Day 7 Notes: Implicit Differentiation:

Ex: $x^2 - 4x + y^2 = 21$ is a circle and has a well-defined slope at nearly every point because it is the union of two graphs $y = \pm\sqrt{-x^2 + 4x + 21}$ which are both differentiable.

The question is how do we find the slopes ? We need to treat y as a differentiable function of x and differentiate both sides with respect to x. Then solve for $\frac{dy}{dx}$.

Ex: $x^2 - 4x + y^2 = 21$ to find the derivative: $2x - 4 + 2y\frac{dy}{dx} = 0$

$$2y\frac{dy}{dx} = -2x + 4$$

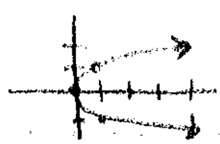
$$\frac{dy}{dx} = \frac{-2x + 4}{2y}$$

Ex: $y^2 = x$ at x = 4 we differentiate the left side using the chain rule. $2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y}$ the y is useful because it shows we have two tangent lines. Write the equations of both:

(4, 2)

(4, -2)



$y - 2 = 1/4(x - 4)$

$y + 2 = -1/4(x - 4)$

Ex: Find the slope of the circle at $x^2 + y^2 = 25$ at the point (3, -4).

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x}{y} = \frac{3}{4}$

2nd derivative.

$\frac{y \cdot 1 - [-x\,dy/dx]}{y^2}$

$\frac{-y + 3/4\,x}{y^2} = \frac{3x - 4y}{4y^2}$

Find the tangent lines at the given point of each curve.

normal ?

Ex: $y^2 - 2x - 4y = 1$ at (-2, 1)

$2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{2}{2y - 4} = -1$

$y - 1 = -1(x + 2)$

Ex: $2xy + \pi\sin(y) = 2\pi$ at $\left(1, \frac{\pi}{2}\right)$

$2x\,dy/dx + 2y + \pi\cos(y)\,dy/dx = 0$

$\frac{dy}{dx} = \frac{-2y}{2x + \pi\cos y} = \frac{-\pi}{2 + 0} = -\pi/2$

$y - \pi/2 = -\pi/2(x - 1)$

Ex: $x\sin(2y) = y\cos(2x)$ at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$x\cos 2y \cdot 2\,dy/dx + \sin(2y) = y \cdot -\sin(2x) \cdot 2 + \frac{dy}{dx}\cos(2x)$

$\frac{dy}{dx} = \frac{-2y\sin(2x) - \sin(2y)}{2x\cos(2y) - \cos(2x)}$

$y - \pi/2 = 0 \cdot (x - \pi/4)$

Ex: $x^2\cos^2(y) - \sin(y) = 0$ at $(0, \pi)$

$x^2 \cdot 2\cos(y)(-\sin(y))\,dy/dx + \cos^2 y \cdot 2x - \cos y\,dy/dx$

$\frac{dy}{dx} = \frac{2x\cos y}{1 + 2x^2\sin y} = 0$

$y = \pi$

Ex: $x + \tan(xy) = 0$

$1 + \sec^2(xy) \cdot [x\,dy/dx + y] = 0$

$x\sec^2(xy) = -1 + y\sec^2(xy)$

$= -1/x\cos^2(xy) - y/x$