① $5y\frac{dy}{dx} + \cos y \cdot \frac{dy}{dx} = 2x$

$$\frac{dy}{dx} = \frac{2x}{10y + \cos y}$$

② $3x^2 + x\frac{dy}{dx} + y - 2 = 0$

$$\frac{dy}{dx} = \frac{2 - y - 3x^2}{x}$$

③ $\frac{1}{2}y^{-1/2} \cdot \frac{dy}{dx} - \cos x = 0$

$$\frac{1}{\sqrt{2y}} \cdot \frac{dy}{dx} = \cos x$$

$$\frac{dy}{dx} = \sqrt{2y} \cdot \cos x$$

④ $x^2\frac{dy}{dx} + 2xy + 3x \cdot 3y^2\frac{dy}{dx} + 3y^3 = 1$

$$x^2\frac{dy}{dx} + 9xy^2\frac{dy}{dx} = 1 - 2xy - 3y^3$$

$$\frac{dy}{dx} = \frac{1 - 3y^3 - 2xy}{x^2 + 9xy^2}$$

⑤ $-x^{-2} + -y^{-2}\frac{dy}{dx} = 0$

$$\frac{-1}{x^2} - \frac{1}{y^2} \cdot \frac{dy}{dx} = 0$$

$$-\frac{1}{y^2}\frac{dy}{dx} = +\frac{1}{x^2}$$

$$\frac{dy}{dx} = \frac{-y^2}{x^2}$$

⑥ $\cos(x^2y^2) \cdot [x^2 \cdot 2y\frac{dy}{dx} + 2xy^2] = 1$

$$\frac{dy}{dx} = \frac{1 - 2xy^2\cos(x^2y^2)}{x^2 \cdot 2y \cdot \cos(x^2y^2)}$$

⑦ $3(\tan(xy^2+y))^2 \cdot \sec^2(xy^2+y) \cdot [x\,2y\frac{dy}{dx} + y^2 + \frac{dy}{dx}] = 1$

$$\frac{dy}{dx} = \frac{1 - 3y^2(\tan(xy^2+y))^2\sec^2(xy^2+y)}{(6xy\tan(xy^2+y))^2\sec^2(xy^2+y) + 3(\tan(xy^2+y))^2 \sec^2(xy^2+y)}$$

8. $\sin(x+y) = y^2\cos x$

$$\cos(x+y) \cdot [1 + \frac{dy}{dx}] = y^2 \cdot (-\sin x) + 2y\frac{dy}{dx}\cos x$$

$$\cos(x+y) - 2y\cos x = -y^2\sin x - \cos(x+y)$$

$$\frac{dy}{dx} = \frac{-y^2\sin x - \cos(x+y)}{\cos(x+y) - 2y\cos x}$$

9. $-4\sin x\sin y + 4\cos x \cdot \cos y\frac{dy}{dx} =$

$$\frac{dy}{dx} = \frac{4\sin x\sin y}{4\cos x\cos y}$$

10. $\frac{1}{3}(x+y)^{-2/3}[1 + \frac{dy}{dx}] = x^2 2y\frac{dy}{dx} + 2x^2y^2$

$$\frac{1}{3\sqrt[3]{(x+y)^2}} + \frac{dy}{dx}\left[\frac{1}{3\sqrt[3]{(x+y)^2}}\right] = x^2 \cdot 2y\frac{dy}{dx} + 2x$$

$$\frac{dy}{dx}\left[\frac{1}{3\sqrt[3]{(x+y)^2}} - x^2 2y\right] =$$

$$\frac{dy}{dx} = \frac{6x^2y^2 3\sqrt[3]{(x+y)^2} - 1}{1 - 6x^2y\sqrt[3]{(x+y)^2}}$$

11. $\frac{1}{2}(xy)^{-1/2} \cdot [x + \frac{dx}{dy}] = x^3 + 3x^2\frac{dx}{dy}$

$$\frac{x}{2\sqrt{xy}} + \frac{dx}{dy}\left[\frac{1}{2\sqrt{xy}}\right] = x^3 + 3x^2\frac{dx}{dy}$$

$$x + \frac{dx}{dy}[1] = 2x^3\sqrt{xy} + 6x^2\sqrt{xy}$$

$$\frac{dx}{dy} = \frac{2x^3\sqrt{xy} - x}{1 - 6x^2\sqrt{xy}}$$

12. $\cos x\frac{dx}{dy} + \cos y = \sin x\cos y + \cos x\frac{dx}{dy} \cdot \sin y$

$$\frac{dx}{dy}[\cos x - \cos x\sin y] = \sin x\cos y - \cos y$$

$$\frac{dx}{dy} = \frac{\sin x\cos y - \cos y}{\cos x - \cos x\sin y}$$

13. $\sec^2\left(\frac{x}{y}\right) \cdot \frac{y \cdot \frac{dx}{dy} - x}{y^2} = \frac{dx}{dy} + 1$

$y\sec^2\left(\frac{x}{y}\right) \cdot \frac{dx}{dy} - x\sec^2\left(\frac{x}{y}\right) = y^2\frac{dx}{dy} + y^2$

$$\frac{dx}{dy} = \frac{y^2 + x\sec^2\left(\frac{x}{y}\right)}{y\sec^2\left(\frac{x}{y}\right) - y^2}$$

14. $\frac{1}{2}y^{-1/2} - \cos x \frac{dx}{dy} = 2\frac{dx}{dy} + 1$

$$\frac{dx}{dy} = \frac{1 - 2\sqrt{y}}{2\sqrt{y}(2 + \cos x)} = \frac{1 - 2\sqrt{y}}{4\sqrt{y} + 2\sqrt{y}\cos x}$$

15. $2x\frac{dx}{dy} + x + \frac{dx}{dy} \cdot y + 2y$

$$\frac{dx}{dy} = \frac{-x - 2y}{2x + y}$$

16. $8x + 4y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{y}$ $\quad \frac{d^2y}{dx^2} = \frac{y \cdot -2 - -2x\frac{dy}{dx}}{y^2}$

$$\frac{d^2y}{dx^2} = \frac{-2y + 2x\frac{dy}{dx}\left[-\frac{2x}{y}\right]}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-2y^2 - 4x^2}{y^3}$$

17. $\frac{dy}{dx} + \cos(y) \cdot \frac{dy}{dx} = 1$

$$\frac{dy}{dx} = \frac{1}{1 + \cos y}$$

$$\frac{d^2y}{dx^2} = \frac{-\sin(y)\frac{dy}{dx}}{(1 + \cos y)^2} = \frac{-\sin(y)}{(1 + \cos(y))^3}$$

18. $x \cdot (-\sin y)\frac{dy}{dx} + \cos y = \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-\cos y}{-x\sin(y) - 1} = \frac{\cos y}{x\sin(y) + 1}$$

$\frac{d^2y}{dx^2} = [x\sin(y) + 1] \cdot -\sin y\frac{dy}{dx} -$

19) $2x\frac{dy}{dx} + 2y - 2y\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-2y}{2x - 2y} = \frac{-y}{x - y}$$

$$\frac{d^2y}{dx^2} = \frac{(x - y)\frac{dy}{dx} - \left[-y\left(1 - \frac{dy}{dx}\right)\right]}{(x - y)^2}$$

$$= \frac{-y - \left[-y + y\frac{dy}{dx}\right]}{(x - y)^2}$$

$$= -y + y - y\left(\frac{-y}{x - y}\right)$$

$$= \frac{y^2}{(x - y)^3}$$

20. $x^3 \cdot 3y^2\frac{dy}{dx} + 3x^2y^3 = 0$

$$\frac{dy}{dx} = \frac{-\cancel{3x^2}y^{\cancel{3}}}{x^{\cancel{3}}\cancel{3y^2}} = \frac{-y}{x}$$

$$\frac{d^2y}{dx^2} = \frac{x \cdot -\frac{dy}{dx} - [-y \cdot 1]}{x^2}$$

$$= \frac{y + y}{x^2} = \frac{2y}{x^2}$$

17. $y + \sin y = x$

$$y' + \cos y \, y' = 1$$

$$y' = \frac{1}{1+\cos y} = (1+\cos y)^{-1}$$

$$y'' = -(1+\cos y)^{-2}(-\sin y)(y')$$

$$y'' = \frac{\sin y \, y'}{(1+\cos y)^2} = \frac{\sin y\left(\frac{1}{1+\cos y}\right)}{(1+\cos y)^2}$$

$$= \frac{\sin y}{(1+\cos y)^3}$$

18. $x\cos y = y$

$$-x\sin y \, y' + \cos y = y'$$

$$\cos y = y' + x\sin y \, y'$$

$$y' = \frac{\cos y}{1+x\sin y}$$

$$y'' = \frac{(1+x\sin y)(-\sin y \, y') - \cos y(x\cos y \, y' + \sin y)}{(1+x\sin y)^2}$$

$$y'' = \frac{(1+x\sin y)\left(-\frac{\sin y\cos y}{1+x\sin y}\right) - \cos y\left(x\cos y\left(\frac{\cos y}{1+x\sin y}\right)+\sin y\right)}{(1+x\sin y)^2}$$

(24) $\frac{dy}{dx} = \frac{2y-x^2}{y^2-2x}$ at $(3,3)$ $M_{tan} = \frac{6-9}{9-6} = -1$

$Y-3 = -1(X-3)$

23. $y^2(y^2-4) = x^2(x^2-5)$ $(0,-2)$

$y^4 - 4y^2 = x^4 - 5x^2$

$4y^3y' - 8yy' = 4x^3 - 10x$

$y' = \frac{4x^3-10x}{4y^3-8y} = \frac{2x^3-5x}{2y^3-4y}$

at $(0,-2)$ $M_{tan} = \frac{10}{8} = \frac{5}{4}$ $y+2 = \frac{5}{4}x$

22. $x^2+2xy-y^2+x=2$ $(1,2)$

$2x+2xy'+y(2)-2yy'+1=0$

$y' = \frac{-2x-1-2y}{2x-2y} = \frac{2x+2y+1}{2y-2x}$

at $(1,2)$

$M_{tan} = \frac{7}{2}$

$Y-2 = \frac{7}{2}(X-1)$

21. $x^2+xy+y^2=3$ at $(1,1)$

$2x+xy'+y+2yy'=0$

$y' = \frac{-2x-y}{x+2y}$

M_{tan} at $(1,1)$

$M = \frac{-3}{3} = -1$

$Y-1 = -1(X-1)$

26. if (1,1) on curve then

$$1 + a = b$$
$$1 + 1/4 = b \quad \boxed{b = 5/4}$$

25. $x^3 + y^3 = 6xy$ at $(3,3)$ HTL

$$3x^2 + 3y^2 \frac{dy}{dx} = 6x \frac{dy}{dx} + y(6)$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x} = \frac{2y - x^2}{y^2 - 2x}$$

HTL $2y - x^2 = 0$ $\quad y = \frac{x^2}{2}$ and on curve so

$$x^3 + \left(\frac{x^2}{2}\right)^3 = 6x\left(x^2/2\right) \Rightarrow x^3 + \frac{x^6}{8} = 3x^3$$

$$0 = 2x^3 - \frac{x^6}{8} = x^3\left(2 - \frac{x^3}{8}\right) = 0$$

$x = 0$ or $16 = x^3$ $\quad x = \sqrt[3]{16}$

$\cancel{(0,0)}$ or $\left(\sqrt[3]{16}, \frac{\sqrt[3]{16^2}}{2}\right)$

✓ $x = \frac{y^2}{2}$ $\quad \frac{y^6}{8} + y^3 = 3y^2 \cdot y$

$$\frac{y^6}{8} + y^3 = 3y^3$$

cusp at $(0,0)$

$$0 = 2y^3 - \frac{y^6}{8} = y^3\left(2 - \frac{y^3}{8}\right)$$

$y = 0$ or $y = \sqrt[3]{16}$ $\quad \cancel{(0,0)}$ or ...

27. (x_1, y_1) on curve

$$m_{tan} = \frac{y_1}{x_1} \quad \text{and} \quad m_{tan} = \frac{2-x_1}{y_1}$$

$$\frac{y_1}{x_1} = \frac{2-x_1}{y_1} \Rightarrow y_1^2 = x_1(2-x_1)$$

$$y_1^2 = 2x_1 - x_1^2$$

$$x_1^2 - 4x_1 + y_1^2 + 3 = 0$$

$$x_1^2 - 4x_1 + 2x_1 - x_1^2 + 3 = 0 \Rightarrow -2x_1 = -3$$

$$x_1 = 3/2$$

$$y_1^2 = 3 - 9/4 \quad y_1^2 = 3/4 \quad y_1 = \frac{\pm\sqrt{3}}{2}$$

$(3/2, \sqrt{3}/2)$ $(3/2, -\sqrt{3}/2)$

$$m_{tan\,(3/2,\sqrt{3}/2)} = \frac{2-3/2}{\sqrt{3}/2}$$

$$y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x - 3/2)$$

or $y = 1/\sqrt{3}\,x$

$$m_{tan} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$y + \frac{\sqrt{3}}{2} = \frac{-1}{\sqrt{3}}(x - 3/2)$$

or $y = -1/\sqrt{3}\,x$

$$m_{tan\,(3/2,-\sqrt{3}/2)} = \frac{2-3/2}{-\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

26. $x^2y + ay^2 = b$ $(1,1)$ on graph, tan line at $(1,1)$

$$x^2\frac{dy}{dx} + y \cdot 2x + 2ay\frac{dy}{dx} = 0$$

$4x + 3y = 7$

$$\frac{dy}{dx} = \frac{-2xy}{x^2 + 2ay}$$

$$m_{tan} = -\frac{4}{3}$$

at $(1,1)$ $m = \frac{-2}{1+2a}$

$$\frac{-4}{3} = \frac{-2}{1+2a}$$

$$-4 - 8a = -6$$

$$-8a = -2$$

$\boxed{a = 1/4}$

30. HTL ONLY

29. $x^4 + y^4 = 16$

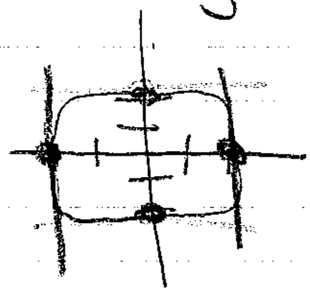
$4x^3 + 4y^3\, dy/dx = 0$

$dy/dx = \dfrac{-x^3}{y^3}$

VTL $y = 0$, $x^4 = 16$, $x = \pm 2$ — (2,0) (-2,0)

HTL $x = 0$, $y^4 = 16$, $y = \pm 2$ — (0,2) (0,-2)

LOOK at PIC



$\dfrac{dy}{dx} = \dfrac{dy}{dt} \cdot \dfrac{dt}{dx}$

28. $2y^3 t + t^3 y = 1$

$$\boxed{\begin{gathered} t = f(x) \\ y = f(t) \\ \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} \end{gathered}}$$

$2y^3 + t \cdot 6y^2 \frac{dy}{dt} + t^3 \frac{dy}{dt} + y \cdot 3t^2 = 0$

$dy/dt = \dfrac{-3yt^2 - 2y^3}{6y^2 t + t^3}$

$dy/dx = \left(\dfrac{3yt^2 - 2y^3}{6y^2 t + t^3}\right)\left(\dfrac{1}{\cos t}\right)$

27. $x^2 - 4x + y^2 + 3 = 0$ 2 tan lines thr (0,0)

$2x - 4 + 2y\, dy/dx = 0$

$\dfrac{dy}{dx} = \dfrac{4 - 2x}{2y} = \dfrac{2 - x}{y}$

need to represent a point on curve

$x^2 - 4x + 4 + y^2 = -3 + 4$

$(x-2)^2 + y^2 = 1$

30. VTL HTL for $2(x^2+y^2)^2 = 25(x^2-y^2)$

$$2(x^4+2x^2y^2+y^4) = 25(x^2-y^2)$$

$$2x^4+4x^2y^2+2y^4 = 25x^2-25y^2$$

$$8x^3+4x^2\cdot 2y\,dy/dx + y^2(8x) + 8y^3\,dy/dx = 50x - 50y\,dy/dx$$

$$\frac{dy}{dx} = \frac{50x-8x^3-8xy^2}{8x^2y+8y^3+50y} = \frac{25x-4x^3-4xy^2}{4x^2y+4y^3+25y}$$

$$HTL \Rightarrow \quad 25x-4x^3-4xy^2 = 0$$

$$y^2 = \frac{-25x+4x^3}{-4x} = \frac{25}{4}-x^2$$

also must be a point on the curve so

$$2(x^2+25/4-x^2)^2 = 25(x^2-(25/4-x^2))$$

$$2(25/4)^2 = 25(2x^2-25/4)$$

$$\frac{625}{8} = 25(2x^2-25/4)$$

$$\frac{25}{8} = 2x^2-25/4$$

$$25/16 = x^2-25/8$$

$$25/16+25/8 = x^2$$

$$75/16 = x^2$$

$$x = \pm\frac{\sqrt{75}}{4} = \pm\frac{5\sqrt{3}}{4}$$

$$\left(\frac{5\sqrt{3}}{4},\ \ \right)\ \left(-\frac{5\sqrt{3}}{4},\ \ \right)$$

Find x coordinates only