5.6 Law of Sines

1. Law of Sines:

$$\frac{a}{b} = \frac{\sin A}{\sin B}$$

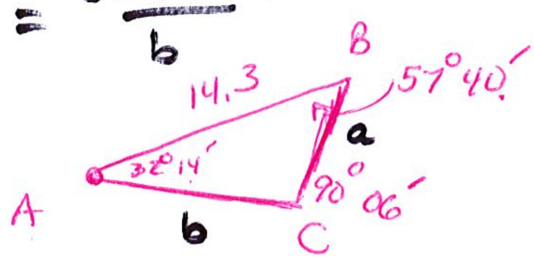
Let triangle ABC be any triangle with a, be and c representing the measures of the sides opposite the angle with measurements A, B, and C respectively. Then the following is true.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

2. Solve triangle ABC if $A = 32°14'$, $B = 57°40'$, and $c = 14.3$. Round angle measures to the nearest minute and side measures to the nearest tenth.

$$\frac{\sin 90°06'}{14.3} = \frac{\sin 57°40'}{b}$$



$$\frac{\sin 90°06'}{14.3} = \frac{\sin 32°14'}{a}$$

$$14.3 \sin 32°14' = a \cdot \sin 90°06'$$

$$\frac{14.3 \sin 32°14'}{\sin 90°06'} = a$$

$$7.63 \approx a$$

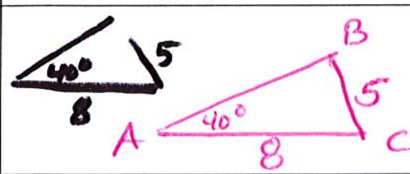
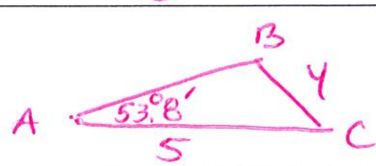
3. Recall the following from Geometry:

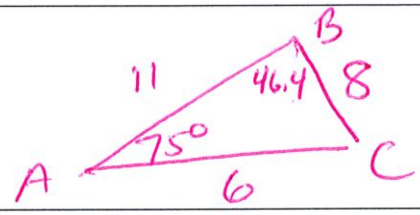
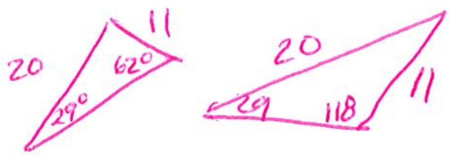
The sum of the lengths of two sides of a triangle is greater than the length of the third side.
SSS, SAS, AAS, ASA, and HL guarantee a unique triangle

SSA (two sides and the angle opposite one of them) *does not guarantee* a unique triangle (or even that a triangle exists). Two angle measures, one, or none may satisfy the value of the sine ratio when you use the law of sines given SSA. When two angle measures satisfy the sine ratio given SSA, two triangles can be determined. This is called the <u>Ambiguous</u> case.

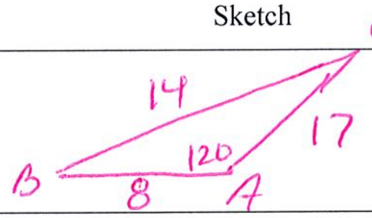
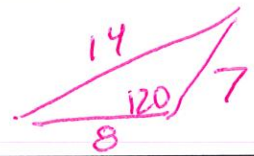
4. When the measures of two sides of a triangle and the measures of the angles opposite one of them are given (SSA), there may not always be one solution. However, one of the following will be true.
   a. No triangle exists. (no solution)
   b. Exactly one triangle exists. (one solution)
   c. Two triangles exist. (two solutions)

   If angle A is acute, there are four possible outcomes:

Number of Possibilites	Sketch	Condition Necessary for Case to Hold
0	 $\frac{\sin 40}{5} = \frac{\sin B}{8}$	$a < b \sin A$
1		$a = b \sin A$

1		$a > b\sin A$ and $a \geq b$
2		$b > a > b\sin A$

If angle A is obtuse, there are two possible outcomes:

Number of Possible Triangles	Sketch	Condition Necessary for Case to Hold
0	 $\frac{\sin 120}{14} = \frac{\sin B}{17}$	$a \leq b$
1		$a > b$

5. Determine whether each triangle has no solution, one solution, or two solutions. Then solve each triangle.

a. $A = 72°12', b = 22, a = 21$

b. $A = 58°, b = 14, a = 14$

$\frac{\sin 58}{14} = \frac{\sin B}{14}$ $B = 58$

CASE 1 $A = 58°$ $B = 58°$ $C = 64°$
$a = 14$ $b = 14$ $c =$

CASE 2 $A = 58°$ $B = 122°$ $c =$

$\frac{\sin 58}{14} = \frac{\sin 64}{c}$

c. $B = 33°, b = 2, a = 3.5$

$\frac{\sin 33}{2} = \frac{\sin A}{3.5}$

$A = 72.4$ $B = 33$ $C = 74.6$
$a = 3.5$ $b = 2$

$A = 107.6$ $B = 33$ $C = 39.4$
$a = 3.5$ $b = 2$

d. $B = 68°, b = 3, a = 5$

$\frac{\sin 68}{3} = \frac{\sin A}{5}$

$\sin A = \frac{5\sin 68}{3}$

$A = \sin^{-1}(\quad)$

NO SOLUTION!

e. $A = 124°, a = 1, b = 2$

NOT AMBIGUOUS

NO SOLUTION

f. $A = 99°, a = 2.5, b = 1.5$

$A = 14°$ $B = 105$
$a = 7$ $b =$

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