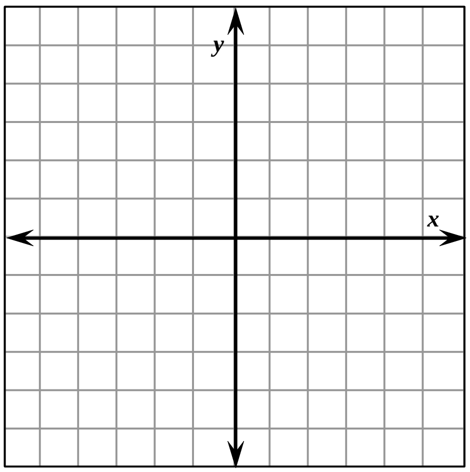
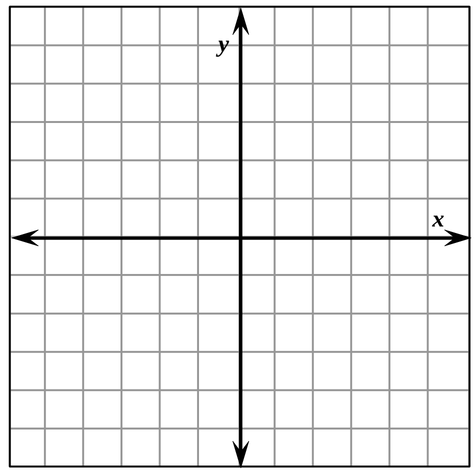
Exponential Function:
Has the form $y = a^x$, where $a > 0, a \neq 1,$ and x is any real number.

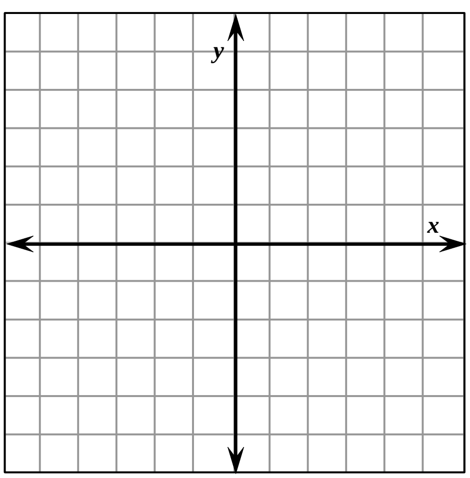
Graph $y = 2^x$



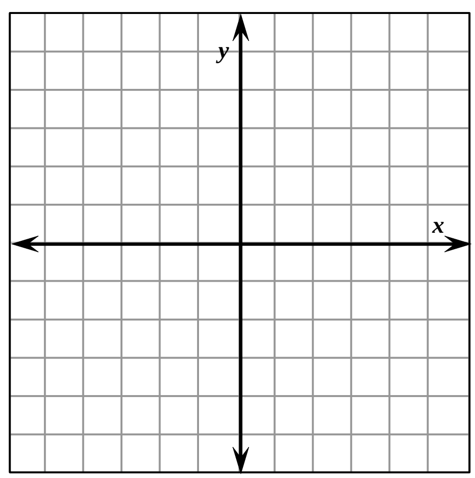
Graph $y = -2^{x+2} + 1$



Graph $y = \frac{1}{2}^{x} - 2$



Graph $y = 3\left(\frac{1}{2}\right)^{x+2} + 1$



The Natural Base *e* (Euler's number):

An irrational number, symbolized by the letter *e*, appears as the base in many applied exponential functions. This irrational number is approximately equal to 2.72. More accurately, $e = 2.71828\ldots$ The number *e* is called the natural base.
The function $f(x) = e^x$ is called the natural exponential function.

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

Evaluate:

a. e   b. e^3 c. e^{-2}

Formulas for Compound Interest:

After t years, the balance A in an account with principal P and annual interest rate r (expressed as a decimal) is given by the following formulas.

For n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

For continuous compounding: $A = Pe^{rt} \backslash$

Continuous is not offered but used to figure the maximum earnings at a given rate.

Ex: The total invested is \$1750 with an annual interest rate of 2% that is compounded yearly. How much money will be in the account after 7 years?

Ex: You have \$10,000 to invest at 4% interest compounded quarterly or continuously for 5 years. What is the difference in the investments?

Ex: Suppose your community has 4512 students this year. The student population is growing 2.5% each year.

Ex: Technetium-99 has a half-life of 6 hours. Suppose a lab has 80 mg of technetium-99. How much technetium-99 is left after 24 hours?

Ex: The half-life of ioding-131 is 8 days. Suppose you start with a 50-millicuries sample of iodine-131. How much iodine-131 is left after one half-life? After two half-lives?

Day 2

Definition of a Logarithmic Function:

The logarithmic function $y = \log_a x$, where a > 0 and $a \neq 1$, is the inverse of the exponential function $y = a^x$.

Logarithms to Exponential and vice versa:

$n = b^p$ <=> **$p = \log_b n$**

n = number
b = base
p = power

Ex: Express $\log_2 8 = 3$ as an exponent

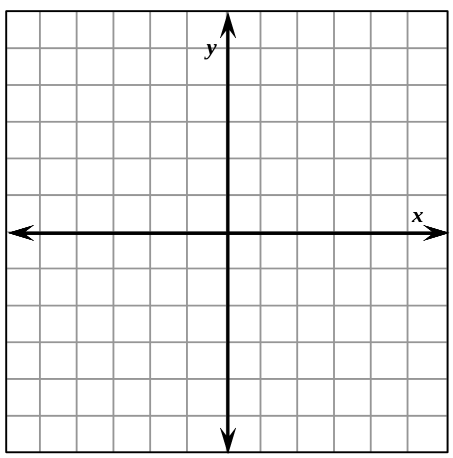
Ex: Express $3^{-4} = \frac{1}{81}$ as a log

Graphs of Exponential and Logarithm:

Exponential has
Horizontal Asymptote y = 0

Logarithm has
Vertical Asymptote x = 0

Composites $y = 3^x$ and $\log_3 x$



Inverse Properties of Logarithms:

For $b > 0$, $\log_b b^x = x$ The logarithm with base b of b raised to a power equals that power.

$b^{\log_b x} = x$ b raised to the logarithm with base b of a number equals that number.

Evaluate each:

a. $\log_4 4^x$ b. $\log_7 7^8$ c. $6^{\log_6 9}$ d. $3^{\log_3 17}$

Ex: Evaluate $\log_3 243 = x$

Ex: Evaluate $\log_x 4 = \frac{1}{2}$

Ex: Evaluate $\log_b 81 = 2$

Ex: Find the $\log_{10} .000001 =$

Ex: $\log_8 x = \frac{2}{3}$

Ex: $\log_3(4 + y) = \log_3(2y)$

Find the domain of each logarithmic function:

a. $f(x) = log_5(x + 4)$ b. $f(x) = log_5(x + 6)$ c. $f(x) = \log(2 - x)$

Solve for x:

Ex: $\log_b(3x - 6) = \log_b(x + 4)$

Ex: $\log_2(2x + 4) = \log_2(x - 5)$

Properties:

<u>**Exponential**</u>	<u>**Logs**</u>
1. $a^m * a^n = a^{m+n}$	1. $\log mn = \log m + \log n$
2. $a^m \div a^n = a^{m-n}$	2. $\log \frac{m}{n} = \log m - \log n$
3. $(a^m)^n = a^{m*n}$	3. $\text{Log } m^3 = 3\log m$
4. $a^{x_1} = a^{x_2} \iff x_1 = x_2$	4. $\log x_1 = \log x_2 \iff x_1 = x_2$

If $b > 0$ and $b \neq 1$ then $\log x_1 = \log_b x_2 \iff x_1 = x_2$
***make sure x_1 and x_2 does not equal a negative.**

Ex: $\log_3 56 - \log_3 8 = \log_3 x$

Ex: $4\log_{10} x = \log_{10} 16$

Ex: $\log_{10}(x^2 + 36) = \log_{10} 100$

Ex: $\log_2 3 + \log_2 7 = \log_2 x$

Day 3

Review Properties:

Ex: Expand $\log_{10} 5x^3y$

Ex: Condense $\frac{1}{3}((\ln 2 + \ln 6)) - \ln 4$

Solve:

Ex: $2 \log_6 4 - \frac{1}{3} \log_6 8 = \log_6 x$

Ex: $2\log_2 x - \log_2 (x + 3) = 2$

Given $\log_2 5 = 2.322$

Find $\log_2 20 =$

$\log_2 25 =$

Given $\log_{12} 9 = .884$ $\log_{12} 18 = 1.163$

Ex: Find $\log_{12} (\frac{3}{4}) =$

Ex: Find $\log_{12} 216 =$

Common log→ $f(x) = \log_{10} x$ base of 10
Natural log→ $f(x) = \log_e x = \ln x$ base of e

All properties of logarithms also hold for natural logarithms.

Graph $f(x) = \log_2 (x - 5)$

How do we graph logs with different bases?

Change of Base Formula: $\log_a n = \frac{\log_{10} n}{\log_{10} a}$

Ex: $\log_4 22 = \frac{\log_{10} 22}{\log_{10} 4}$

Ex: $\log_{12} 95 =$

Day 4

Exponential Equations:

Ex: $4^x = 24$

Ex: $7^x = 20$

Ex: $7^{x-2} = 5^{3-x}$

Ex: $e^x = 72$

Ex: $4e^{2x} = 5$

Ex: $e^{2x} - 3e^x + 2 = 0$

Ex: $5 + 2\ln x = 4$

Ex: $2\ln 3x = 4$

Ex: A population of a town increases according to the model

$$P(t) = 2500e^{.0293t} \quad \text{with } t = 0 \text{ corresponding to 1990}$$

What will the population be in 2010?

When will the population increase beyond 8,000 people?

Ex: The number of trees per acre N of a certain species is

$$N(t) = 68(10^{-04x}) \qquad 5 \le x \le 40$$

where x is the average diameter of trees 3 feet above the ground. Use the model to approximate the average diameter when N = 21.

Day 5
Exponential and Logarithmic Models

1. Exponential Growth and Decay Models

$$f(t) = A_0 e^{kt}$$

2. The table shows the growth of the minimum wage from 1970 through 2000. In, 1970, the minimum wage was $1.60 per hour. By 2000, it had grown to $5.15 per hour.

1970	1975	1980	1985	1990	1995	2000
$1.60	$2.10	$3.10	$3.35	$3.80	$4.25	$5.15

   a. Find the exponential growth function that models the date for 1970 through 2000.

   b. By which year will the minimum wage reach $7.50 per hour?

3. In 1990, the population of Africa was 643 million and by 2000 it had grown to 813 million.
   a. Use the exponential growth model $A = A_0 e^{kt}$, in which t is the number of years after 1990, to find the exponential growth function that models the data.

   b. By which year will Africa's population reach 2000 million, or two billion?

4. Use the fact that after 5715 years a given amount of carbon-14 will have decayed to half the original amount to find the exponential decay model for carbon-14.

   a. In 1947, earthenware jars containing what are known as the Dead Sea Scrolls were found by an Arab Bedouin herdsman. Analysis indicated that the scroll wrappings contained 76% of their original carbon-14. Estimate the age of the Dead Sea Scrolls.

5. Gaussian Models

$$y = ae^{-\frac{(x-b)^2}{c}}$$

This type of model is commonly used in probability and statistics to represent populations that are normally distributed. The graph of a Gaussian model is called a bell-shaped curve.

6. Logistic Growth Models

This model grows rapidly then has a declining rate of growth (growth slows down).

$$y = \frac{a}{1+be^{-rx}}$$

7. The function $f(t) = \frac{30{,}000}{1+20e^{-1.5t}}$ describes the number of people, $f(t)$, who have become ill with influenza t weeks after its initial outbreak in a town with 30,000 inhabitants.

   a. How many people became ill with the flu when the epidemic began?

   b. How many people were ill by the end of the fourth week?

   c. What is the limiting size of $f(t)$, the population that becomes ill?

8. The table represents the federal minimum wages that were not adjusted for inflation from 1970 through 2000.

x, Number of Years after 1969	y, Federal Minimum Wage
1	1.60
6	2.10
11	3.10
16	3.35
21	3.80
26	4.25
31	5.15

a. Use the graphing calculator to find an exponential model for this data.

b. Use the graphing calculator to find a logarithmic model for this data.

c. Use the graphing calculator to find a linear model for this data.

d. Use the graphing calculator to find a power model for this data.

e. Which model fits the data the best? Use this model to predict which year the minimum wage will reach $7.50. How does this answer compare to the answer we found in number 1?