Integration by Parts

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= uv' + vu'$$

$$uv = \int uv'\,dx + \int vu'\,dx$$

$$uv = \int u\,dv + \int v\,du$$

$$\int u\,dv = uv - \int v\,du$$

Choices of u and dv are critical.

1) Let dv be the most complicated. then u will be the remaining factor.

Ex: $\int x^2 \ln x \, dx$

$u = \ln x \qquad \int dv = \int x^2\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^3}{3} + C$

$$\int \underset{dv}{x^2}\,\underset{u}{\ln x}\,dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int \overset{\frac{x^2}{3}}{\frac{x^3}{3}}\left(\frac{1}{x}\right)dx$$

$$= \frac{x^3}{3}\ln x - \frac{1}{9}x^3 + C$$

Check! $\cancel{\frac{x^3}{3}\cdot\frac{1}{x}} + \ln x \cdot x^2 - \cancel{\frac{1}{3}x^2} = x^2 \ln x$

Ex! $\int x e^x \, dx$

$u = e^x \qquad \int dv = \int x$

$du = e^x dx \qquad v = \frac{1}{2}x^2 + C$

$= \frac{1}{2}x^2 e^x - \int \frac{1}{2}x^2 \cdot e^x$

$u = x \qquad \int dv = \int e^x$

$du = 1\, dx \qquad v = e^x + C$

$\int x e^x dx = x e^x - \int e^x (1)\, dx$

$= x e^x - e^x + C$

Check! $x \cdot e^x + \cancel{1e^x} - \cancel{e^x}$

Ex! $\int x^2 \sin x \, dx$

$u = x^2 \qquad \int dv = \int \sin x \, dx$

$du = 2x \qquad v = -\cos x + C$

$= -x^2 \cos x - \int -2x \cos x$

$= -x^2 \cos x + \int 2x \cos x \, dx$

$u = 2x \qquad \int dv = \int \cos x \, dx$

$du = 2\, dx \qquad v = +\sin x + C$

$\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x - \int 2 \sin x \, dx$

$= -x^2 \cos x + 2x \sin x + 2\cos x + C$

① Pick Logarithm
② Algebraic before trig. } Picking "u"
③ Exponential

Ex $\int_0^1 \arcsin x \, dx$

$u = \arcsin x \qquad \int dv = \int dx$

$du = \frac{1}{\sqrt{1-x^2}} dx \qquad v = x + C$

$\int_0^1 \arcsin x \, dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$

$u = 1 - x^2$

$du = -2x \, dx$

$= x \arcsin x + \frac{1}{2} \int u^{-1/2} du$

$= x \arcsin x + \frac{1}{2}(2) \cdot u^{1/2} + C$

$= x \arcsin x + \sqrt{1-x^2} \Big]_0^1$

$\pi/2 - 1 \approx .571$

Ex! $\int \sec^3 x \, dx$

$u = \sec x \qquad \int dv = \int \sec^2 x \, dx$

$du = \sec x \tan x \, dx \qquad v = \tan x + \cancel{C}$

$(\sec^2 x - 1)$

$\int \sec^3 x \, dx = \sec x \tan x - \int \sec x \tan^2 x \, dx$

$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$

$2 \int \sec^3 x = \sec x \tan x + \int \sec x \, dx$

$\int \sec^3 x \, dx = \frac{1}{2} \left[\sec x \tan x + \ln|\sec x + \tan x| + C \right]$

Ex! Find the avg value of $x^3 \cos(2x)$

$[0, \pi/2]$

$$\int x^3 \cos(2x)\, dx$$

$u = x^3 \qquad \int dv = \int \cos(2x)$

$du = 3x^2\, dx \qquad v = \frac{1}{2}\sin(2x) + C$

$$= \frac{1}{2}x^3 \sin(2x) - \int \frac{3}{2}x^2 \sin(2x)\, dx$$

$u = 3/2 x^2 \qquad \int dv = \int \sin(2x)$

$du = 3x \qquad v = -\frac{1}{2}\cos(2x)$

$$= \frac{1}{2}x^3 \sin(2x) - \left[-3/4 x^2 \cos(2x) - \int -\frac{3}{2}x \cos(2x)\, dx \right]$$

$$= \frac{1}{2}x^3 \sin(2x) + 3/4 x^2 \cos(2x) - \int \frac{3}{2}x \cos(2x)\, dx$$

$u = 3/2 x \qquad dv = \cos(2x)$

$du = 3/2\, dx \qquad v = \frac{1}{2}\sin(2x) + C$

$$= \frac{1}{2}x^3 \sin(2x) + 3/4 x^2 \cos(2x) - 3/4 x \sin(2x) + \int \frac{3}{4}\sin(2x)\, dx$$

$$= \frac{\left.\frac{1}{2}x^3 \sin(2x) + 3/4 x^2 \cos(2x) - 3/4 x \sin(2x) - 3/8 \cos(2x)\right]_0^{\pi/2}}{\pi/2 - 0}$$