Review Ch 4

1. Evaluate the integral: $\int \frac{4 + 5x^{3/2}}{\sqrt{x}}\,dx$.

(a) $2x^{-3/2} + \frac{5}{2}x^2 + C$ (b) $8\sqrt{x} + \frac{5}{2}x^2 + C$ (c) $-2x^{-3/2} + 5 + C$

(d) $2\sqrt{x} + \frac{5}{2}x^2 + C$ (e) None of these

2. Use $a(t) = -32$ ft/s² as the acceleration due to gravity. A ball is thrown vertically upward from the ground with an initial velocity of 96 feet per second. How high will the ball go?

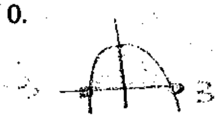
(a) 32 feet (b) 64 feet (c) 24 feet

(d) 144 feet (e) None of these

$-144 + 288$ 144 f

3. Write the definite integral for the area of the region bounded by the graphs of $y = 9 - x^2$ and $y = 0$.

$0 = 9 - x^2$



$\int_{-3}^{3}(9-x^2)\,dx$ $2\left[9x - \frac{x^3}{3}\right]_0^3$ $2[27-9]$

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4. Choose the correct statement given that $\int_0^7 f(x)\,dx = 8$ and $\int_1^7 f(x)\,dx = -3$.

(a) $\int_7^1 f(x)\,dx = -3$ (b) $\int_0^1 f(x)\,dx = 5$ (c) $\int_1^0 f(x)\,dx = 11$

(d) $\int_0^1 f(x)\,dx = 11$ (e) None of these

5. If $\int_2^5 f(x)\,dx = 5$ and $\int_4^5 f(x)\,dx = \pi$, find:

a. $\int_5^5 f(x)\,dx$ 0 b. $\int_5^4 f(x)\,dx$ $-\pi$ c. $\int_2^4 f(x)\,dx$ $5-\pi$

6. evaluate $\int_1^2 \frac{1}{x^2}\,dx$.

(a) $-\frac{1}{x} + C$ (b) $-\frac{3}{4}$ (c) $\frac{1}{2}$

(d) $-\frac{3}{2}$ (e) None of these

7. Evaluate the integral: $\frac{1}{2}\int x(x^2 - 1)^4\,dx$.

$u = x^2 - 1$
$du = 2x\,dx$

$\frac{1}{2}\int u^4\,du$

$\frac{1}{2}\frac{u^5}{5} + C$

$\frac{1}{10}(x^2-1)^5 + C$

(a) $\frac{1}{10}(x^2)(x^2 - 1)^5$ (b) $\frac{1}{10}(x^2 - 1)^5 + C$ (c) $\frac{1}{5}(x^3 - x)^5 + C$

(d) $\frac{1}{5}(x^2 - 1)^5 + C$ (e) None of these

8. Evaluate the integral: $\frac{1}{3}\int 3\sin^3 3x \cos 3x\,dx$.

$\frac{1}{3}\int u^3\,du$ $\frac{1}{12}u^4 + C$

(a) $\frac{1}{8}\sin^4 3x \cos^2 3x + C$ (b) $\frac{1}{4}\sin^4 3x + C$ (c) $3\sin^2 3x(3\cos^2 3x - \sin^2 3x) + C$

(d) $\frac{1}{12}\sin^4 3x + C$ (e) None of these

$u = \sin(3x)$
$du = 3\cos(3x)\,dx$

$\frac{1}{12}(\sin(3x))^4 + C$

9. Determine which of the following is not equal to the definite integral $\int_2^7 axf(x)\,dx$.

(a) $a\int_2^7 xf(x)\,dx$ (b) $x\int_2^7 af(x)\,dx$ (c) $-\int_7^2 axf(x)\,dx$

(d) $\int_2^4 axf(x)\,dx + \int_4^7 axf(x)\,dx$ (e) None of these

$x = 1 - u$
$u = 1 - x$
$du = -1\,dx$

10. Evaluate the integral: $\int x\sqrt{1 - x}\,dx$.

$= \int (1-u)u^{1/2}\,du$

$= -\int (u^{1/2} - u^{3/2})\,du$

$\frac{2}{3}u^{3/2} + \frac{2}{5}u^{5/2}$

(a) $-\frac{x^2}{3}(1 - x)^{3/2} + C$ (b) $\frac{2 - 3x}{2\sqrt{1 - x}} + C$ (c) $\frac{x^2}{3}(1 - x)^{3/2} + C$

(d) $-\frac{2}{15}(2 + 3x)(1 - x)^{3/2} + C$ (e) None of these

$(1-x^{3/2})\left(-\frac{2}{3}+\frac{2}{5}(1-x)\right)$ $-\frac{2}{3}(1-x)^{3/2} + \frac{2}{5}(1-x)^{5/2} + C$

$(1-x^{3/2})\left(-\frac{2}{3}+\frac{2}{5}-\frac{2}{5}x\right) = (1-x^{3/2})\left(-\frac{4}{15}-\frac{2}{5}x\right) = -\frac{2}{15}(1-x)^{3/2}(2+3x)$

$\frac{-10+6}{15} = -\frac{4}{15}$

11. An object has a constant acceleration of 72 feet per second squared, an initial velocity of 17 feet per second, and an initial position of 10 feet. Find the position function describing the motion of this object.

(a) $s = 36t^2 + 17t + 10$ (b) $s = 36t^2 + 27$ (c) $s = 72t^2 + 10$

(d) $s = 72t^2 + 17t + 10$ (e) None of these

$a(t) = 72$
$v(t) = 72t + 17$
$s(t) = 36t^2 + 17t + 10$

12. Evaluate the integral: $\int 5\sec x \tan x\,dx$.

(a) $5\sec^3 x \tan x + C$ (b) $5\sec x + C$ (c) $\frac{1}{5}\sec^3 x \tan x + C$

(d) $5[\sec^3 x + \sec x \tan^2 x] + C$ (e) None of these

$u = 3 - 7x^2$

$du = -14x\,dx$

13. evaluate the integral: $\int x\sqrt{3 - 7x^2}\,dx$.

(a) $-\frac{1}{21}(3 - 7x^2)^{3/2} + C$ (b) $-\frac{4}{21}(3 - 7x^2)^{3/2} + C$ (c) $-\frac{1}{14}(3 - 7x^2)^{3/2} + C$

(d) $\frac{2}{3}(3 - 7x^2)^{3/2} + C$ (e) None of these

$-\frac{1}{14}\int u^{1/2}\,du$

$-\frac{1}{14}\cdot\frac{2}{3}u^{3/2}$

$-\frac{1}{21}(3-7x^2)^{3/2} + C$

14. Evaluate the integral: $\int x \sec^2 x^2\,dx$.

(a) $\frac{1}{6}x^3 \sec^3 x^2 + C$ (b) $\frac{1}{2}\tan x^2 + C$ (c) $\frac{1}{2}x^2 \tan x^2 + C$

(d) $\tan x^3 + C$ (e) None of these

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}\int \sec^2 u\,du$

$\frac{1}{2}\tan u + C$

$\frac{1}{2}\tan x^2 + C$

15. Evaluate the integral: $\int \frac{5x}{\sqrt{x+2}}\,dx$.

(a) $\frac{2}{3}\sqrt{x+2}(x - 4) + C$ (b) $\frac{5}{2}\left(x - 4\ln\sqrt{x+2}\right) + C$ (c) $\frac{10}{3}\sqrt{x+2}(x - 4) + C$

(d) $5\left(x - 4\ln\sqrt{x+2}\right) + C$ (e) None of these

$u = x+2$

$du = dx$

$(x+2)^{1/2}\left[\frac{10}{3}(x+2) - 4\right]$

16. Find the value of c guaranteed by the Mean Value Theorem for Integrals for $f(x) = \frac{4}{x^2}$ on the interval [1, 4].

$\frac{\int_1^4 \frac{4}{x^2}\,dx}{3} = \frac{\int_1^4 4x^{-2}\,dx}{3}$

Av value = 1

$1 = \frac{4}{x^2}$

$\boxed{2}$

17. evaluate the integral: $\int_3^5 \frac{5 + 6x + x^2}{5 + x}\,dx$.

(a) -10 (b) -8 (c) 10

(d) 0 (e) None of these

$\frac{(x+5)(x+1)}{(x+5)}$

18. Find the function, $y = f(x)$, if $f'(x) = 2x - 1$ and $f(1) = 3$.

$y = x^2 - x + C$

$3 = 1 - 1 + C$

$3 = C$

$y = x^2 - x + 3$

19. Find the average value of $f(x) = 3x^2 - 2$ on the interval $[0, 2]$.

$$\frac{\int_0^2 (3x^2-2)\,dx}{2} =$$

(a) 1 (b) 5 (c) 6

(d) 2 (e) None of these

$$\frac{x^3 - 2x \Big]_0^2}{2}$$

$$\frac{8-4}{2}$$

20. Find the average value of $f(x) = \sin x$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

$$\frac{\int_{\pi/4}^{\pi/2} \sin x \cdot dx}{\pi/2 - \pi/4} = \frac{-\cos x \Big]_{\pi/4}^{\pi/2}}{\pi/4}$$

$$\frac{-\cos\frac{\pi}{2} - -\cos\frac{\pi}{4}}{\frac{\pi}{4}} = \frac{0 + \frac{\sqrt{2}}{2}}{\frac{\pi}{4}} \quad \frac{\sqrt{2}}{2} \cdot \frac{4}{\pi}$$

$$\frac{2\sqrt{2}}{\pi}$$

21. evaluate the integral: $\int_{\pi/2}^{5\pi/4} \sin x\,dx$.

(a) $-\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}} - 1$ (c) $\frac{\sqrt{3}}{2}$

(d) $\frac{1}{\sqrt{2}}$ (e) None of these

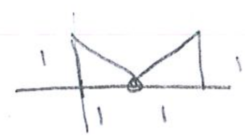
$$\int_{\pi/2}^{5\pi/4} \sin x\,dx \quad -\cos x \Big]_{\pi/2}^{5\pi/4} = -\cos 5\pi/4 - -\cos \pi/2$$

$$-\frac{\sqrt{2}}{2} - -0 \quad -\sqrt{2}/2$$

22. evaluate: $\int_0^2 |x-1|\,dx$.

(a) 0 (b) 1 (c) $\frac{1}{2}$

(d) 2 (e) None of these



23. Consider $F(x) = \int_x^1 \sqrt{1+t^2}\,dt$. Find $F'(x)$.

(a) $\sqrt{1+x^2}$ (b) $\frac{1}{\sqrt{2}} - \frac{x}{\sqrt{1+x^2}}$ (c) 1

(d) $-\sqrt{1+x^2}$ (e) None of these

$$-\sqrt{1+x^2}$$

24. Consider $F(x) = \int_1^{x^2} (t^3 + \sqrt{t})\,dt$. Find $F'(x)$.

$$2x\,(x^6 + |x|)$$

25. evaluate the integral: $\int x\sqrt{4-9x^2}\,dx$.

(a) $-\frac{1}{27}(4-9x^2)^{3/2} + C$ (b) $-\frac{1}{18}(4-9x^2)^{3/2} + C$ (c) $\frac{2}{3}(4-9x^2)^{3/2} + C$

(d) $-\frac{4}{27}(4-9x^2)^{3/2} + C$ (e) None of these

$$\boxed{-\frac{1}{27}(4-9x^2)^{3/2} + C}$$

$u = 4 - 9x^2$

$du = -18x\,dx$

$$\frac{2}{3} \quad -\frac{1}{18}\int u^{1/2}\,du = -\frac{2}{54}u^{3/2} + C$$

1. A particle moves along the x-axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position is $x(1) = 9$.

(a) Write an expression for the velocity of the particle $v(t)$. $v(t) = 6t^2 - 18t + 12$

(b) At what values of t does the particle change direction? $t = 1, 2$

(c) Write an expression for the position $x(t)$ of the particle. $x(t) = 2t^3 - 9t^2 + 12t - 4$

(d) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 6$.

$x(3/2) = 1/2$ $\quad > 1/2$

$x(2) = 0$ $\quad$ Total distance $= 176.5$

$x(6) = 176$ $\quad > 176$

2. **Let the function f be defined as follows:**

$f(x) = x^3$ **for** $x > 1$,

$f(x) = 3x - 2$ **for** $x \leqq 1$.

(a) Show that f is continuous at $x = 1$.

(b) Use the definition of derivative to show that $f'(1)$ exists.

(c) Is f' continuous at $x = 1$? Why?

(d) Evaluate $\int_0^2 f(x)\,dx$.

$$\int_0^2 f(x)\,dx = \int_0^1 x^3\,dx + \int_1^2 (3x-2)\,dx$$

$$= \frac{x^4}{4}\Big]_0^1 + \frac{3x^2}{2} - 2x\Big]_1^2 = (6-4) - (3/2 - 2)$$

$$4 - 3/2$$

$$= 1/4 + 5/2 \quad 2\,3/4 = 11/4$$

3. The average value of $f(x) = 3 + |x|$ ~~on the interval~~ $[-2, 4]$ is

(A) $2\frac{2}{3}$ (B) $3\frac{1}{3}$ (C) $4\frac{2}{3}$ (D) $5\frac{1}{3}$ (E) 6

$$\frac{\int_{-2}^4 (3+|x|)\,dx}{4 - -2} = \frac{28}{6} =$$

4. $\int_0^3 \sqrt{9-x^2}\,dx \qquad \frac{1}{4}(\pi \cdot 9) = \frac{9}{4}\pi$

$$\frac{9\pi}{4}$$