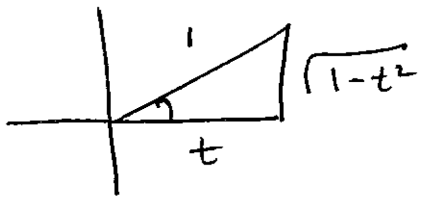
49. $\sin(\arccos t) = \sqrt{1-t^2}$

$\cos(\cancel{\arccos t}) \cdot \frac{-1}{\sqrt{1-t^2}} = \frac{-t}{\sqrt{1-t^2}}$



$f(x) = \sqrt{1-t^2}$

$f'(x) = 1/2\,(1-t^2)^{-1/2} \cdot -2t$

$\sin^{-1}(u) = \int \frac{u'}{\sqrt{1-u^2}}$ $\qquad = \frac{-t}{\sqrt{1-t^2}}$

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$

$\int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \sec^{-1} \frac{|u|}{a} + C$

1) $\int \frac{dx}{\sqrt{\frac{4-x^2}{4}}}$ or $\int \frac{1}{2} \frac{dx}{\sqrt{1-\frac{x^2}{4}}} = \frac{1}{2} \int \frac{dx}{\sqrt{1-\frac{x^2}{4}}} = \frac{1}{2} \cdot 2 \int \frac{du}{\sqrt{1-u^2}}$

$\sqrt{4(1-x^2/4)}$ $\qquad = \sin^{-1}\left(\frac{x}{2}\right) + C$ $\qquad u^2 = \frac{x^2}{4}$, $u = \frac{x}{2}$, $du = 1/2\,dx$

$\int \frac{du}{\sqrt{a^2-u^2}} = \int \frac{dx}{\sqrt{4-x^2}} = \sin^{-1}\left(\frac{x}{2}\right) + C$

$a = 2 \quad u = x$

$$\int \frac{dx}{\frac{2+9x^2}{2}} = \frac{1}{2}\int \frac{dx}{1+\frac{9}{2}x^2} = \frac{\sqrt{2}}{3}\cdot\frac{1}{2}\int\frac{du}{1+u^2}$$

$u^2 = \frac{9}{2}x^2$

$u = \frac{3}{\sqrt{2}}x$

$du = \frac{3}{\sqrt{2}}dx$

$$= \frac{\sqrt{2}}{6}\tan^{-1}\left(\frac{3}{\sqrt{2}}x\right)+C$$

$x = 3\theta$

$$\frac{du}{a^2+u^2} = \frac{du}{2+9x^2}$$

$a = \sqrt{2}$

see!

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$\cancel{\frac{1}{3}}\ \boxed{\cancel{2}}\ \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x}{\sqrt{2}}\right)+C\cdot\frac{1}{3}$$

$$\int\frac{dx}{x\sqrt{\frac{4x^2-9}{9}}} = \int\frac{dx}{x\sqrt{\frac{4}{9}x^2-1}} = \frac{1}{3}\cdot\frac{3}{2}\int\frac{du}{\frac{3}{2}u\sqrt{u^2-1}}$$

$u^2 = \frac{4}{9}x^2$

$u = \frac{2}{3}x$ $\quad x = \frac{3}{2}u$ $\quad = \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\int\frac{du}{u\sqrt{u^2-1}}$

$\frac{|2x|}{3} = \frac{2}{3}|x|$ $\quad du = \frac{2}{3}dx$

$$= \frac{1}{3}\sec^{-1}\frac{|2x|}{3}+C$$

$$\int\frac{dx}{\sqrt{e^{2x}-1}} = \int\frac{du}{e^x\sqrt{u^2-1}} = \int\frac{du}{u\sqrt{u^2-1}}$$

$u^2 = e^{2x}$

$u = e^x$

$du = e^x\,dx \quad dx = \frac{du}{ex}$

$$= \sec^{-1}|e^x|+C$$

$$= \sec^{-1}(e^x)+C$$

Find. $\int \frac{x-1}{\sqrt{9-x^2}}$

$$\int \frac{x}{\sqrt{9-x^2}}\,dx - \int \frac{1}{\sqrt{\frac{9-x^2}{9}}}\,dx$$

$u = 9-x^2$

$du = -2x\,dx$

$$-\frac{1}{3}\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}$$

$$\frac{-1}{2}\int u^{-1/2}\,du$$

$u^2 = \frac{x^2}{9}$

$u = x/3$

$du = 1/3\,dx$

$$-\frac{1}{2}\cdot 2\,u^{1/2}$$

$$-\frac{1}{3}\cdot\frac{3}{1}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$-\sqrt{9-x^2} - \sin^{-1}\left(\frac{x}{3}\right) + C$$

$$\int \frac{dx}{x^2-6x+11}$$

$(x^2-6x+9)+11-9$

$(x-3)^2+2$

$$\int \frac{dx}{\frac{(x-3)^2}{2}+\frac{2}{2}} = \frac{\;}{2}\int \frac{dx}{u^2+1} \qquad \frac{\sqrt{2}}{2}\int \frac{du}{u^2+1}$$

$u^2 = \frac{(x-3)^2}{2}$

$u = \frac{x-3}{\sqrt{2}}$

$du = 1/\sqrt{2}\,dx$

$$\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{x-3}{\sqrt{2}}\right)+C$$