below to find the values θ of for which $\cos\theta = -1$



$\theta = \pi \quad \theta = -\pi$

$\theta = 3\pi \quad \theta = -3\pi$

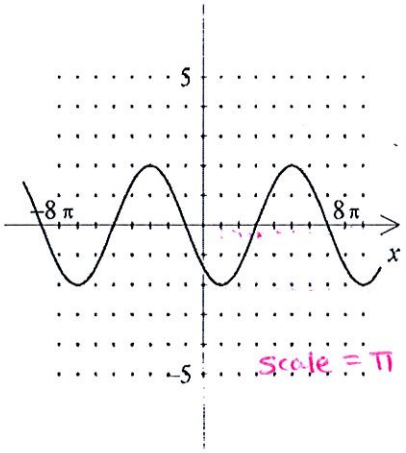
$\theta = \pi \pm 2k\pi$

36. State the amplitude and period for the function $y = -5\sin\frac{1}{5}\theta$.

$a = 5$

period $= 10\pi$

37. Find a cosine function of the form $y = A\cos(Bx + C), A > 0$, a portion of whose graph is:



$\frac{2\pi}{b} = 8\pi \quad b = 1/4$

$y = -2\cos\frac{1}{4}(x - 3\pi)$ or

$y = 2\cos\frac{1}{4}(x - 5\pi)$

$y = 2\cos 1/4\,(x + \pi)$

38. The function below determines the amount of yearly income tax a person must pay based on the amount of money they earn in a year.

$$T(x) = \begin{cases} 0.10x & 0 \le x < 12{,}750 \\ 0.07(x - 12{,}750) + 765 & 12{,}750 \le x < 60{,}000 \\ 0.0775(x - 60{,}000) + 4{,}072.50 & x \ge 60{,}000 \end{cases}$$

a) Describe what x and T(x) represent in the context of the problem.

x = earnings
T(x) = tax

b) Write the domain and range in interval notation.

D: $[0, \infty)$
R: $[0, \infty)$

c) Explain your tax rate (i.e. the % you pay) if you make:

i) less than \$12,750 per year 10 %

ii) \$12,750 to \$59,999 per year 7 %

iii) \$60,000 per year or more 7.75 %

Make sure to review your Unit Circle. Be ready for no calculator questions on Regular Trigonometry

AFM Fall 2014 Graphing Calculator Active

Midterm review II

1. Solve for x: $5x^3+13x^2-5x+3=0$.

P = 3 ±1, ±3 P/Q = ±1, ±1/5, ±3, ±3/5

Q = 5 ±1, ±5

-3 | 5 13 -5 3

-15 6 -3

5 -2 1 | 0

$5x^2-2x+1$

$x=\frac{2\pm\sqrt{4-[4(5)(1)}}{10}$

$x=\frac{2\pm 4i}{10}$

x = -3

$x=\frac{1\pm 2i}{5}$

2. Determine the equation whose roots are 2, 9 and -4.

$(x-2)(x-9)=x^2-11x+18$

$x^3-7x^2-26x+72$

$x+4$

$4x^2-44x-72$

$-11x^2-18x$

3. Suppose $f(x)=x^3-2x^2+13x+k$ The remainder when $f(x)$ is divided by (x + 1) is -8. What is the remainder when $f(x)$ is divided by (x – 1)?

-1 | 1 -2 13 K

-1 3 -16

1 -3 16 K = 8

1 | 1 -2 13 8

1 -1 12

1 -1 12 20

R = 20

4. Which of the following is NOT a factor of $2x^4-3x^2+1$?

A. x + 1 B. x – 1 C. 2x – 1

-1 | 2 0 -3 0 1

-2 2 1 -1

2 -2 -1 1 | 0

1 | 2 0 -3 0 1

2 2 -1 -1

2 2 -1 -1 | 0

1/2 | 2 0 -3 0 1

1 1/2 -5/4 -5/8

2 1 -5/2 -5/4 | 3/8

5. Write the polynomial equation of least degree for the roots 2*i*, -2*i* and -4.

$(x+4)(x^2+4)$

$f(x)=x^3+4x^2+4x+16$

$-2i \mid (2i)(-2i)$

0 4

6. Find the domain for: $y=\frac{2x-1}{4x^2+3x-3}$

$(2x-1)(2x+3)$

$x\neq 1/2$

$(-\infty,-3/2)\cup(-3/2,1/2)\cup(1/2,\infty)$

7. Find all asymptotes and holes for #6.

Hole (1/2, 4)

V.A. x = -3/2 HA: y = 0

8. Find the x and y intercepts of $y=\frac{3(x-4)(x+5)}{(x+7)(x-9)}$.

x int (4, 0) (-5, 0)

y int = (0, -60/-63) = (0, 20/21)

9. Find the smallest positive co-terminal angle with an angle measuring -213°.

-213 + 360 = 147°

10. Change -62° 11' 45" to radian measure. Round answer to 4 places.

$-62°11'45''\cdot\frac{\pi}{180}=-1.0855$

11. Change 1.24 radians to degrees. Round to 4 places.

$1.24\cdot\frac{180}{\pi}=\frac{223.2}{\pi}=71°2'48''$

12. Find the degree measure to the nearest tenth, of a central angle whose intercepted arc measures 16in for a circle with radius 12in.

$16=12\theta$

$4/3=\theta$

$4/3\cdot\frac{180}{\pi}=$

$\theta=76.4°$

13. Find the area to the nearest tenth, of a sector whose central angle measures 105° if the radius of the circle is 4.2 in.

$A=\frac{1}{2}(4.2)^2(7\pi/12)$

$105\cdot\frac{\pi}{180}=$

$A=16.2\ in^2$

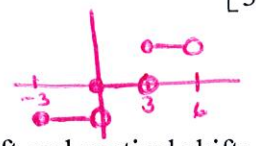
14. Find the reference angle for an angle measuring -124°. 56°

15. Describe the transformations in order then graph: $f(x) = -2|x+3| - 4$

left 3
reflected over x-axis
vertically stretched
down 4

16. Describe the transformations in order then graph: $y = \left[\frac{1}{3}x\right]$

horizontal stretch by 3



17. Find the amplitude, period, phase shift and vertical shift: $y = -\frac{1}{3}\cos\left(\frac{1}{2}x - \frac{3}{2}\right) + 5$

Amp = 1/3
Period = 4π
Phase = right 3
Vertical = up 5

18. Find the amplitude, period, phase shift and vertical shift: $f(x) = 4\sin(3x+2)$

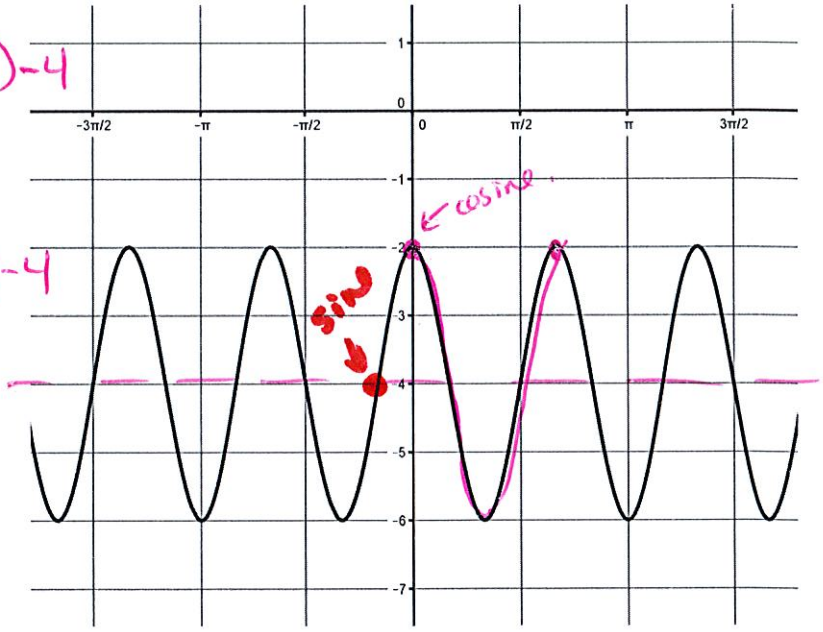
Amp = 4
Period = 2π/3
Phase = left 2/3
V.S = none

19. Write a sine and cosine equation for the following graph:

Period 2π/3

$y = 2\cos 3(x) - 4$

$y = 2\sin 3(x + \pi/6) - 4$



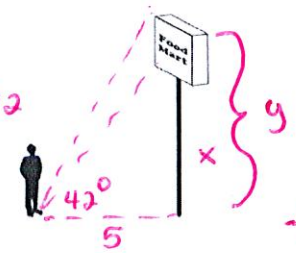
20. The sign for a resturant is mounted on a pole. From a position 5 m from the base of the pole, Mike has to look up 42° to see the bottom of the sign, and 52° to see the top of the sign. How tall is the sign?

$\tan 42 = \frac{x}{5}$ $x = 4.502$

$\tan 52 = \frac{y}{5}$ $y = 6.4$

height = Y − X

6.4 − 4.5 = 1.9 m



21. Graph on graph paper:

$$f(x) = \begin{cases} x^2; & x < -2 \\ -2; & -2 \le x < 1 \\ x+1; & x \ge 1 \end{cases}$$