5 points each

1. $\int \frac{e^x}{1+e^x}\,dx$ $u = 1+e^x$, $du = e^x dx$ $\int \frac{du}{u}$ = $\boxed{\ln(1+e^x)+C}$

2. $\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$ $u=\ln x$, $du=\frac{1}{x}dx$ $\int \frac{du}{\sqrt{1-u^2}}$ $\boxed{\sin^{-1}(\ln x)+C}$

3. $\frac{1}{2}\int \frac{2\,dx}{\sqrt{1-4x^2}}$ $u=2x$, $du=2dx$ $\frac{1}{2}\int\frac{du}{\sqrt{1-u^2}}$ $\boxed{\frac{1}{2}\sin^{-1}(2x)+C}$

4. $-\frac{1}{6}\int -6x^2 e^{-2x^3}\,dx$ $u=-2x^3$, $du=-6x^2dx$ $\boxed{-\frac{1}{6}e^{-2x^3}+C}$

5. $-\int \frac{-\sin x\,dx}{\cos^2 x+1}$ $u=\cos x$, $du=-\sin x\,dx$ $-\int\frac{du}{1+u^2}$ $\boxed{-\tan^{-1}(\cos x)+C}$

6. $\frac{1}{5}\int \frac{dx}{\frac{5+x^2}{5}}$ $\frac{\sqrt{5}}{1}\cdot\frac{1}{\sqrt{5}}$ $u=\frac{x}{\sqrt{5}}$, $du=\frac{1}{\sqrt{5}}dx$ $\frac{1}{\sqrt{5}}\int\frac{du}{1+u^2}$ $\boxed{\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+C}$

7. $\int \frac{e^x\,dx}{\sqrt{1-e^{2x}}}$ $u=e^x$, $du=e^x dx$ $\int\frac{du}{\sqrt{1-u^2}}$ $\boxed{\sin^{-1}e^x+C}$

8. $\frac{1}{2}\int_1^3 \frac{2(x+2)}{\sqrt{x^2+4x+7}}$ $u=x^2+4x+7$, $du=2x+4$, $\frac{1}{2}\int u^{-1/2}du$ $u^{1/2}$ $\sqrt{x^2+4x+7}\Big]_1^3 = \sqrt{28}-\sqrt{12}$ ($2\sqrt{7}-2\sqrt{3}$)

9. $-\frac{1}{4}\int_0^{\ln 5} -4e^x(3-4e^x)\,dx$ $-\frac{1}{4}\int u\,du$ $-\frac{(3-4e^x)^2}{8}\Big]_0^{\ln 5}$ $\frac{-288}{8} = -\frac{144}{4}$ $-\frac{72}{2}$
$u=3-4e^x$, $du=-4e^x dx$ $-\frac{1}{8}(3-4(5))^2-\frac{1}{8}(3-4)^2 = -\frac{1}{8}(-17)^2+\frac{1}{8}(1)$ $\frac{-289}{8}+\frac{1}{8}$ (-36)

10. $\int_e^{e^2}\frac{\ln x}{x}\,dx$ $u=\ln x$, $du=\frac{1}{x}dx$ $\int u\,du$ $\frac{(\ln x)^2}{2}\Big]_e^{e^2}$ $\frac{(\ln e^2)^2}{2}-\frac{(\ln e)^2}{2}$ $\frac{4}{2}-\frac{1}{2}$ ($3/2$)

Find dy/dx

11. $y = 3^x \ln x$ $y' = 3^x\left(\frac{1}{x}\right)+\ln x\cdot 3^x\ln 3$ ($\frac{3^x}{x}+\ln x\cdot\ln 3\, 3^x$)

12. $y = \log_7(x^2+3x) = \frac{\ln(x^2+3x)}{\ln 7}$ ($y' = \frac{2x+3}{\ln 7\,(x^2+3x)}$)

13. $y = \sqrt{\tan^{-1}x}$ $\frac{\frac{1}{2}(\tan^{-1}x)^{-1/2}}{1+x^2}$ $\boxed{\frac{1}{2\sqrt{\tan^{-1}x}\,(1+x^2)}}$

14. $y = \cos^{-1}(e^{2x})$ ($\frac{-2e^{2x}}{\sqrt{1-e^{4x}}}$)

15. Find average value of $f(x) = \frac{1}{x}$ on [1,4]

$\frac{\int_1^4 \frac{1}{x}dx}{3}$ $\frac{\ln 4 - \ln 1}{3}$ $\frac{2}{3}\ln 2$ or $\frac{1}{3}\ln 4$

16. Solve $y' = 2x\sqrt{1-y^2}$ $y(0)=0$

$\int \frac{dy}{\sqrt{1-y^2}} = \int 2x\,dx$

$\sin^{-1} y = x^2 + C$

$y = \sin(x^2+C)$

$0 = \sin C$

$C=0$

$y = \sin x^2$

2 N.i. 100/108

17. The half life of radium 226 is 1590 yrs

If a sample has mass of 100 mg,

a) Find mass after 1000 yrs. $100\, e^{\frac{-\ln 2}{1590}(1000)}$

b) When will mass be 30 mgs.

$\frac{1}{2} = e^{1590K}$

$\frac{-\ln 2}{1590} = K$

$P = 100\, e^{\frac{-\ln 2}{1590}t}$

$30 = 100\, e^{\frac{-\ln 2}{1590}t}$

$.3 = e^{\frac{-\ln 2}{1590}t}$

$\ln .3 = \frac{-\ln 2}{1590}t$

$t = \frac{-1590 \ln .3}{\ln 2}$

18. The number of puppies in a population grows at a rate proportional to the square root of the number present at time t. If it starts with 16 puppies, and in 2 yrs there are 144 then when will there be 400 puppies?

$\frac{dP}{dt} = K\sqrt{P}$

$\int \frac{dP}{\sqrt{P}} = \int K\,dt$

$2\sqrt{P} = Kt + C$

$2 \cdot 4 = C \Rightarrow C = 8$

$2\sqrt{P} = Kt + 8$

$2\sqrt{144} = K(2) + 8$

$24 = 2K + 8$

$16 = 2K$

$8 = K$

$2\sqrt{P} = 8t + 8$

$\sqrt{P} = 4t + 4$

$P = (4t+4)^2$

$400 = (4t+4)^2$

4yrs

$4t + 4 = 20$

Kuta Software - Infinite Calculus

Name___________________________________

Differentiation - Inverse Trigonometric Functions

Date__________________ Period____

Differentiate each function with respect to *x*.

1) $y = \cos^{-1} -5x^3$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1 - (-5x^3)^2}} \cdot -15x^2$$
$$= \frac{15x^2}{\sqrt{1 - 25x^6}}$$

2) $y = \sin^{-1} -2x^2$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - (-2x^2)^2}} \cdot -4x$$
$$= -\frac{4x}{\sqrt{1 - 4x^4}}$$

3) $y = \tan^{-1} 2x^4$

$$\frac{dy}{dx} = \frac{1}{(2x^4)^2 + 1} \cdot 8x^3$$
$$= \frac{8x^3}{4x^8 + 1}$$

4) $y = \csc^{-1} 4x^2$

$$\frac{dy}{dx} = -\frac{1}{|4x^2|\sqrt{(4x^2)^2 - 1}} \cdot 8x$$
$$= -\frac{2}{x\sqrt{16x^4 - 1}}$$

5) $y = (\sin^{-1} 5x^2)^3$

$$\frac{dy}{dx} = 3 \cdot (\sin^{-1} 5x^2)^2 \cdot \frac{1}{\sqrt{1 - (5x^2)^2}} \cdot 10x$$
$$= \frac{30x \cdot (\sin^{-1} 5x^2)^2}{\sqrt{1 - 25x^4}}$$

6) $y = \sin^{-1} (3x^5 + 1)^3$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - ((3x^5 + 1)^3)^2}} \cdot 3(3x^5 + 1)^2 \cdot 15x^4$$
$$= \frac{45x^4(3x^5 + 1)^2}{\sqrt{1 - (3x^5 + 1)^6}}$$

7) $y = (\cos^{-1} 4x^2)^2$

$$\frac{dy}{dx} = 2\cos^{-1} 4x^2 \cdot -\frac{1}{\sqrt{1 - (4x^2)^2}} \cdot 8x$$
$$= -\frac{16x\cos^{-1} 4x^2}{\sqrt{1 - 16x^4}}$$

8) $y = \cos^{-1} (-2x^3 - 3)^3$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1 - ((-2x^3 - 3)^3)^2}} \cdot 3(-2x^3 - 3)^2 \cdot -6x^2$$
$$= \frac{18x^2(-2x^3 - 3)^2}{\sqrt{1 - (-2x^3 - 3)^6}}$$

Kuta Software - Infinite Calculus

Name________________________________

Integration

Date______________ Period____

Evaluate each indefinite integral.

1) $\int \frac{1}{\sqrt{16 - x^2}}\,dx$

2) $\int \frac{1}{4 + x^2}\,dx$

3) $\int \frac{1}{x\sqrt{x^2 - 1}}\,dx$

4) $\int \frac{1}{16 + x^2}\,dx$

5) $\int \frac{1}{x\sqrt{x^2 - 4}}\,dx$

6) $\int \frac{1}{\sqrt{25 - x^2}}\,dx$

7) $\int \frac{1}{x\sqrt{x^2 - 81}}\,dx$

8) $\int \frac{1}{4 + x^2}\,dx$

Kuta Software - Infinite Calculus

Name________________________________

Integration

Date______________ Period____

Evaluate each indefinite integral.

1) $\int \frac{1}{\sqrt{16 - x^2}}\,dx$

$\sin^{-1} \frac{x}{4} + C$

2) $\int \frac{1}{4 + x^2}\,dx$

$\frac{1}{2} \cdot \tan^{-1} \frac{x}{2} + C$

3) $\int \frac{1}{x\sqrt{x^2 - 1}}\,dx$

$\sec^{-1} |x| + C$

4) $\int \frac{1}{16 + x^2}\,dx$

$\frac{1}{4} \cdot \tan^{-1} \frac{x}{4} + C$

5) $\int \frac{1}{x\sqrt{x^2 - 4}}\,dx$

$\frac{1}{2} \cdot \sec^{-1} \frac{|x|}{2} + C$

6) $\int \frac{1}{\sqrt{25 - x^2}}\,dx$

$\sin^{-1} \frac{x}{5} + C$

7) $\int \frac{1}{x\sqrt{x^2 - 81}}\,dx$

$\frac{1}{9} \cdot \sec^{-1} \frac{|x|}{9} + C$

8) $\int \frac{1}{4 + x^2}\,dx$

$\frac{1}{2} \cdot \tan^{-1} \frac{x}{2} + C$

Kuta Software - Infinite Calculus Name________________

Integration by Substitution

Date________________ Period____

Evaluate each indefinite integral. Use the provided substitution.

1) $\int \frac{20x^3}{\sqrt{25 - 25x^8}}\, dx;\ u = 5x^4$

2) $\int \frac{10x^4}{9 + 4x^{10}}\, dx;\ u = 2x^5$

3) $\int -\frac{2 \cdot \csc^2 2x}{\cot(2x) \cdot \sqrt{\cot^2 2x - 1}}\, dx;\ u = \cot 2x$

4) $\int \frac{1}{x\sqrt{25 - (\ln -2x)^2}}\, dx;\ u = \ln -2x$

Evaluate each indefinite integral.

5) $\int \frac{8x}{\sqrt{9 - 16x^4}}\, dx$

6) $\int \frac{3x^2}{x^3\sqrt{x^6 - 1}}\, dx$

7) $\int \frac{10x}{16 + 25x^4}\, dx$

8) $\int -\frac{4\sin 4x}{\sqrt{9 - \cos^2 4x}}\, dx$

Kuta Software - Infinite Calculus Name________________

Integration by Substitution

Date________________ Period____

Evaluate each indefinite integral. Use the provided substitution.

1) $\int \frac{20x^3}{\sqrt{25 - 25x^8}}\, dx;\ u = 5x^4$

$\sin^{-1} \frac{5x^4}{5} + C$

2) $\int \frac{10x^4}{9 + 4x^{10}}\, dx;\ u = 2x^5$

$\frac{1}{3} \cdot \tan^{-1} \frac{2x^5}{3} + C$

3) $\int -\frac{2 \cdot \csc^2 2x}{\cot(2x) \cdot \sqrt{\cot^2 2x - 1}}\, dx;\ u = \cot 2x$

$\sec^{-1} |\cot 2x| + C$

4) $\int \frac{1}{x\sqrt{25 - (\ln -2x)^2}}\, dx;\ u = \ln -2x$

$\sin^{-1} \frac{\ln -2x}{5} + C$

Evaluate each indefinite integral.

5) $\int \frac{8x}{\sqrt{9 - 16x^4}}\, dx$

$\sin^{-1} \frac{4x^2}{3} + C$

6) $\int \frac{3x^2}{x^3\sqrt{x^6 - 1}}\, dx$

$\sec^{-1} |x^3| + C$

7) $\int \frac{10x}{16 + 25x^4}\, dx$

$\frac{1}{4} \cdot \tan^{-1} \frac{5x^2}{4} + C$

8) $\int -\frac{4\sin 4x}{\sqrt{9 - \cos^2 4x}}\, dx$

$\sin^{-1} \frac{\cos 4x}{3} + C$