

## Trigonometric Identities

## Precalculus

Ex: Use Pythagorean identities to find all six functions.

$$\csc \theta = \frac{-5}{2} \quad \cos > 0$$

Ex: Simplify a)  $\cot \theta \sin \theta = \cos \theta$

Ex: Simplify  $\frac{\sin^2 \theta}{\sec^2 \theta - 1}$

Ex: Factor a)  $\sec^2 \theta - 1$

Ex: Factor  $4\sin^2 \theta + \sin \theta - 3$

Ex: Simplify  $\sin \theta + \cot \theta \cos \theta$

Ex: Verify  $\frac{\cot \theta}{\sec \theta} = \csc \theta - \sin \theta$

Ex: Simplify:  $(\sec x - \tan x)(1 + \sin x)$

Ex: Simplify:  $\frac{\csc \theta - \sin \theta}{\cot \theta}$

Ex: Simplify:  $\frac{\sin^2 \theta - \cos^2 \theta + 1}{2\sin \theta \cos \theta}$

Ex: Verify:  $\sin \theta + \cot \theta \cos \theta = \csc \theta$

Ex: Verify  $\frac{\sin \theta}{1 + \cos \theta} + \frac{\cos \theta}{\sin \theta} = \csc \theta$

Ex: Verify  $\tan x + \cot x = \csc x \sec x$

Ex: Show proof of  $\tan^2 x + 1 = \sec^2 x$

Ex: Verify:  $\frac{\sec^2 B - 1}{\sec^2 B} = \sin^2 B$

Ex: Verify:  $\frac{\cot^2 x}{1 + \csc x} = \frac{1 - \sin x}{\sin x}$

Ex:  $\frac{1 - \sin \theta}{1 + \sin \theta} = 2 \tan^2 \theta - 2 \sec \theta \tan \theta + 1$

Ex:  $\cos(x) * (\tan^2 x + 1) = \sec x$

Ex:  $\frac{\cot^2 x}{1 + \cot^2 x} = 1 - \sin^2 x$

$$\text{Ex: } \sec x + \tan(x) = \frac{\cos(-x)}{1 - \sin x}$$

$$\text{Ex: } \frac{\cos\left(\frac{\pi}{2} - x\right)}{1 + \cos x} = \csc x - \cot x$$

$$\text{Ex: } \left( \sin(-x) - \sin\left(\frac{\pi}{2} - x\right) \right)^2 = 1 + 2 \sin x \cos x$$

### **Solving Trig Equations:**

We will use standard algebraic techniques such as collecting like terms and factoring.

$$\text{Ex: Solve } 2\sin x - 1 = 0$$

$$\text{Ex: Solve } \cos x + \sqrt{2} = -\cos x$$

$$\text{Ex: Solve } 3 \tan^2 x - 1 = 0$$

$$\text{Ex: Solve } \cot x \cos^2 x = 2 \cot x$$

$$\text{Ex: Solve } 2\sin^2 x - \sin x - 1 = 0 \text{ in interval } [0, 2\pi]$$

$$\text{Ex: Use Quadratic Formula to solve } \sec^2 x - 3\sec x - 2 = 0$$

**Rewriting equations with single function:**

Ex:  $2\sin^2x + 3\cosx - 3 = 0$

Ex: Solve  $\cosx + 1 = \sinx$

Ex: Solve  $2\cos(3t) - 1 = 0$

Ex: Solve  $3\cot\left(\frac{x}{2}\right) + 3 = 0$

Ex:  $\sec^2x - 2\tanx = 4$

Ex: Solve  $2\sin(4x) - 1 = 0$

**Extra examples-**

1)  $3\csc^2\theta + 2\csc\theta = 1$

2)  $8\cos^2\theta + 3\cos\theta = 0$

3)  $3\sin\theta + \frac{1}{\sin\theta} = 4$

4)  $\sin\theta - 2\csc\theta + 1 = 0$

5)  $4\cos^2\theta + 4\sin\theta = 5$

6)  $6 - \tan\theta = 2\sec^2\theta$

7)  $\tan\theta = \frac{1}{\csc\theta}$

8)  $\cot^2\theta = \csc\theta + 5$

\*Using arc trig functions

Ex: Solve  $\csc^2 x - 2\cot x = 4$

Ex: : Solve  $\csc^2 x + .5\cot x = 5$

**Sum & Difference Formulas:**

$$\sin (A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\sin (A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

$$\cos (A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos (A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

Ex: Find exact value of  $\sin 105^\circ$

Ex: Find exact value of  $\cos \frac{5\pi}{12}$

Ex:  $\cos 48^\circ \cos 12^\circ - \sin 48^\circ \sin 12^\circ$

Ex: evaluate  $\sin (\arctan 2 + \arcsin x)$

Ex:  $\cos (15^\circ) = \cos (60 - 45)$

Ex:  $\sin 60 \cos 30 - \cos 60 \sin 30 = \sin (60 - 30)$

Ex: Verify the cofunction identity

$$\sin \left( \frac{\pi}{2} - x \right) = \cos (x)$$

### Deriving Reduction Formulas:

Simplify  $\tan(\theta + 2\pi) =$

Ex: Solve trig equation: Find all solutions

$$\cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4} - (\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4})$$

Show proof of  $(u - v) = \cos u \cos v + \sin u \sin v$

Why? We can now prove  
 $\cos(-B) = \cos(0 - B)$

Show  $\sin(-\theta) = -\sin \theta$

**Double Angle Formulas:**

Ex:  $\sin 2u = 2\sin u \cos u$

Ex:  $\sin^2 x + \cos 2x = 0$

Ex:  $2\cos x + \sin 2x = 0$

Ex: Find  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$   
if  $\sin \theta = -3/8$  and  $\pi < \theta < 3\pi/2$

1. draw reference angle
2. find all sides

**Half Angle Formulas:**

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$$

**Decide the sign based on what quadrant  $\frac{\theta}{2}$  is in**Find exact value of  $\cos 105^\circ$ Find exact value of  $\tan 165^\circ$ Find exact value of  $\sin -75^\circ$ Find exact value of  $\cos 157.5^\circ$ **Derive Triple Angle:**

$$\cos 3x = \cos (2x + x)$$

What is  $\tan 3x = ?$